Optical phase conjugation in difference-frequency generation

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We present a theoretical treatment of difference-frequency generation in conditions of phase mismatch for seed and generated fields that propagate at different angles to the pump. The analytical solution in the case of nondepleted plane-wave pump and experiments performed with seed fields strongly phase and amplitude modulated show that the pump-field wave fronts behave as efficient phase-conjugating mirrors.

1. INTRODUCTION

In the nonlinear optics literature, optical phase conjugation (OPC), a process whose earliest experimental realization was by stimulated Brillouin scattering, is more often associated with four-wave mixing than three-wave mixing interactions. As new photoactive materials were developed, OPC by four-wave mixing at frequency degeneracy became a more and more popular technique for dynamic (real-time) holography. Similarly the increased availability of highly nonlinear materials with fast responses stimulated many applications to ultrashort pulses. Among these we mention the correction of the temporal distortions caused by either dispersion at different orders or self-phase modulation of pulses during their propagation and amplification. However, since the 1970s it has been recognized that difference-frequency generation (DFG), a phenomenon that relies on nonlinear material, is a step forward in the use of DFG for image detection through scattering media. In fact complete cancellation of the distortion effects induced by a nonabsorbing scatterer is expected if the OPC wave is generated without loss or gain at an infinite phase-conjugate mirror.

2. GENERAL THEORY

The type I interaction depicted in Fig. 1 that occurs in a nonlinear medium among three noncollinearly propagating amplitude-modulated plane waves, \( \mathbf{E}_1(\mathbf{r}, t) \), \( \mathbf{E}_2(\mathbf{r}, t) \), and \( \mathbf{E}_3(\mathbf{r}, t) \), with wave vectors \( \mathbf{k}_{1,2,3} \) (unit vectors \( \hat{k}_{1,2,3} \)) can be described by Eqs. (2) in the paper by Bondani and Andreoni. In the parametric approximation, in which the amplitude of the field at the highest frequency (pump field \( \mathbf{E}_2 \) at \( \omega_2 \)) is virtually independent of \( \mathbf{r} \), we write the signal and idler fields, \( \mathbf{E}_1(\mathbf{r}, t) \) and \( \mathbf{E}_2(\mathbf{r}, t) \), with ordinary polarization parallel to \( \hat{\mathbf{x}} \), as

\[
\mathbf{E}_1(\mathbf{r}, t) = \frac{\hat{\mathbf{x}}}{2} \left( \frac{2 \eta_0 \hbar \omega_1}{n_1} \right)^{1/2} \times \{a_1(\mathbf{r}) \exp[-i(\mathbf{k}_1 \cdot \mathbf{r} - \omega_1 t)] + \text{c.c.}\},
\]

where \( a_1(\mathbf{r}) \) is the spatial distribution of the signal field, and \( \mathbf{k}_1 \) is the wave vector of the pump field. The spatial variation of the signal field is given by

\[
a_1(\mathbf{r}) = \int d\mathbf{r}' a_1(\mathbf{r}') \exp[-i(\mathbf{k}_1 \cdot (\mathbf{r} - \mathbf{r}'))],
\]

with \( a_1(\mathbf{r}) \) being a slowly varying function of \( \mathbf{r} \).

The nondepleted plane-wave pump field is provided by a frequency-doubled amplified Nd:YLF with a coherence length >3 m. We show that OPC in a low-gain regime allows full recovery of a beam at the fundamental frequency that passes through a diffusing glass plate. Our research, together with the availability of nonlinear crystals with noticeable cross-sectional sizes and wide acceptance angles, made of materials with high \( \chi^{(2)} \) nonlinearity, is a step forward in the use of DFG for image detection through scattering media. In fact complete cancellation of the distortion effects induced by a nonabsorbing scatterer is expected if the OPC wave is generated without loss or gain at an infinite phase-conjugate mirror.
\[ E_2(r, t) = \frac{\hbar}{2n_2} \left( \frac{2\eta_0 \hbar \omega_2}{n_2} \right)^{1/2} \times \{ a_2(r) \exp[-i(k_2 \cdot r - \omega_2 t)] + c.c. \}, \]

where \( \omega_1 + \omega_2 = \omega_3 \) for energy conservation, \( \eta_0 \) is the vacuum impedance, and \( n_j \) are the refractive indices at the corresponding frequencies \( \omega_j \). Note that \( |a_j(r)|^2 \) has the dimension of a photon flux density. The equations that describe the interaction reduce to

\[ \hat{k}_1 \cdot \nabla a_1(r) = i g_{\text{eff}} a_0(0) a_2^*(r) \exp(-i\Delta k \cdot r), \quad (2.1) \]

\[ \hat{k}_2 \cdot \nabla a_2(r) = i g_{\text{eff}} a_0(0) a_1^*(r) \exp(-i\Delta k \cdot r), \quad (2.2) \]

where \( r \) is the position inside the crystal with respect to an origin located on the entrance face where the constant pump-field amplitude, \( a_0(0) = a_2(r = 0) \), is taken. As shown in Fig. 1, with the \( z \) axis perpendicular to the crystal entrance face, \( r = (0, y, z) \) is in the \((y, z)\) plane, which is also the principal plane. Thus the gradient in Eqs. (2.1) and (2.2) is \( \nabla = \hat{\mathbf{y}} \partial/\partial y + \hat{\mathbf{z}} \partial/\partial z \) with \( \hat{\mathbf{y}} \) and \( \hat{\mathbf{z}} \) as unit vectors along the axes. The fact that wave vectors \( \mathbf{k}_{1,2,3} \) are at different angles \( \theta_{1,2,3} \) shown in Fig. 2 with respect to the \( z \) axis gives rise, in general, to an error in the phase-matching (PM) condition given by \( \Delta k = k_3 - k_1 - k_2 \). Finally \( g_{\text{eff}} \) is the effective coupling coefficient that takes into account that the pump field enters the crystal as a transverse wave.\(^{10}\) For \( \beta\text{-BaB}_2\text{O}_4 \) in type I (BBO I) PM conditions, \( g_{\text{eff}} \) has the following expression:

\[ g_{\text{eff}} = (d_{22} \cos \alpha + d_{31} \sin \alpha) \left( \frac{2\hbar \omega_1 \omega_2 \omega_3 \eta_0}{n_1 n_2 n_3} \right)^{1/2}, \]

where \( \alpha \) is the angle between the optical axis and the pump wave vector \( \mathbf{k}_0 \) (see Fig. 1) and \( d \) represents the relevant coupling coefficients.\(^{13}\) It is worth noting that Eqs. (2) include the conservation law, which is our Manley–Rowe relation:

\[ \nabla |a_1(r)|^2 \cdot \hat{k}_1 = \nabla |a_2(r)|^2 \cdot \hat{k}_2, \]

which links the amplitude square magnitude of the amplified (AMP) signal field \( E_1(r, t) \) with that of the DFG field \( E_2(r, t) \).

The solutions to Eqs. (2) for boundary conditions \( a_1(0) \neq 0 \) and \( a_2(0) = 0 \) are

\[ a_1(r) = a_1(0) \left[ \cosh \left( \frac{Q}{2} \frac{\Delta k}{2} \cdot r \right) \right. \]

\[ + \left. \frac{i\Delta k}{Q} \sinh \left( \frac{Q}{2} \frac{\Delta k}{2} \cdot r \right) \right] \times \exp \left[ -i \left( \frac{\Delta k}{2} \cdot r \right) \right], \quad (5.1) \]

\[ a_2(r) = a_1^*(0) \frac{2i A_3}{(\Delta \hat{k} \cdot \hat{k}_2) Q} \sinh \left[ Q \left( \frac{\Delta k}{2} \cdot r \right) \right] \]

\[ \times \exp \left[ -i \left( \frac{\Delta k}{2} \cdot r \right) \right], \quad (5.2) \]

where

\[ Q = \left[ \frac{4|A_3|^2}{(\Delta \hat{k} \cdot \hat{k}_1)(\Delta \hat{k} \cdot \hat{k}_2)} - \Delta k^2 \right]^{1/2}. \]

with \( A_3 = g_{\text{eff}} a_3(0) \) and \( \Delta \hat{k} = \Delta k/\Delta k \). To demonstrate the space-dependent phases of the fields we rewrite them [see Eqs. (1)] in the following form:

\[ E_1(r, t) = \hat{x} \left( \frac{2\eta_0 \hbar \omega_1}{n_1} \right)^{1/2} |a_1(r)| \times \cos[\varphi_1(r) + \omega_1 t], \quad (7.1) \]

\[ E_2(r, t) = \hat{x} \left( \frac{2\eta_0 \hbar \omega_2}{n_2} \right)^{1/2} |a_2(r)| \times \cos[\varphi_2(r) + \omega_2 t], \quad (7.2) \]

so that for the phases from Eqs. (5) we get

\[ \varphi_1(r) = \varphi_1(0) - \mathbf{k}_1 \cdot \mathbf{r} - \frac{\Delta k}{2} \cdot \mathbf{r} \]

\[ + \tan^{-1} \left[ \frac{\Delta k}{Q} \sinh \left( \frac{Q}{2} \frac{\Delta k}{2} \cdot \mathbf{r} \right) \right], \quad (8.1) \]

\[ \varphi_2(r) = -\varphi_1(0) + \varphi_3(0) + \frac{\pi}{2} - \mathbf{k}_2 \cdot \mathbf{r} - \frac{\Delta k}{2} \cdot \mathbf{r} \quad (8.2) \]

whose sum is
where \( \varphi_3(\mathbf{r}) = \varphi_3(0) - \mathbf{k}_3 \cdot \mathbf{r} \) is the phase of \( \mathbf{E}_3(\mathbf{r}, t) \) and, for the magnitudes of the amplitudes,

\[
|a_1(\mathbf{r})| = \left| a_1(0) \right| \frac{Q^2 + 4|A_3|^2}{(\Delta k \cdot \mathbf{k}_1)(\Delta k \cdot \mathbf{k}_2)} \sinh^2 \left[ Q \left( \frac{\Delta k}{2} \cdot \mathbf{r} \right) \right]^{1/2},
\]

(9.1)

\[
|a_2(\mathbf{r})| = \left| a_1(0) \right| \frac{2|A_3|}{(\Delta k \cdot \mathbf{k}_2)} \sinh \left[ Q \left( \frac{\Delta k}{2} \cdot \mathbf{r} \right) \right].
\]

(9.2)

The solutions represent two waves, AMP and DFG, that amplify while they propagate in the medium for real \( Q \). After squaring Eqs. (9) and summing we obtain

\[
|a_1(\mathbf{r})|^2 = |a_1(0)|^2 + \frac{\Delta k \cdot \mathbf{k}_2}{\Delta k \cdot \mathbf{k}_1} |a_2(\mathbf{r})|^2
\]

(10)

in agreement with Eq. (6). In fact, by calculating \( \nabla|a_{1,2}(\mathbf{r})|^2 \) from Eqs. (9) we obtain

\[
\nabla|a_1(\mathbf{r})|^2 = \left| a_1(0) \right|^2 \frac{4|A_3|^2}{Q} \frac{\Delta k \cdot \mathbf{k}_1}{(\Delta k \cdot \mathbf{k}_1)(\Delta k \cdot \mathbf{k}_2)} \sinh \left[ Q \left( \frac{\Delta k}{2} \cdot \mathbf{r} \right) \right] \frac{\Delta k}{\Delta k \cdot \mathbf{k}_2}
\]

(11.1)

\[
\nabla|a_2(\mathbf{r})|^2 = \left| a_1(0) \right|^2 \frac{4|A_3|^2}{Q} \frac{\Delta k \cdot \mathbf{k}_2}{(\Delta k \cdot \mathbf{k}_2)^2} \sinh \left[ Q \left( \frac{\Delta k}{2} \cdot \mathbf{r} \right) \right] \frac{\Delta k}{\Delta k \cdot \mathbf{k}_1}
\]

(11.2)

To find the propagation directions of the constant-phase surfaces we calculated \( \nabla \varphi_1(\mathbf{r}) \) and \( \nabla \varphi_2(\mathbf{r}) \). From Eq. (8.1) with the help of Eq. (9.1) we obtained

\[
\nabla \varphi_1(\mathbf{r}) = -\mathbf{k}_1 - \frac{\Delta \mathbf{k}}{2} + \frac{\Delta \mathbf{k}}{2} \times \frac{|a_1(0)|^2}{|a_1(0)|^2}
\]

(12.1)

and conclude that the fields in Eqs. (7), whose space-dependent complex amplitudes are constant in magnitude on planes perpendicular to \( \Delta \mathbf{k} \) [see Eqs. (9) with constant \( \Delta \mathbf{k} \cdot \mathbf{r} \) values and Eqs. (11)], are such that field \( \mathbf{E}_1(\mathbf{r}, t) \) is always a plane wave with wave fronts that propagate along wave vector \( \mathbf{k}_2 + (\Delta \mathbf{k}/2) \) and that field \( \mathbf{E}_2(\mathbf{r}, t) \) also tends to become a plane wave with wave fronts that propagate along wave vector \( \mathbf{k}_3 + (\Delta \mathbf{k}/2) \) when the gain increases. Note that the sum of these wave vectors matches the pump-field wave vector \( \mathbf{k}_3 \) for any \( \Delta \mathbf{k} \).

Moreover, after calculation of the time-averaged Poynting vectors, that is (see Appendix A),

\[
S_1(\mathbf{r}) = \frac{\hbar \omega_1}{k_1} |a_1(\mathbf{r})|^2 \left[ \mathbf{k}_1 + \frac{\Delta \mathbf{k}}{2} - \frac{\Delta \mathbf{k}}{2} \times \frac{|a_1(0)|^2}{|a_1(\mathbf{r})|^2} \right],
\]

(13.1)

\[
S_2(\mathbf{r}) = \frac{\hbar \omega_2}{k_2} |a_2(\mathbf{r})|^2 \left[ \mathbf{k}_2 + \frac{\Delta \mathbf{k}}{2} \right],
\]

(13.2)

we find that for each field the power flows normally to the wave front although it grows maximally in a different direction [\( \mathbf{r} \equiv r \Delta \mathbf{k} \), see Eqs. (11)]. We finally note that, by inserting Eq. (10) into Eq. (13.1), this can be rewritten as

\[
S_1(\mathbf{r}) = \frac{\hbar \omega_1}{k_1} |a_1(0)|^2 \left[ \mathbf{k}_1 + \frac{\Delta \mathbf{k}}{2} \right],
\]

(14)

in which the former term is the contribution of the seed field at the crystal entrance and the latter is that due to seed amplification.

It is worth noting at this point that, as in Ref. 10, in deriving Eqs. (2) from the Helmholtz equations with sources, we chose to neglect the second derivatives of \( a_{1,2}(\mathbf{r}) \) with respect to \( y \) and \( z \). We must then check \textit{a posteriori} under which conditions the solutions in Eqs. (5) verify such a slowly varying envelope approximation. This is done in Appendix B where we show that such a condition is

\[
\Delta \mathbf{k} / 2 \ll k_1, k_2,
\]

(15)

provided that both \( \Delta \mathbf{k} \cdot \mathbf{k}_1 \) and \( \Delta \mathbf{k} \cdot \mathbf{k}_2 \) are sufficiently different from zero. In the solutions in Eqs. (5), because of boundary condition \( a_1(0) = 0 \), \( \Delta \mathbf{k} \) appears as a free parameter, with \( \mathbf{k}_2 \) univocally linked to \( \Delta \mathbf{k} \) as depicted in Fig. 2. This means that, given a pump field \( |A_3| \) and \( \mathbf{k}_3 \) assigned for any seed field that enters the crystal with certain \( a_1(0) \) and \( \mathbf{k}_1 \), the system of Eqs. (2.1) and (2.2) provides solutions associated with any orientation of \( \mathbf{k}_2 \) (angle \( \theta_2 \)) corresponding to a real \( Q \). Such an indetermination in \( \mathbf{k}_2 \) is obviously not encountered in PM conditions where \( \mathbf{k}_2 = \mathbf{k}_3 - \mathbf{k}_1 \). When we solved Eqs. (2) in PM, we found \( |a_2(\mathbf{r})| \) of the form

\[
|a_2(\mathbf{r})| = |a_1(0)| |\sinh \left[ \frac{|A_3|}{\frac{1}{2}} \sin \left( \frac{\theta_1 - \theta_2}{2} \right) \cos \left( \frac{\theta_1}{2} \right) \right]
\times \left( \sin \frac{\theta_1}{2} + \sin \frac{\theta_2}{2} \right),
\]

(16)

where, as shown in Fig. 2, \( (\theta_1 + \theta_2)/2 \) is the angle formed by the bisector of the \( \mathbf{k}_1 \)-to-\( \mathbf{k}_2 \) angle with the \( x \) axis and \( (\theta_1 - \theta_2)/2 \) is one-half of the \( \mathbf{k}_1 \)-to-\( \mathbf{k}_2 \) angle. If we write Eq. (9.2) for \( \Delta \mathbf{k} = 0 \),
3. EXPERIMENTAL RESULTS AND DISCUSSION

The experiments presented in this paper were performed at frequency degeneracy \((\omega_1 = \omega_2 = \omega)\). The intense plane wave to be used as the pump field \((\omega_3 = 2\omega)\) was provided by the frequency-doubled output of a high-coherence pulsed Nd:YLF laser (Green Star, Neva Technologies, Ltd., St. Petersburg, Russia). The residual fundamental pulse provided the seed field. This Q-switched source emits 25-ns pulses at the fundamental (1053 nm) and includes a double-pass Nd:glass amplifier. A phase-conjugating mirror based on Brillouin scattering reflects the pulse back to the amplifier between the two passes. After frequency doubling in a KTP crystal the coherence length is >3 m and the energy is well above 1 J. Both fundamental and frequency-doubled beams have flat-top profiles with 9-mm diameter. The polarization of the output at 527 nm (pump field at \(\omega_3\)) was in the horizontal plane and that of the 1053 nm (seed field at \(\omega_1\)) was in the vertical plane, so that the \((y, z)\) plane in Fig. 1 was horizontal. By using the setup in Fig. 4, we performed two distinct series of experiments in BBO. The first, which we designate as exp-I in what follows, was based on a type I interaction and used the configuration depicted in Fig. 4(a). The second, exp-II, was based on a type II interaction and implied the superimposition of a strong phase disturbance on the signal through the scheme sketched in Fig. 4(b). In exp-I we used a BBO crystal (Fujian Castech Crystals, Inc., Fuzhou, China) with a 5 mm \(	imes\) 5 mm cross section and thickness of \(d = 2\) mm, cut for phase-matched collinear second-harmonic generation at normal incidence by use of Nd:YAG [BBO I in Fig. 4(a)]. In exp-II the BBO crystal (10 mm \(	imes\) 10 mm cross section, 3-mm thickness, same manufacturer) was cut for collinear Nd:YAG second-harmonic generation at normal incidence in a type II PM condition [BBO II in Fig. 4(b)]. The space filter on the

![Diagram](image-url)
beam at the fundamental frequency. To detect fluence lenses L2 and L3 and pinhole PH1 produced a similar greater than any relevant distance for our experiments. PH2 shapes the laser output into a Gaussian beam of aperture 527-nm beam consisting of lenses L4 and L5 plus pinhole PH1 with suitable color filters in front to reject the green pump-field intensities, the pump beam at $2\omega$ measured the ratio of the energy of the DFG pulse to that of $A_1(0)$, whereas we measured integral photon fluxes in the spot formed by the DFG field on the CCD sensor when many $\hat{k}_1$ unit vectors symmetrically spread in angle about the single $\hat{k}_1$ of the pump impinged onto the crystal. Moreover we normalized these integral photon fluxes to those of the seed field entering the crystal. Such normalized integral fluxes were found to increase linearly with $|A_3|^2$ as shown in Fig. 5. Since we set the CCD sensor perpendicular to $\hat{k}_3$, we use Eq. (A1.4) with $\hat{A} = \hat{k}_3 = \hat{z}$ and Eq. (13.2), together with Eq. (21), with $\mathbf{r}$ such that $\Delta \hat{k} \cdot \mathbf{r} = \hat{z} \cdot \mathbf{r} = d$ is constant to calculate the photon flux density $\Phi_2(\mathbf{r})$ generated in response to a single spatial Fourier component (i.e., single $\hat{k}_1$ at angle $\vartheta_1$ to $\hat{k}_3$) of seed field 1. At degeneracy ($k_1 = k_2 = k_\omega$), we find

$$|\Phi_2(\mathbf{r})|^2 = |A_1(0)|^2|A_3|^2 \frac{d^2}{\cos^2 \vartheta_1} \left\{ \frac{\sin \left[ \frac{Q}{2} (\hat{z} \cdot \mathbf{r}) \right]}{Q \left[ \frac{1}{2} (\hat{z} \cdot \mathbf{r}) \right]} \right\}^2 \times \left( \frac{\hat{k}_2 + \frac{\Delta k}{2k_\omega} \hat{z}}{\hat{k}_2} \right) \cdot \hat{z} \equiv |A_1(0)|^2|A_3|^2 \frac{d^2}{\cos^2 \vartheta_1} \left\{ \frac{\sin \left[ \frac{Q}{2} (\hat{z} \cdot \mathbf{r}) \right]}{Q \left[ \frac{1}{2} (\hat{z} \cdot \mathbf{r}) \right]} \right\}^2 \hat{k}_1 \cdot \hat{z},$$

(22)

since $\Delta k/2 \ll k_\omega$ and $\hat{k}_2 \cdot \hat{z} = \hat{k}_2 \cdot \hat{z}$ (namely, $\vartheta_2 = -\vartheta_1$) according to Eqs. (15) and (18), respectively. By recognizing that $|A_1(0)|^2 \hat{k}_1 \cdot \hat{z} = \Phi_1(0)$ is the photon flux density associated with our single plane-wave component at angle $\vartheta_1$, we can recast Eq. (22) into the following form:

![Fig. 5. Photon flux in the DFG field at BBO I output $\Phi_2(\mathbf{r})$, normalized to seeded photon flux $\Phi_1(0)$, as a function of pump photon flux density $|A_3|^2$ (lower scale). Upper scale, pump intensity: least-squares fit of experimental points; inset, typical fluence map of a DFG spot obtained with the highest pump intensity.](image-url)
\[ \Phi_3(r) \]
\[ = \frac{|A_3|^2d^2}{\cos^2 \vartheta_1 \left( \frac{4|A_3|^2}{\cos^2 \vartheta_1} - 4k^2_0(1 - \cos \vartheta_1)^2 \right)^{1/2} \frac{d}{2}} \]
\[ \sinh^2 \left( \sqrt{\frac{4|A_3|^2}{\cos^2 \vartheta_1} - 4k^2_0(1 - \cos \vartheta_1)^2} \frac{d}{2} \right) \]
\[ = \frac{|A_3|^2d^2}{\cos^2 \vartheta_1 \left( \frac{4|A_3|^2}{\cos^2 \vartheta_1} - 4k^2_0(1 - \cos \vartheta_1)^2 \right)^{1/2} \frac{d}{2}} \]
\[ \frac{\sinh^2 \left( \sqrt{\frac{4|A_3|^2}{\cos^2 \vartheta_1} - 4k^2_0(1 - \cos \vartheta_1)^2} \frac{d}{2} \right)} \]
\[ \text{(23)} \]

where we made use of Eq. (6) with \( k_1 = k_2 = k_\omega \). The maximum deviation of the fraction on the right-hand side of Eq. (23) from the value 1 occurs for \( \vartheta_1 = 0 \). For the \( |A_3| \) value corresponding to the highest pump intensity used in the experiment (203 MW/cm\(^2\)), the fraction value is \( \sim 1.06 \). It is then reasonably correct to fit the experimental points in Fig. 5 by a straight line and interpret its slope as \( d^2 \). Our fitting gives a \( d \) value of 2.13 mm, to be compared with the nominal thickness of 2 mm. The fact that the fitting line in Fig. 5 is slightly shifted with respect to the origin of the plot might be due to (small) offsets either in the measurements of the pump-pulse energies from which we calculated the \( |A_3| \) values (abscissa in Fig. 5) or in those performed with CCD and photodiodes on the DFG and seed beams that were necessary to construct the ratios \( \Phi_3(r)/\Phi_1(0) \) that were plotted on the vertical scale. Lefort and Barthelemy conducted a similar experiment\( ^9 \) and, although they worked with a 7-mm-thick KTP crystal in type II PM conditions, they obtained a conversion efficiency that agrees with Eq. (23).

As to the phase relations among the three interacting fields we note that the space-dependent phase of pump, DFG, and AMP fields are linked to each other by Eq. (8.3), which is well approximated by

\[ \varphi_1(r) + \varphi_2(r) = \varphi_3(r) + \pi/2, \]
\[ \text{(24)} \]

when \( \tan^{-1}[(\Delta k/Q)\tanh(Q(\Delta k/2)\cdot r)/2]] \ll \pi/2 \). In this condition, if point \( r \) inside the nonlinear crystal is chosen on plane II of Fig. 6, perpendicular to \( \hat{k}_3 \), we would find spatial phase reversal between fields 1 and 2 on II. This property leads to a number of effects that are observable outside the crystal. We note, for example, that the DFG field (field 2), after propagation from \( r \in \Pi \) to \( r' \in \Pi \) in the absence of interaction, would achieve a phase value \( \varphi_2(r') = \varphi_2(r) - k_2 \cdot (r' - r) \). By substituting Eq. (8.2), it is easy to show that, if \( r' \) coincides with the origin (see Figs. 4 and 1), then \( \varphi_2(0) = -\varphi_1(0) + \varphi_3(0) + \pi/2 - (\Delta k/2) \cdot r \). Since within the limit \( Q(\Delta k/2) \cdot r \rightarrow 0 \), that is, in a regime of linear amplification [see Eqs. (5)], the condition for the validity of Eq. (24) amounts to a request for \( \tan^{-1}[(\Delta k/2) \cdot r] \ll \pi/2 \). Under this condition, if the DFG field 2 is backpropagated by means of a plane mirror, it will reach the origin with a phase value \( \varphi_2(0) = -\varphi_1(0) + \text{const} \). Note that the above condition for frequency degeneracy and for \( \hat{k}_3 = \hat{z} \) becomes \( \tan^{-1}[(\Delta k/2) \cdot (\hat{z} \cdot r)] = \tan^{-1}[(\Delta k/2) \cdot (1 - \cos \vartheta_1) \cdot d] \ll \pi/2 \), which is easily fulfilled. For the experiment presented below (\( d = 3 \) mm, \( \vartheta_1 = 0.25 \) deg), we calculated a maximum value of \( \sim 0.087\pi \).

Although by operating with a different laser source we already demonstrated that the simple fact in Eq. (5.2), \( \alpha(r) \propto \alpha^2 \varphi_1(0) \), allows us to obtain DFG fields that are capable of reconstructing holographic replicas of the wave fronts of the seed field with high spatial/angular resolution.\( ^{10,11,14} \) The experiment (exp-II) we present here is aimed at demonstrating that the phase reversal expected from Eq. (24) is so accurate as to allow corrections of remarkable phase disturbances deliberately introduced in the seed field that enters the crystal. We substituted pinhole PH1 (see Fig. 4) with a micromask that, in the absence of interaction, produced a beam at \( \omega \) with the intensity distributions displayed in Figs. 7(a) and 7(b). The two maps were detected by the CCD at distances of 51 and 291 cm from the nonlinear crystal, respectively (see below for details). With the beams at \( \omega \) and at \( 2\omega \) aligned as in the previous experiment, in exp-II we passed such a seed beam through a diffusing glass plate, DIFF in Fig. 4(b), which was located 55 mm in front of the nonlinear crystal. The DIFF plate introduced a beam divergence of 0.5 deg (full angle at \( 1/e^2 \)) and produced the intensity distribution shown in Fig. 8. The ex-
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4. CONCLUSIONS

By working with pump intensities below the damage threshold of BBO for long pulses we obtained OPC idler fields with 10% photon conversion efficiency per millimeter of BBO thickness (see Fig. 5). The high quality of the phase conjugation should allow one to use the technique to correct severe phase distortions imprinted on fields that carry image information by passage through scattering media or by transmission through fiber optics and optical components that cause aberrations. The feasibility of such an OPC with pulses of tens of nanosecond duration is relevant for application to the correction of phase aberrations brought about by laser amplification. 8

Our theory shows that the OPC quality can be preserved on increasing Δk if a pump beam of suitable intensity were available. In fact, the error in phase conjugation [see the last term in Eq. (8.3)], which in the linear-amplification regime of our experiments reduces to tan⁻¹(Δk/d2), becomes at most tan⁻¹(Δk/Q) in the high-gain regime. Also this expression can represent a negligible phase error for sufficiently high pump intensities. Thus suitably short laser pulses could be used to obtain space–phase conjugation in DFG processes in which the seed-field spectrum may have a wider angular spread than that in our experiments. Our theory should still apply and could also be fully tested: properties that concern the DFG wave-vector propagation [see Eq. (18) and Fig. 3], the magnitude and propagation of the Poynting vectors [see Eqs. (13) and (14)], and the limit of validity in Eq. (20) can be investigated only at high pump intensity. Finally we note that, although we are interested in investigating space–phase conjugation by DFG with shorter pulses for the above reasons, others have recently proposed temporal–phase conjugation by DFG in BBO as an efficient method for compensating the effects of dispersion and nonlinearity on ultrashort pulses that propagate through optical fibers. 15 This is an application that could greatly benefit from the use of a χ(2) instead of a χ(3) nonlinear interaction, 5 with the further advantage that χ(2) media with negligible group-velocity dispersion are available and that they can often be operated in conditions of exact group-velocity matching. 16,17 From this standpoint, BBO is an ideal material. 17

APPENDIX A

The time-averaged Poynting vectors, \( \mathbf{S}_1 \) and \( \mathbf{S}_2 \), are given by

\[
\mathbf{S}_{1,2}(r) = \text{Re} \left[ \frac{1}{2} \mathbf{E}_{1,2}(r) \times \mathbf{H}_{1,2}^* \right],
\]

\[(A1)\]

traordinarily polarized DFG field that emerged from BBO II was reflected back to the DIFF plate by a plane mirror (M4) located behind the BBO II crystal as near as possible (2 mm) to its exit face and perpendicular to the pump beam. Obviously the phase distortions operated by the difusing plate on the reflected DFG field are identical to those operated on the beam at \( \omega \) used as the seed for the nonlinear interaction. 12 The reflected DFG field, behind the DIFF plate, was deflected by a beam splitter [BS2 in Fig. 4(b)] toward the CCD camera. To prevent the DFG light transmitted by BS2 from reentering the laser amplifier, we chose PM II and inserted a plastic polarizer (P in the figure) to stop it. The cross-sectional intensity distribution of the reflected DFG field after backpropagation through the diffusing plate detected at distances of 51 and 291 cm from BBO II are displayed in Figs. 9(a) and 9(b), respectively. The corresponding measurements performed on the seed beam reflected by mirror M4 after removal of the diffusing plate are those in Figs. 7(a) and 7(b). We note that the DFG beam in Figs. 9(a) and 9(b) is well collimated and has a profile similar in shape and size to that of the nondifused seed beam in Figs. 7(a) and 7(b). Note that after propagation over meters, the diffused seed beam would exhibit a speckle pattern similar to that in Fig. 8 but enlarged to many-centimeter diameters.

The accurate restoration produced by OPC of the non-difused beam profile shows that, at the crystal output, the DFG field has phase values that are point by point equal to the opposite of those of the AMP seed beam [see \( \varphi_2(P) = -\varphi_1(P) \) in Fig. 6]. In fact it is only in this case that backpropagation through the diffusing plate allows the DFG field to recover the phase disturbances of the seed-field wave front that the nonlinear interaction has oppositely copied onto the DFG wave front.

Fig. 8. Speckle pattern of the seed field measured at a distance of 6 cm from the diffusing glass plate.

Fig. 9. Fluence maps of the DFG field after reflection by mirror M4 in Fig. 4(c) and backpropagation through DIFF. The CCD camera is at distances of (a) 51 and (b) 291 cm from BBO II.
where $E_{1,2}(r)$ is the space-dependent part of fields $E_{1,2}(r, t)$ and $H^*_{1,2}(r)$ is the complex conjugate of $H_{1,2}(r)$, with
\[ \nabla \times E_{1,2}(r) = -i \omega_{1,2} \mu_0 H_{1,2}(r). \] (A2)

Since, according to Eqs. (2) and (7)
\[ E_{1,2}(r) = \frac{2 \eta_0 h \omega_{1,2}}{n_{1,2}} \left| a_{1,2}(r) \right| \exp\left[ i \varphi_{1,2}(r) \right], \] (A3)

if $H_{1,2}(r)$ is calculated from Eq. (A2) and substituted into Eq. (A1), we obtain the $S_1$ and $S_2$ vectors reported in Eqs. (13.1) and (13.2), respectively. Note that, according to Eq. (A1), the intensities, in watts per square meter, on planes perpendicular to the directions of $S_1$ and $S_2$ are given by $|S_{1,2}|^2$ whereas the photon flux densities $\Phi_{1,2}(r)$ through an elementary area $dA$ located at $r$ with arbitrary orientation are given by
\[ \Phi_{1,2}(r) = S_{1,2}(r) \cdot \hat{A}/h \omega_{1,2}, \] (A4)

where $\hat{A}$ is the unit vector perpendicular to area $dA$.

APPENDIX B

The complete equations would be
\[ \frac{1}{2i k_1} \nabla^2 a_{1}(r) + \hat{k}_1 \cdot \nabla a_{1}(r) = i g_{45} a_{3}(0) a_{2}^*(r) \exp(-i \Delta k \cdot r), \] (B1)

\[ \frac{1}{2i k_2} \nabla^2 a_{2}(r) + \hat{k}_2 \cdot \nabla a_{2}(r) = i g_{45} a_{3}(0) a_{1}^*(r) \exp(-i \Delta k \cdot r). \] (B2)

A more safe criterion for accepting our $a_{1,2}(r)$ solutions is the request
\[ \left| \frac{1}{2i k_{1,2}} \nabla^2 a_{1,2}(r) \right| \ll |\hat{k}_{1,2} \cdot \nabla a_{1,2}(r)|, \] (B3)

which, after substitution of Eqs. (5) and squaring, corresponds to
\[ (Q^2 + \Delta k^2) \sinh^2 \left[ Q \left( \frac{\Delta k}{2} \cdot r \right) \right] + 16 Q^2 \left( \frac{\Delta k}{2} \cdot \Delta k \right)^2 \ll 16(\Delta \hat{k} \cdot k_{1,2})^2 (Q^2 + \Delta k^2) \sinh^2 \left[ Q \left( \frac{\Delta k}{2} \cdot r \right) \right] \]
\[ + 16 Q^2 (\Delta \hat{k} \cdot k_{1,2})^2. \] (B4)

This ordering relation is certainly true if we simultaneously require that
\[ (Q^2 + \Delta k^2) \ll 16(\Delta \hat{k} \cdot k_{1,2})^2, \]
\[ \left( \frac{\Delta k}{2} \cdot \Delta k \right)^2 \ll (\Delta \hat{k} \cdot k_{1,2})^2. \] (B5)

If we take $a_{1,2}(r)$ solutions that correspond to $\Delta \hat{k}/2 \ll k_{1,2}$ we fulfill the latter relation provided that $\Delta \hat{k}$ is at angles to $k_1$ and $k_2$, sizably different from $\pi/2$. As to the former we observe that, according to Eq. (6),
\[ (Q^2 + \Delta k^2) = 4|A_3|^2/(|\Delta \hat{k} \cdot k_1| |\Delta \hat{k} \cdot k_2|). \] Considering the damage threshold intensities of nonlinear crystals the former inequality cannot be violated in any practical situation. We can safely conclude that the $a_{1,2}(r)$ solutions in Eqs. (5) that correspond to $\Delta \hat{k}/2$ values negligible with respect to $k_1$ and $k_2$ fulfill the slowly varying envelope approximation in which Eq. (B1) can be substituted by Eqs. (2),

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