Quantum Ratchets and Wave Packet Collapse in Dissipative Chaotic Systems

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Motivations and Outline

• Study the effect of quantum noise on open quantum chaotic systems

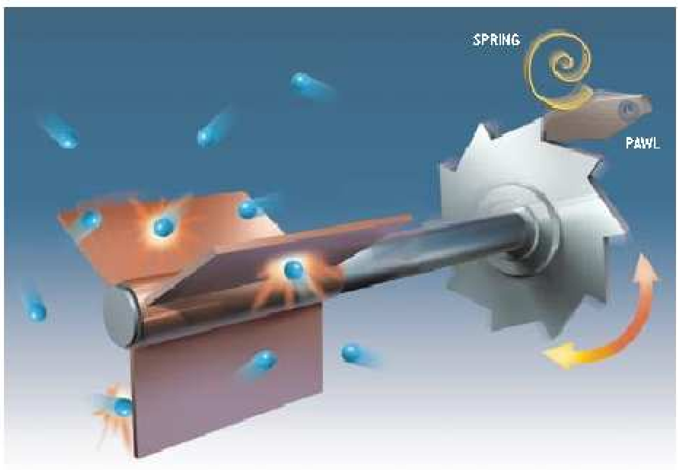
• Investigate the possibilities opened by optical lattices for the quantum simulation of complex dissipative systems

• A model for quantum directed transport in a periodic chaotic systems with dissipation, in presence of lattice asymmetry and unbiased driving
Possible experimental implementation with cold atoms in optical lattices

• Is it possible to recover classical-like chaotic dynamics (positive Lyapunov exponent) in a dissipative system?
Transition from wave packet collapse to explosion

The Feynman ratchet

Can useful work be extracted out of unbiased microscopic random fluctuations if all acting forces and temperatures gradients average out to zero?



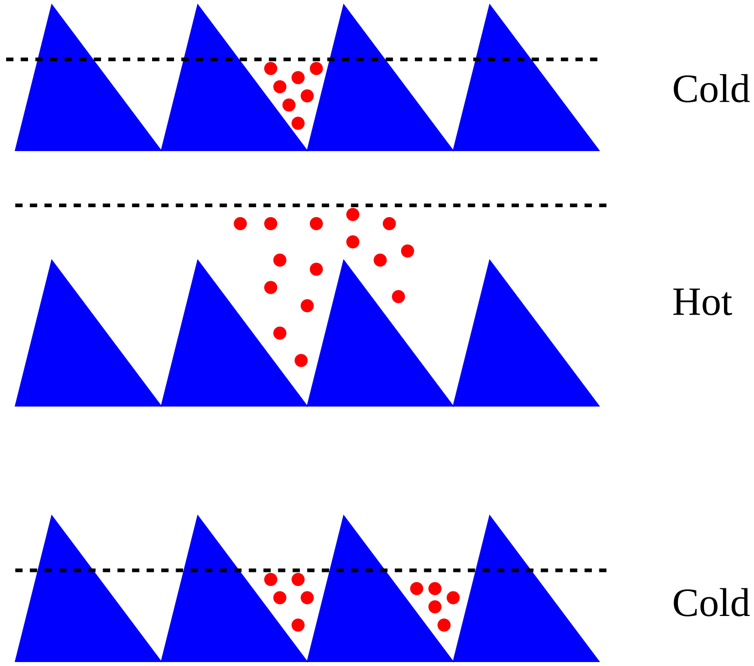
(taken from D.Astumian, Scientific American, July 2001)

Thermal equilibrium: the gas surrounding the paddles and the ratchet (plus the pawl) are at the same temperature

In spite of the built asymmetry no preferential direction of motion is possible. Otherwise, we could implement a perpetuum mobile, in contradiction with the second law of thermodynamics

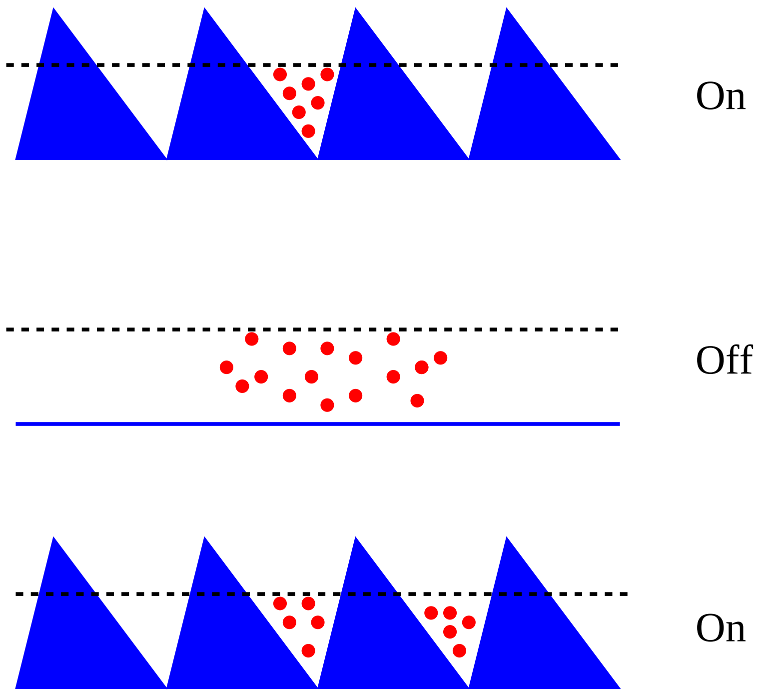
Brownian motors

To build a Brownian motor drive the system out of equilibrium

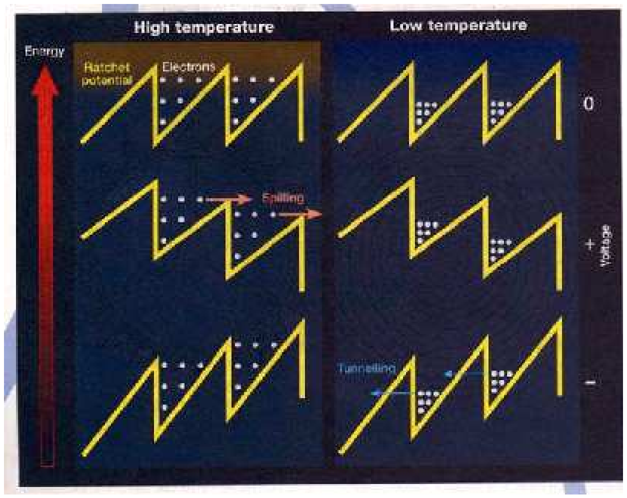


Working principle of a Brownian motor driven by temperature oscillation

Another model of Brownian motor: a pulsating (flashing) ratchet



Quantum ratchets



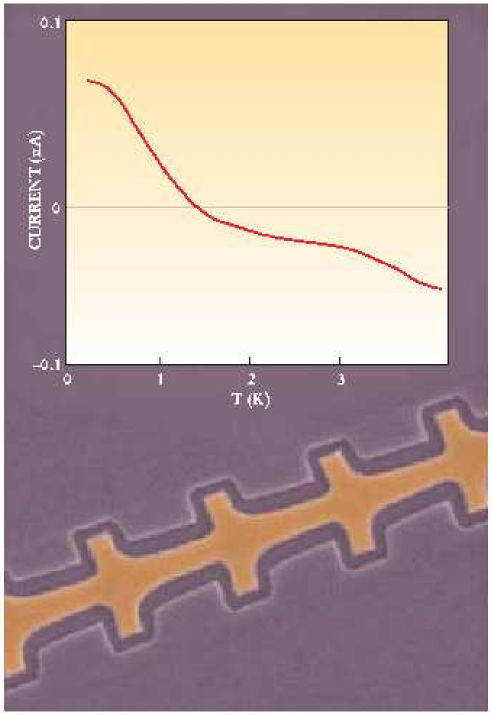
A rocking ratchet: the ratchet potential is tilted symmetrically and periodically

Due to the asymmetry of the barriers, a thermally activated net current (to the right) is generated (after averaging over both tilt directions)

Tunneling electrons, however, prefer the thinner barriers that are the result of tilt to the left

Electrons powered by ac signals could run against a static electric field ("electrons going uphill")

Quantum tunneling provides a second mechanism (the first being the thermal activation) to overcome energy barriers and lead to directed motion



(String of triangular quantum dots, Linke *et al* experiments, Science, 1999)

Rectification of fluctuations in optical lattices

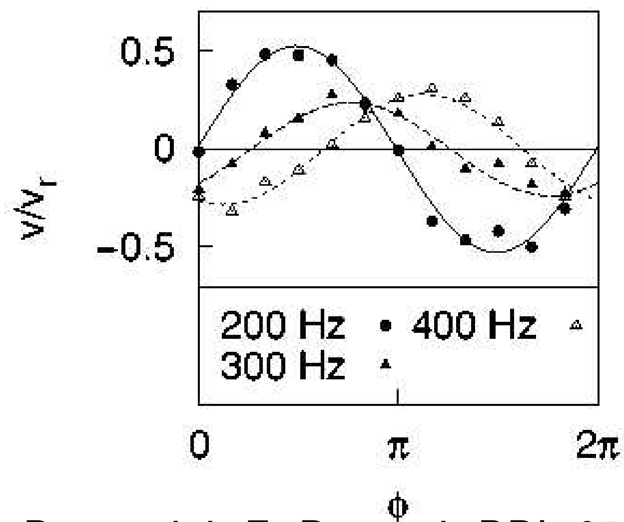
Optical pumping: transition between two ground state sublevels of atoms in optical lattices - As this is a stochastic process, fluctuations in the atomic dynamics are introduced, resulting in a random walk through the optical lattice

Apply a zero-mean ac force breaking all relevant system's symmetry:

$$F(t) = F_0[A\cos(\omega t) + B\cos(2\omega t - \phi)]$$

This force is obtained (in the accelerated frame in which the optical lattice is stationary) by means of a phase-modulated beam:

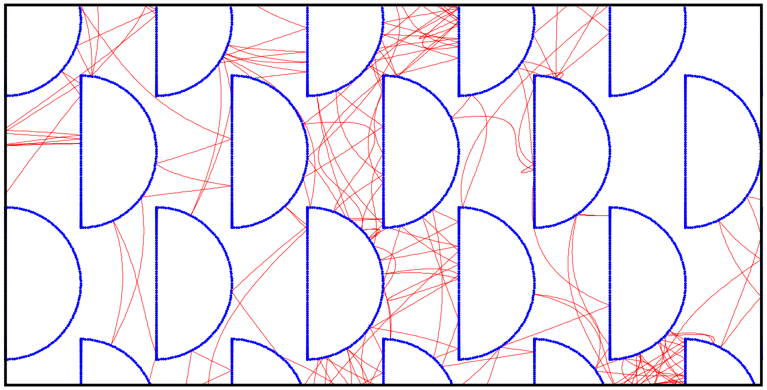
$$\alpha(t) = \alpha_0[A\cos(\omega t) + \frac{B}{4}\cos(2\omega t - \phi)]$$



[R. Gommers, S. Bergamini, F. Renzoni, PRL 95, 073003 (2005)]

Phase lag due to dissipation (Sisyphus cooling)

Directed transport in asymmetric antidot lattices



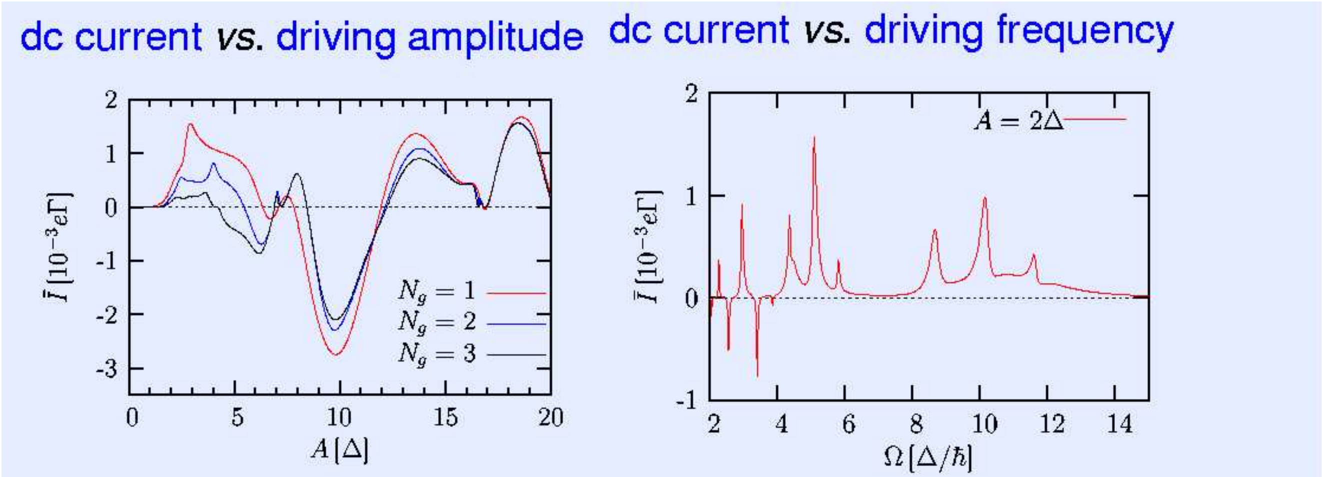
[A.D. Chepelianskii and D.L. Shepelyansky, PRB **71**, 052508 (2005)]

The semidisk Galton board (with chaotic classical dynamics) is subjected to microwave polarized radiation, at finite temperature

Directed transport with antidots of micron size up to about 100 GHz - possible application as new type of highly sensitive detectors of polarized radiation, useful for instance in the field of radioastronomy

Ratchet effect in molecular wires

Molecular wire in an asymmetric potential, subjected to effective dissipation from leads and to a laser field (Motivations: molecular electronics, self-assembly,...)



[J. Lehmann, S. Kohler, P. Hänggi, and A. Nitzan PRL **88**, 228305 (2002)]

Ratchet current exhibits resonances: coherent transport

The multiple current reversals open prospects to pump and shuttle electrons on the nanoscale in an *a priori* manner

Quantum ratchets in dissipative chaotic systems,
Phys. Rev. Lett. **94**, 164101 (2005)

A deterministic model of quantum chaotic dissipative ratchet

Particle moving in a kicked periodic asymmetric potential $[H = \frac{I^2}{2} + V(x,\tau)]$

$$V(x,\tau) = k\left[\cos(x) + \frac{a}{2}\cos(2x+\phi)\right]\sum_{m=-\infty}^{+\infty}\delta(\tau - mT),$$

Classical evolution in one period described by the map

$$\begin{cases} \overline{I} = (1-\gamma)I + k(\sin(x) + a\sin(2x+\phi)), \\ \overline{x} = x + T\overline{I}, \end{cases}$$

$0 < \gamma < 1$ dissipation parameter (velocity proportional damping):
$\gamma = 1$ overdamping – $\gamma = 0$ Hamiltonian evolution

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Quantum ratchets in dissipative chaotic systems,
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Study of the quantized model

Quantization rules: $x \to \hat{x}$, $I \to \hat{I} = -i(d/dx)$ (we set $\hbar = 1$)

Since $[\hat{x}, \hat{p}] = iT$, the effective Planck constant is $\hbar_{\text{eff}} = T$

In order to simulate a dissipative environment in the quantum model we consider a master equation in the Lindblad form for the density operator $\hat{\rho}$ of the system:

$$\dot{\hat{\rho}} = -i[\hat{H}_s, \hat{\rho}] - \frac{1}{2}\sum_{\mu=1}^{2}\{\hat{L}_\mu^\dagger \hat{L}_\mu, \hat{\rho}\} + \sum_{\mu=1}^{2} \hat{L}_\mu \hat{\rho} \hat{L}_\mu^\dagger$$

$\hat{H}_s = \hat{I}^2/2 + V(\hat{x}, \tau)$ system Hamiltonian
$\hat{L}_\mu$ Lindblad operators
$\{\,,\,\}$ denotes the anticommutator

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The dissipation model

We assume that dissipation is described by the lowering operators

$$\hat{L}_1 = g \sum_I \sqrt{I+1}\, |I\rangle \langle I+1|,$$
$$\hat{L}_2 = g \sum_I \sqrt{I+1}\, |-I\rangle \langle -I-1|, \quad n = 0, 1, ...$$

These Lindblad operators can be obtained by considering the interaction between the system and a bosonic bath. The master equation is then derived, at zero temperature, in the usual weak coupling and Markov approximations

Requiring that at short times $\langle p \rangle$ evolves like in the classical case, as it should be according to the Ehrenfest theorem, we obtain $g = \sqrt{-\ln(1-\gamma)}$

Simulation of quantum dissipation with quantum trajectories

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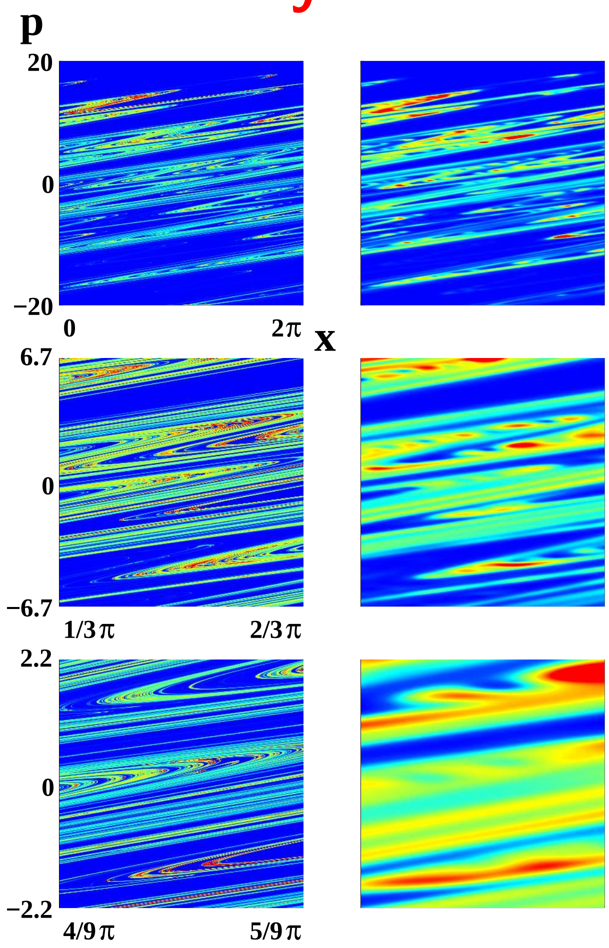
Asymmetric quantum strange attractor



Phase space pictures for $K = 7$, $\gamma = 0.3$, $\phi = \pi/2$, $a = 0.7$, after 100 kicks: classical Poincaré sections (left) and quantum Husimi functions at $\hbar_{\rm eff} = 0.012$ (right)

$p = TI$ rescaled momentum
$K = Tk$ rescaled kicking strength

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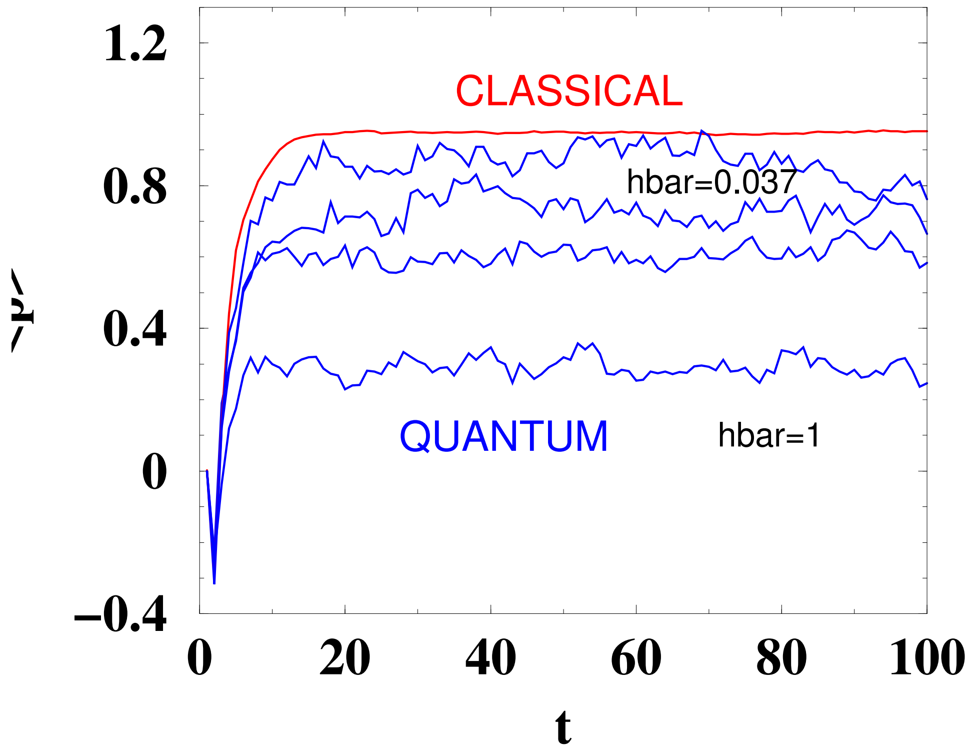
Ratchet effect



Average momentum $\langle p \rangle$ as a function of time t (measured in number of kicks)

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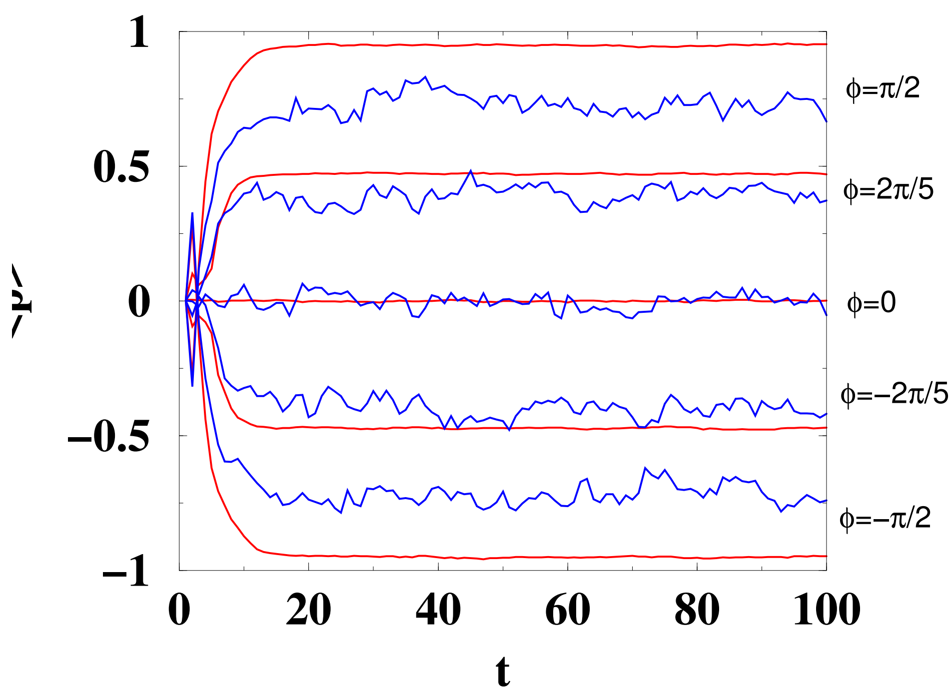
Control the direction of transport



Zero net current for $\phi = n\pi$, due to the space symmetry $V(x,\tau) = V(-x,\tau)$

In general $\langle p \rangle_{-\phi} = -\langle p \rangle_{\phi}$, due to the symmetry $V_{\phi}(x,\tau) = V_{-\phi}(-x,\tau)$

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Quantum ratchets in dissipative chaotic systems,
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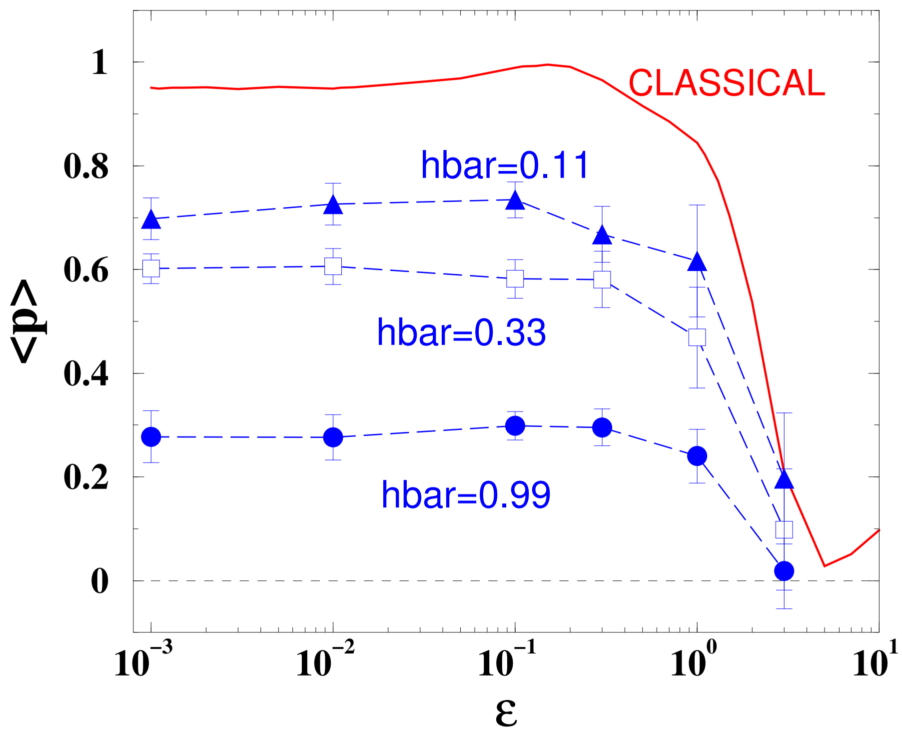
Stability under noise effects



Memoryless fluctuations in the kicking strength: $K \to K_\epsilon(t) = K + \epsilon(t)$, $\epsilon(t) \in [-\epsilon, +\epsilon]$

The ratchet effect survives up to a noise strength ϵ of the order of the kicking strength K

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Possible experimental implementation

Possible experimental implementations with cold atoms in a periodic standing wave of light

Values $K = 7$, $\hbar_{\text{eff}} \sim 1$ were used in the experimental implementations of the kicked rotor model

Friction force can be implemented by means of Doppler cooling techniques

State reconstruction techniques could in principle allow the experimental observation of a quantum strange ratchet attractor

The ratchet effect is robust when noise is added; due to the presence of a strange attractor, the stationary current is independent of the initial conditions

Quantum trajectories approach

An open quantum system becomes, in general, entangled with its environment, and therefore its state is mixed and described by a density matrix. Its evolution is ruled, under the assumption that the environment is Markovian, by a master equation. Solving this equation for a complex many-level system is a prohibitive task in terms of memory cost

Quantum Trajectories allow us to store only a stochastically evolving state vector, instead of a density matrix

This has an enormous advantage in memory requirements: if the Hilbert space has size N, we store only a state vector of size N instead of a density matrix of size $N \times N$

By averaging over many runs we get the same probabilities (within statistical errors) as the ones obtained by solving the density matrix directly

Quantum trajectories in the Markov approximation

If a system interacts with the environment, its state is described by a density operator ρ. Under the Markov assumption, the dynamics of the system is described by a (Lindblad) master equation:

$$\dot{\rho} = -\frac{i}{\hbar}[H_s, \rho] - \frac{1}{2}\sum_k \{L_k^\dagger L_k, \rho\} + \sum_k L_k \rho L_k^\dagger,$$

H_s is the system's Hamiltonian, $\{\, , \,\}$ denotes the anticommutator and L_k are the Lindblad operators, with $k \in [1, \ldots, M]$ (the number M depending on the particular model of interaction with the environment)

• The first two terms of the above equation can be regarded as the evolution performed by an effective non-hermitian Hamiltonian, $H_{\text{eff}} = H_s + iK$, with

$K = -\hbar/2 \sum_k L_k^\dagger L_k$:

$$-\frac{i}{\hbar}[H_s, \rho] - \frac{1}{2}\sum_k \{L_k^\dagger L_k, \rho\} = -\frac{i}{\hbar}[H_{\text{eff}}\rho - \rho H_{\text{eff}}^\dagger].$$

- The last term is the one responsible for the so called quantum jumps

If the initial density matrix describes a pure state ($\rho(t_0) = |\phi(t_0)\rangle\langle\phi(t_0)|$), then, after an infinitesimal time dt, it evolves into the statistical mixture

$$\begin{aligned}
\rho(t_0 + dt) \quad &= \rho(t_0) - \tfrac{i}{\hbar}[H_{\text{eff}}\rho(t_0) - \rho(t_0)H_{\text{eff}}^\dagger]dt + \textstyle\sum_k L_k\rho(t_0)L_k^\dagger dt \\
&\approx (I - \tfrac{i}{\hbar}H_{\text{eff}}dt)\rho(t_0)(I + \tfrac{i}{\hbar}H_{\text{eff}}^\dagger dt) + \textstyle\sum_k L_k\rho(t_0)L_k^\dagger dt \\
&= (1 - \textstyle\sum_k dp_k)\, |\phi_0\rangle\langle\phi_0| \, + \textstyle\sum_k dp_k |\phi_k\rangle\langle\phi_k|,
\end{aligned}$$

where the probabilities $dp_k = dt\langle\phi(t_0)|L_k^\dagger L_k|\phi(t_0)\rangle$, and the (normalized) new states are defined by

$$|\phi_0\rangle = \frac{(I - iH_{\rm eff}dt/\hbar)|\phi(t_0)\rangle}{\sqrt{1-\sum_k dp_k}}, \quad |\phi_k\rangle = \frac{L_k|\phi(t_0)\rangle}{||L_k|\phi(t_0)\rangle||}.$$

Therefore with probability dp_k a jump occurs and the system is prepared in the state $|\phi_k\rangle$. With probability $1-\sum_k dp_k$ there are no jumps and the system evolves according to the effective Hamiltonian $H_{\rm eff}$. (normalization is included because the evolution is non-hermitian)

Numerical method (Monte Carlo wave function approach)

• Start the time evolution from a pure state $|\phi(t_0)\rangle$

• At intervals dt much smaller than the time scales relevant for the evolution of the system, choose a random number ϵ from a uniform distribution in the unit interval $[0, 1]$

• 1) If $\epsilon \leq dp$, where $dp = \sum_k dp_k$, the state of the system jumps to one of the states $|\phi_k\rangle$ (to $|\phi_1\rangle$ if $0 \leq \epsilon \leq dp_1$, to $|\phi_2\rangle$ if $dp_1 < \epsilon \leq dp_1 + dp_2$, and so on)
2) if $\epsilon > dp$ the evolution with the non-hermitian Hamiltonian H_{eff} takes place and we end up in the state $|\phi_0\rangle$

• Repeat this process as many times as $n_{\text{steps}} = \Delta t/dt$, where Δt is the total evolution time

This procedure describes a stochastically evolving wave vector, and we say that a single evolution is a quantum trajectory

• Average over different runs to recover, up to statistical errors, the probabilities obtained using the density operator. Given an operator A, we can write the mean value $\langle A\rangle_t = \mathrm{Tr}[A\rho(t)]$ as the average over $\mathcal{N}$ trajectories:

$$\boxed{\langle A\rangle_t = \lim_{\mathcal{N}\to\infty} \frac{1}{\mathcal{N}} \sum_{i=1}^{\mathcal{N}} \langle \phi_i(t)|A|\phi_i(t)\rangle}$$

There is also an advantage in computation time: in general $\mathcal{N} \approx 100-500$ trajectories are needed in order to obtain a satisfactory statistical convergence, so that there is an advantage in computer time provided that $N > \mathcal{N}$

Quantum trajectories and stochastic Schrödinger equation

A quantum trajectory represents a single member of an ensemble whose density operator satisfies the corresponding master equation. This picture can be formalized by means of the nonlinear stochastic Schrödinger equation

$$
\begin{aligned}
|d\phi\rangle &= -iH|\phi\rangle dt - \frac{1}{2}\sum_k (L_k^\dagger L_k - \langle\phi|L_k^\dagger L_k|\phi\rangle)|\phi\rangle dt \\
&\quad + \sum_k \left(\frac{L_k}{\sqrt{\langle\phi|L_k^\dagger L_k|\phi\rangle}} - I \right) |\phi\rangle \, dN_k .
\end{aligned}
$$

The stochasticity is due to the measurement results: we think that the environment is actually measured (as it is the case in indirect measurement models) or simpler, that the contact of the system with the environment produces an effect similar to a continuous (weak) measurement

The nonlinearity appears due to the renormalization of the state vector after each measurement process

The stochastic differential variables dN_k are statistically independent and represent measurement outcomes. Their ensemble mean is given by $M[dN_k] = \langle\phi|L_k^\dagger L_k|\phi\rangle dt$. The probability that the variable dN_k is equal to 1 during a given time step dt is $\langle\phi|L_k^\dagger L_k|\phi\rangle dt = dp_k$. Therefore, most of the time the variables dN_k are 0 and as a consequence the system evolves continuously by means of H_{eff}. However, when a variable dN_k is equal to 1, a quantum jump occurs. Differently from the master equation for the density operator, the stochastic Schrödinger equation represents the evolution of an individual quantum system, as exemplified by a single run of a laboratory experiment

An example from quantum optics: spontaneous emission

Let us consider the simplest, zero temperature instance of the quantum optics master equation

$$\dot{\rho} = -\frac{i}{\hbar}[H, \rho] - \frac{\gamma}{2}(\sigma_- \sigma_+ \rho + \rho \sigma_- \sigma_+) + \gamma \sigma_+ \rho \sigma_-,$$

where the Hamiltonian $H = \frac{1}{2}\hbar\omega_0\sigma_z$ describes the free evolution of a two-level atom, γ is the atom-field coupling constant and $\sigma_\pm = \frac{1}{2}(\sigma_x \pm \sigma_y)$

In this case there is a single Lindblad operator $L_1 = \sqrt{\gamma}\sigma_+$ and a jump is a transition from the excited state ($|1\rangle$) to the ground state ($|0\rangle$) of the atom

Starting from an initial pure state $|\phi(t_0)\rangle = \alpha|0\rangle + \beta|1\rangle$ and evolving it for an infinitesimal time dt, the probability of a jump in a time dt is given by

$$dp = \langle \phi(t_0)|L_1^\dagger L_1|\phi(t_0)\rangle dt = \gamma \langle \phi(t_0)|\sigma_- \sigma_+|\phi(t_0)\rangle dt = \gamma p_e(t_0) dt,$$

where $p_e(t_0) = |\beta|^2$ is the population of the excited state $|1\rangle$ at time t_0

If a jump occurs, the new state of the atom is

$$|\phi_1\rangle = \frac{L_1|\phi(t_0)\rangle}{||L_1|\phi(t_0)\rangle||} = \frac{\sqrt{\gamma}\,\sigma_+(\alpha|0\rangle + \beta|1\rangle)\sqrt{dt}}{\sqrt{dp}} = \frac{\beta}{|\beta|}|0\rangle.$$

In this case, the transition $|1\rangle \rightarrow |0\rangle$ takes place and the emitted photon is detected. As a consequence, the atomic state vector collapses onto the ground state $|0\rangle$

If instead there are no jumps, the system's evolution is ruled by the non-Hermitian effective Hamiltonian $H_{\text{eff}} = H - i\frac{\hbar}{2}L_1^\dagger L_1 = H - i\frac{\hbar}{2}\gamma\sigma_-\sigma_+$, so that the state of the atom at time $t_0 + dt$ is

$$|\phi_0\rangle = \frac{\left(I - \frac{i}{\hbar}H_{\text{eff}}dt\right)|\phi(t_0)\rangle}{\sqrt{1-dp}} = \frac{\left(1 - i\frac{\omega_0}{2}dt\right)\alpha|0\rangle + \left(1 + i\frac{\omega_0}{2}dt - \frac{\gamma}{2}\right)\beta|1\rangle}{\sqrt{1-\gamma|\beta|^2 dt}}$$

The normalization factor $\frac{1}{\sqrt{1-dp}}$ is due to the fact that, if no counts are registered by the photodetector, then we consider more probable that the system is unexcited

To see this, let us consider the evolution without jumps in a finite time interval, from t_0 to $t_0 + t$. We obtain

$$|\phi_0(t_0+t)\rangle = \frac{\alpha\exp\left[-i\frac{\omega_0}{2}(t-t_0)\right]|0\rangle + \beta\exp\left[\left(i\frac{\omega_0}{2} - \frac{\gamma}{2}\right)(t-t_0)\right]|1\rangle}{\sqrt{|\alpha|^2 + |\beta|^2\exp[-\gamma(t-t_0)]}}$$

Note that as $t \to +\infty$ the state $|\phi_0(t)\rangle \to |0\rangle$ (up to an overall phase factor). That is, if after a long time we never see a count, we conclude that we have been in the ground state $|0\rangle$ from the beginning

Dissipative quantum chaos: transition from wave packet collapse to explosion,
Phys. Rev. Lett. **95**, 164101 (2005)

Dissipative quantum chaos: transition from wave packet collapse to wave packet explosion

The instability of classical dynamics leads to exponentially fast spreading of the quantum wave packet on the logarithmically short Ehrenfest time scale

$$t_E \sim \frac{|\ln \hbar|}{\lambda}$$

λ Lyapunov exponent, $\hbar$ effective Planck constant

After the logarithmically short Ehrenfest time a description based on classical trajectories is meaningless for a closed quantum system

What is the interplay between wave packet explosion (delocalization) induced by chaotic dynamics and wave packet collapse (localization) caused by dissipation?

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A model of dissipative chaotic dynamics

Markovian master equation $\dot{\hat{\rho}} = -i[\hat{H}, \hat{\rho}] - \frac{1}{2}\sum_{\mu}\{\hat{L}_{\mu}^{\dagger}\hat{L}_{\mu}, \hat{\rho}\} + \sum_{\mu}\hat{L}_{\mu}\hat{\rho}\hat{L}_{\mu}^{\dagger}$

Kicked rotator Hamiltonian $\hat{H} = \frac{\hat{I}^2}{2} + k\cos(\hat{x}) \sum_{m=-\infty}^{+\infty} \delta(\tau - mT)$

Dissipation described by the Lindblad operators

$$\hat{L}_1 = g\sum_{I}\sqrt{I+1}\,|I\rangle\,\langle I+1|,\;\; \hat{L}_2 = g\sum_{I}\sqrt{I+1}\,|-I\rangle\,\langle -I-1|$$

At the classical limit, the evolution of the system in one period is described by the Zaslavsky map

$$\begin{cases} I_{t+1} = (1-\gamma)I_t + k\sin x_t, \\ x_{t+1} = x_t + TI_{t+1}, \end{cases}$$

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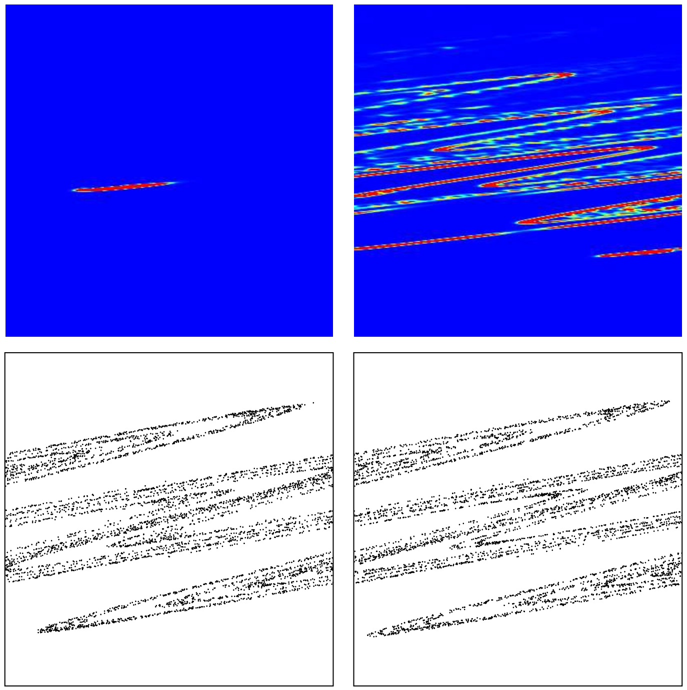
Collapse to explosion transition (going from strong to weak dissipation)



$K = 7$, $\hbar = 0.012$, $\gamma = 0.5$ and $\gamma = 0.01$

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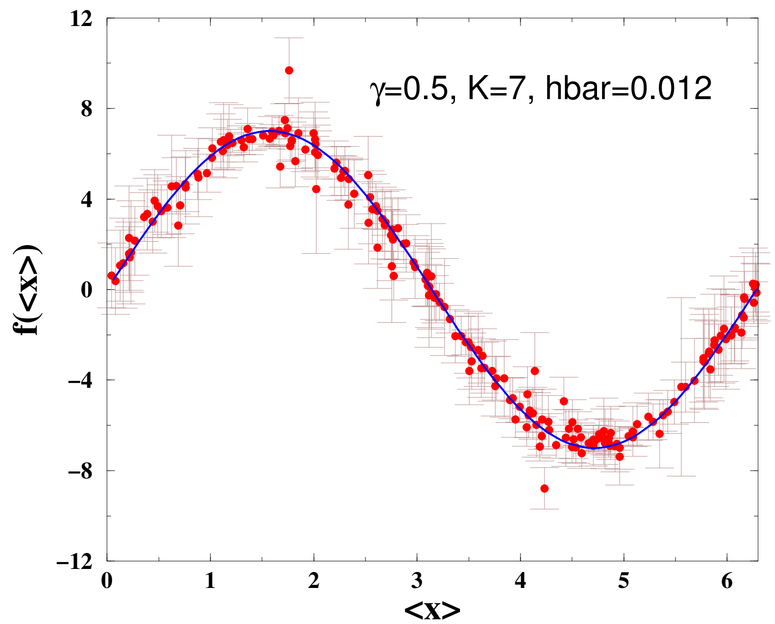
Classical-like evolution of quantum trajectories



$$f \equiv \langle p \rangle_{t+1} - (1-\gamma)\langle p \rangle_t, \quad \langle p \rangle_t = \langle x \rangle_t - \langle x \rangle_{t-1}$$

From classical dynamics we expect $f(x) = K \sin x$ - Quantum fluctuations $\propto \sqrt{\hbar}$

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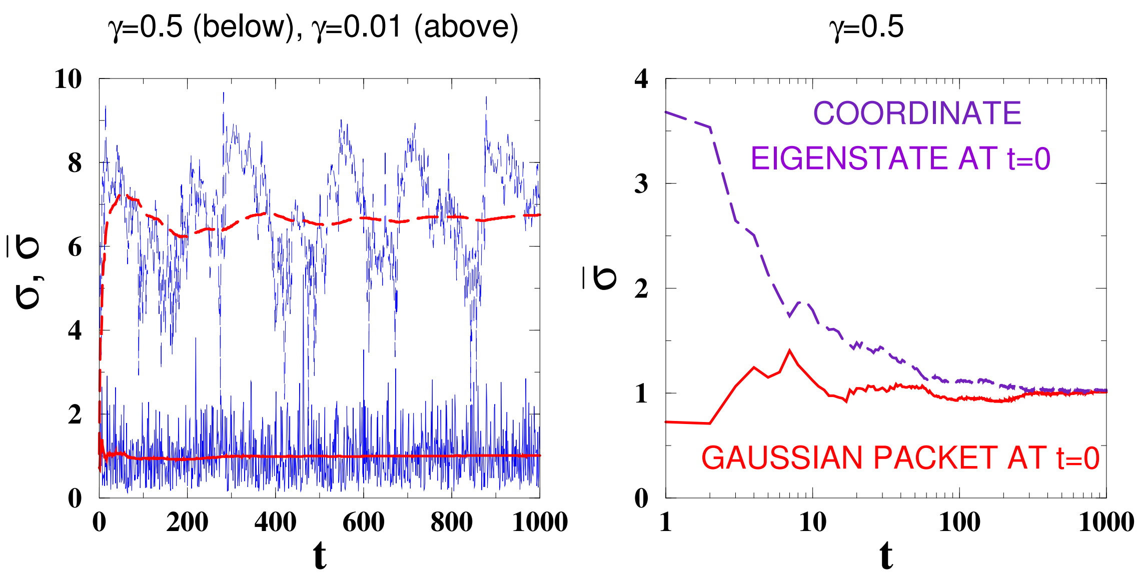
Wave packet dispersion

$$\sigma_t = \sqrt{(\Delta x)_t^2 + (\Delta p)_t^2}, \quad \text{cumulative average } \overline{\sigma}_t \equiv \frac{1}{t}\sum_{j=1}^{t} \sigma_j$$



$$(K = 7,\, \hbar = 0.012)$$

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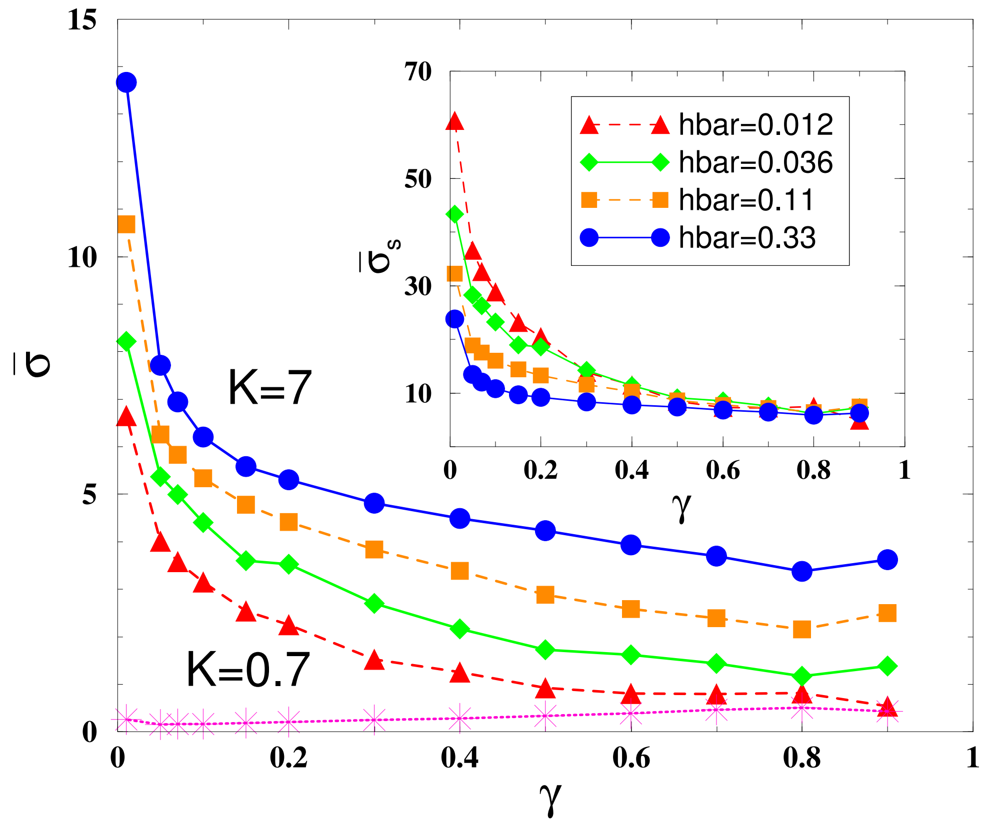
Localization - delocalization crossover



$\overline{\sigma}_s \equiv \overline{\sigma}/\sqrt{\hbar}$ scaled dispersion

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Ehrenfest explosion

Due to the exponential instability of chaotic dynamics the wave packet spreads exponentially and for times shorter than the Ehrenfest time we have $\sigma_t \sim \sqrt{\hbar}\exp(\lambda t)$

The dissipation localizes the wave packet on a time scale of the order of $1/\gamma$

Therefore, for $1/\gamma \ll t_E \sim |\ln\hbar|/\lambda$, we obtain $\overline{\sigma} \sim \sqrt{\hbar}\exp(\lambda/\gamma) \ll 1$

In contrast, for $1/\gamma > t_E$ the chaotic wave packet explosion dominates over dissipation and we have complete delocalization over the angle variable

In this case, the wave packet spreads algebraically due to diffusion for $t > t_E$: for $t \gg t_E$ we have $\sigma_t \sim \sqrt{D(K)t}$, $D(K) \approx K^2/2$ being the diffusion coefficient; this regime continues up to the dissipation time $1/\gamma$, so that $\overline{\sigma} \sim \sqrt{D(K)/\gamma}$

G.G. Carlo, G.Benenti, D.L.Shepelyansky

Dissipative quantum chaos: transition from wave packet collapse to explosion,
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Tthe transition from collapse to explosion (Ehrenfest explosion) takes place at

$$t_E \sim \frac{|\ln \hbar|}{\lambda} \sim \frac{1}{\gamma}$$

Therefore, even for infinitesimal dissipation strengths the quantum wave packet is eventually localized when $\hbar \to 0$: we have $\lim_{\hbar \to 0} \overline{\sigma} = 0$; in contrast, in the Hamiltonian case $(\gamma = 0)$ $\lim_{\hbar \to 0} \overline{\sigma} = \infty$

Only for open quantum systems the classical concept of trajectory is meaningful for arbitrarily long times; on the contrary, for Hamiltonian systems a description based on wave packet trajectories is possible only up to the Ehrenfest time scale

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Conclusions and prospects

- Cold atoms and Bose-Einstein condensates exposed to time-dependent standing waves of light provide an ideal test bed to explore dissipative quantum chaos

- Quantum ratchets: study thermal effects on the ratchet current; adapt the proposed ratchet model to condensates (including Gross-Pitaevsky nonlinearity effects); study the impact of dynamical effects such as bifurcations

- Ehrenfest explosion: investigate the dynamical stability of condensates subjected to chaotic dynamics and dissipation

- From simple models to complex solid state and biological samples: understand charge-transfer phenomena in molecular wires and biomolecules: study conductance under laser excitation, current control, ...

- Strategies for the directed transport of quantum information?