Recovering Entanglement by Local Operations

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Outline

Problem: entanglement revivals without non-local operations

Quantification of entanglement based on the density operator formalism vs ensemble description

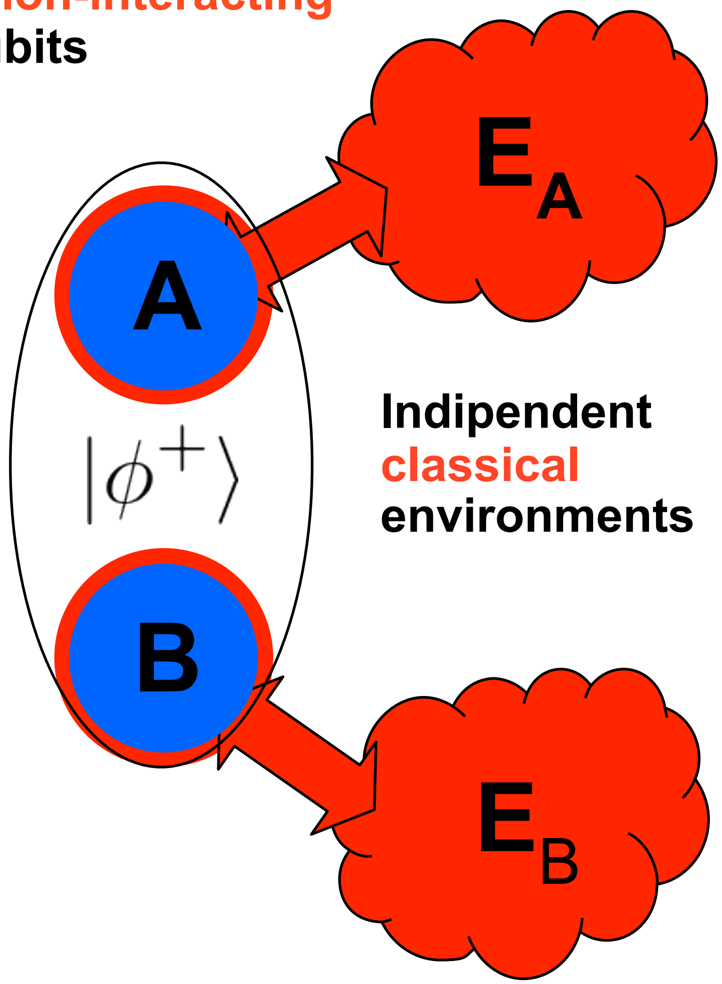
Definition of hidden entanglement

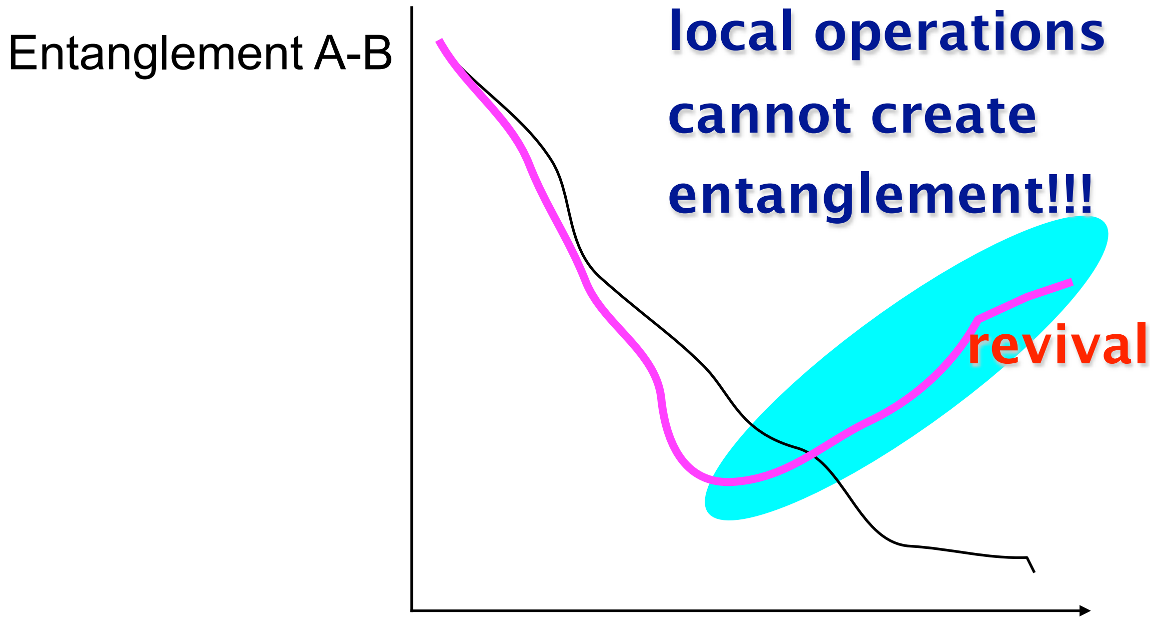
Examples: Random local fields, solid state system affected by low frequency noise, amplitude damping channel, etc.

Non-Markovian dynamics and non-monotonous behavior of entanglement

Problem: Entanglement revivals

2 non-interacting qubits





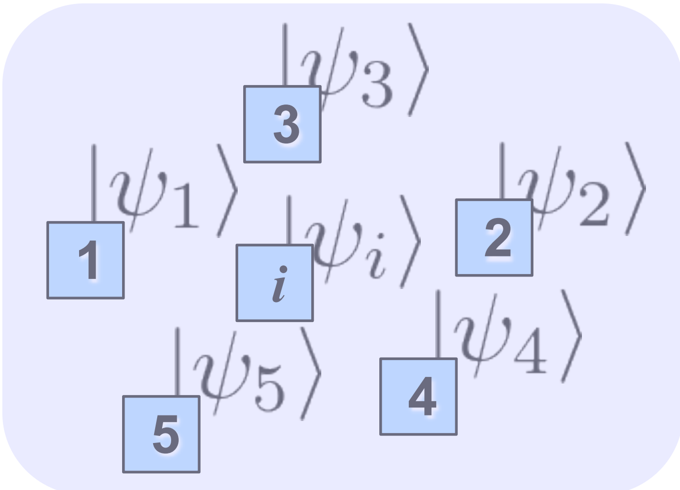
How does entanglement revises?

No non–local operations

No exchange of quantum correlations system–environment

Entanglement of a mixed state

Charlie



Charlie sends the prepared systems to Alice and Bob

Alice and Bob

$|\psi_3\rangle$

$|\psi_1\rangle$ $|\psi_2\rangle$

$|\psi_i\rangle$

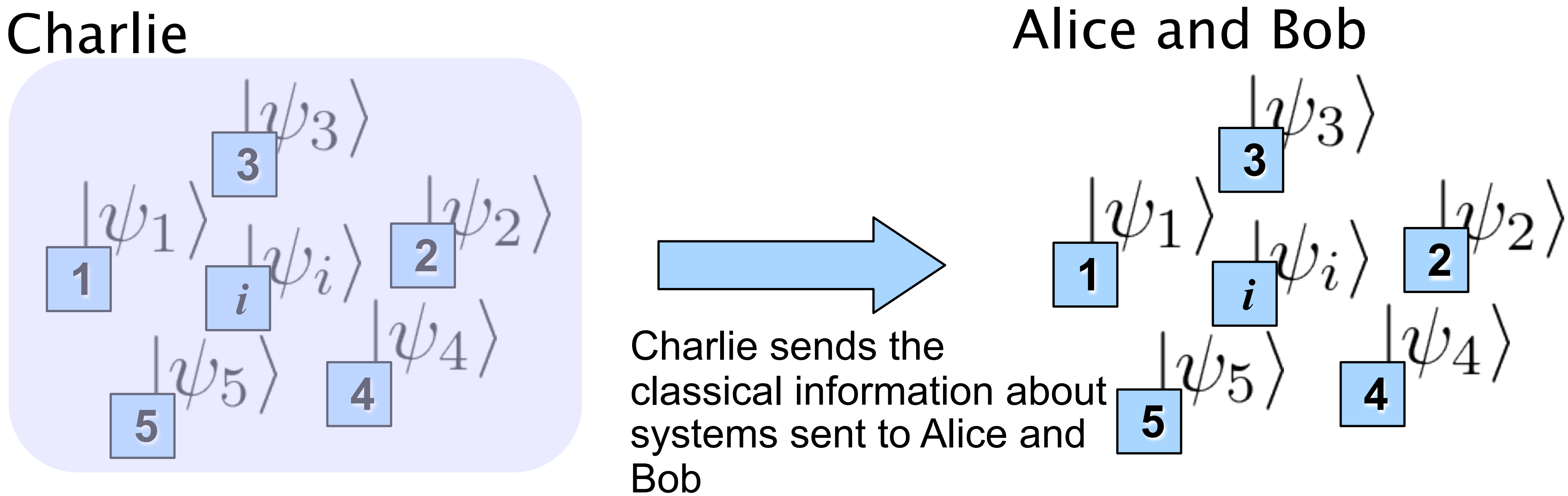
$|\psi_5\rangle$ $|\psi_4\rangle$

$$\rho = \sum_i p_i |\psi_i\rangle\langle\psi_i|$$

How much is the entanglement shared by Alice and Bob?

$$E(\rho)$$

Entanglement of a quantum ensemble



What changes if Alice and Bob try do distill now entanglement?

$$E_{av}(\mathcal{A}) = \sum p_i E(|\psi_i\rangle\langle\psi_i|)$$

$$\mathcal{A} = \{(p_i, |\psi_i\rangle)\}$$

Definition: Hidden entanglement

$$E_h(\mathcal{A}) \equiv E_{av}(\mathcal{A}) - E(\rho) =$$
$$= \sum_i p_i E(|\psi_i\rangle\langle\psi_i|) - E\left(\sum_i p_i |\psi_i\rangle\langle\psi_i|\right)$$

E is any convex entanglement measure, so that $E_h \geq 0$

E_h: Entanglement not exploitable, lack of classical information

$\Downarrow$

It may be **<u>recovered</u>** **without non-local operations**

No complete knowledge...

$|\psi_3\rangle$

$|\psi_1\rangle$ $|\psi_i\rangle$ $|\psi_2\rangle$

$|\psi_5\rangle$ $|\psi_4\rangle$

$$\rho = \sum_i p_i |\psi_i\rangle\langle\psi_i|$$

classical
information
about systems

$|\psi_3\rangle$ 3

$|\psi_1\rangle$ 1 $|\psi_i\rangle$ *i* $|\psi_2\rangle$ 2

$|\psi_5\rangle$ 5 $|\psi_4\rangle$ 4

$$\mathcal{A} = \{(p_i, |\psi_i\rangle)\}$$

$$E(\rho) = E\Big(\sum p_i |\psi_i\rangle\langle\psi_i|\Big)$$

no non-local operations

no quantum correlations tranfer

only classical information

$$E_{av}(\mathcal{A}) = \sum p_i E(|\psi_i\rangle\langle\psi_i|)$$

$$E_{av}(\mathcal{A}) - E(\rho) \geq 0$$

Entanglement increases

Example: random local field

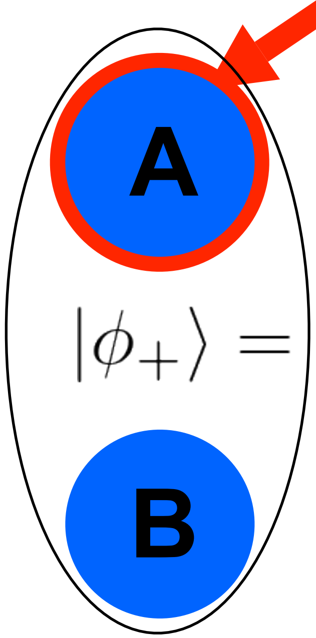
Random Local Field

$$p_x = \frac{1}{2}, \qquad U_x = e^{-i\sigma_x \omega t/2}$$

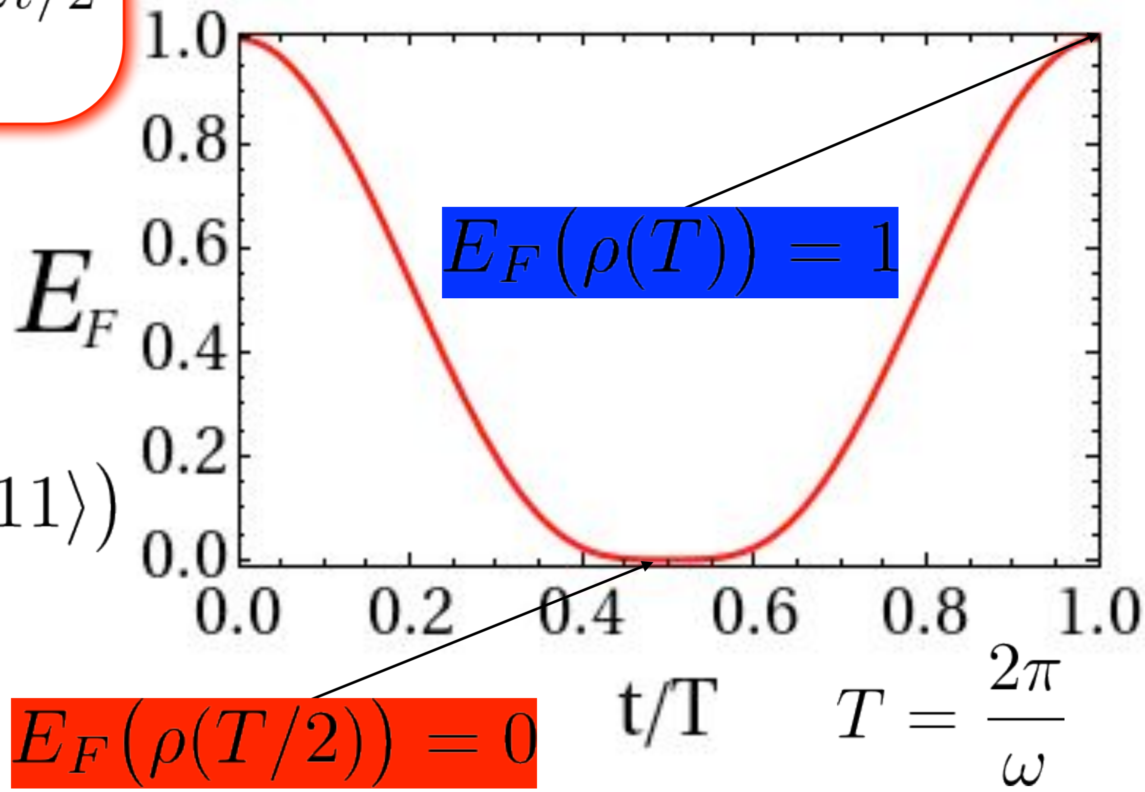
$$p_z = \frac{1}{2}, \qquad U_z = e^{-i\sigma_z \omega t/2}$$

entanglement increases

without **non-local operations**



$$|\phi_+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$



$$T = \frac{2\pi}{\omega}$$

$$E_F(\rho(T/2)) = 0 \xrightarrow{\;]0.5T,\ T]\;} E_F(\rho(T)) = 1$$

Manifestation of **quantum correlations**

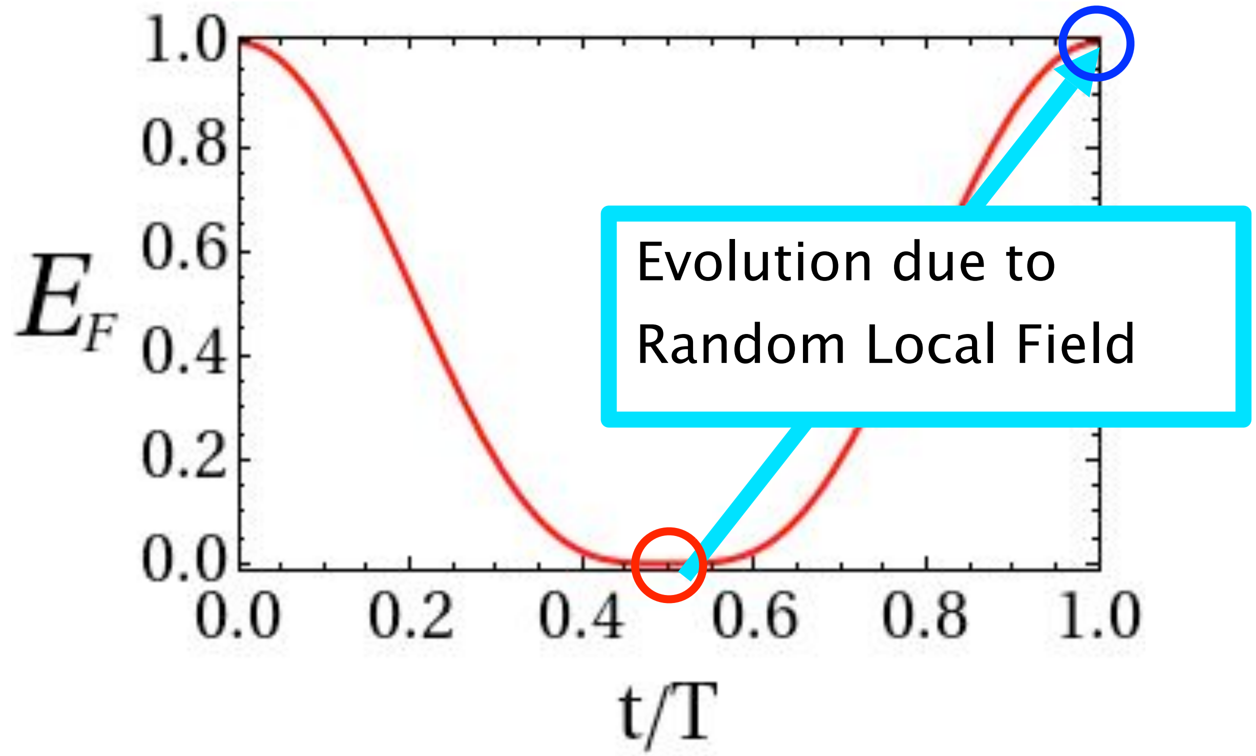
that must **be present before**

Quantum ensemble description

$$p_x, \quad |\psi_x(t)\rangle = U_x|\phi_+\rangle$$

$$p_z, \quad |\psi_z(t)\rangle = U_z|\phi_+\rangle$$

$$\mathcal{A}(t) = \{p_i, |\psi_i(t)\rangle\}$$



Density operator formalism

$$\rho(t) = \sum_i p_i U_i|\phi_+\rangle\langle\phi_+|U_i^\dagger$$

mixture

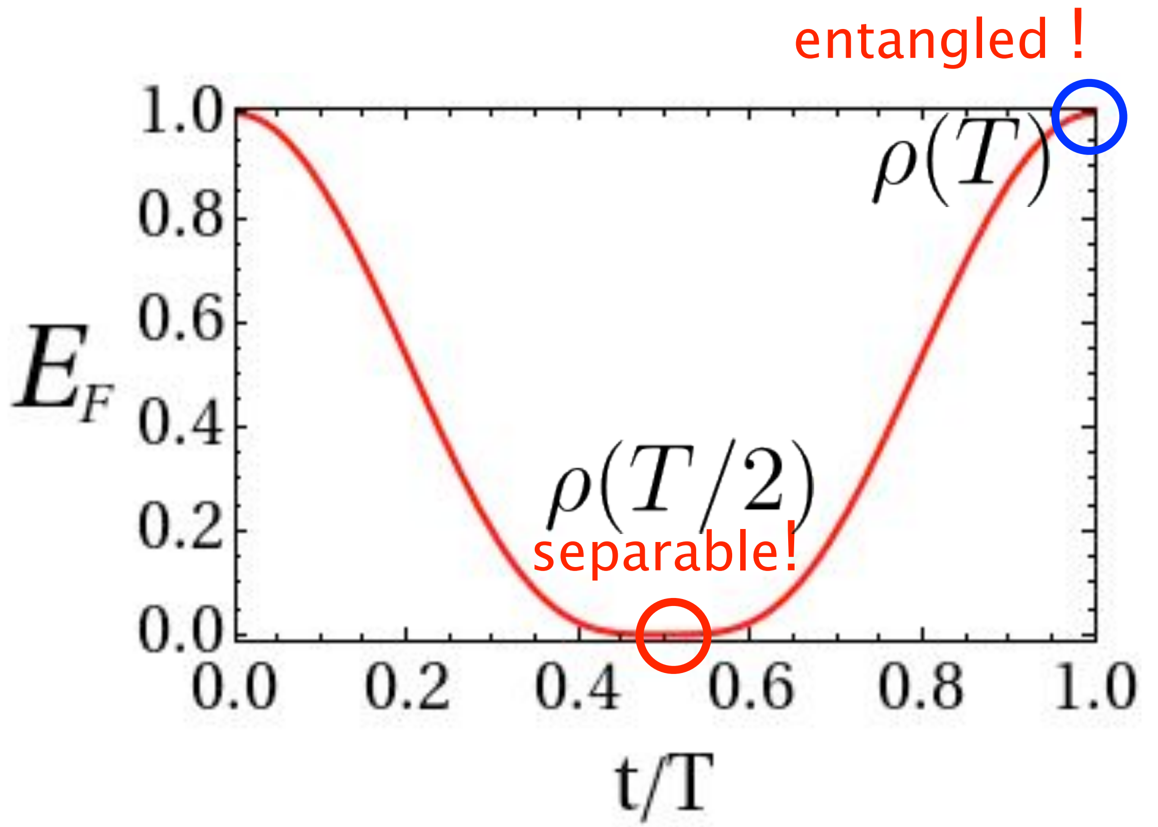
$$E_{av}(\mathcal{A}) = \sum_i p_i \, E\big(|\psi_i(t)\rangle\big) = \sum_i p_i \, E\big(U_i|\phi_+\rangle\big) = 1 \quad \forall \, t$$

$U_x(T/2) = -i\sigma_x \quad U_z(T/2) = -i\sigma_z$

$\mathcal{A}(T/2) = \left\{ (\frac{1}{2}, |\phi_-\rangle), (\frac{1}{2}, |\psi_+\rangle) \right\}$

$$E(\rho(T/2)) = 0$$

$$E_h(\mathcal{A}(T/2)) = 1$$



$U_x(T) = U_z(T) = I_d$

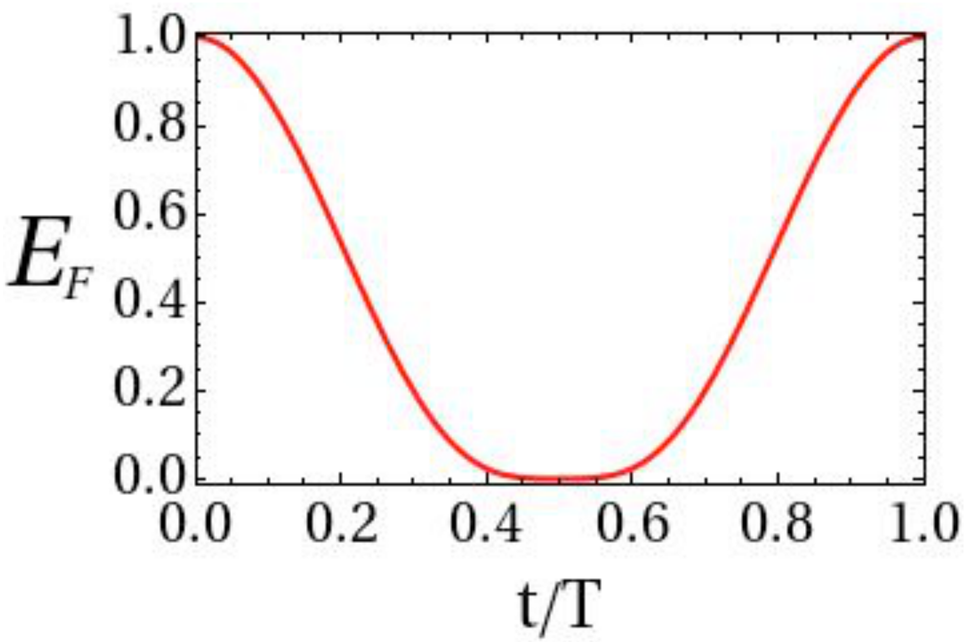
$\mathcal{A}(T) = \{1, |\phi_+\rangle\}$

$$E(\rho(T)) = 1$$

Violation of the monotonicity axiom?

Monotonicity axiom:
Entanglement cannot increase under LOCC

The entanglement recovery is induced by local operations that are not LOCC (the evolution of the density operator from T/2 to T is not a CPT map)



Purely dephasing random telegraph noise

$$\mathcal{H} = \mathcal{H}_0 + \delta\mathcal{H},$$

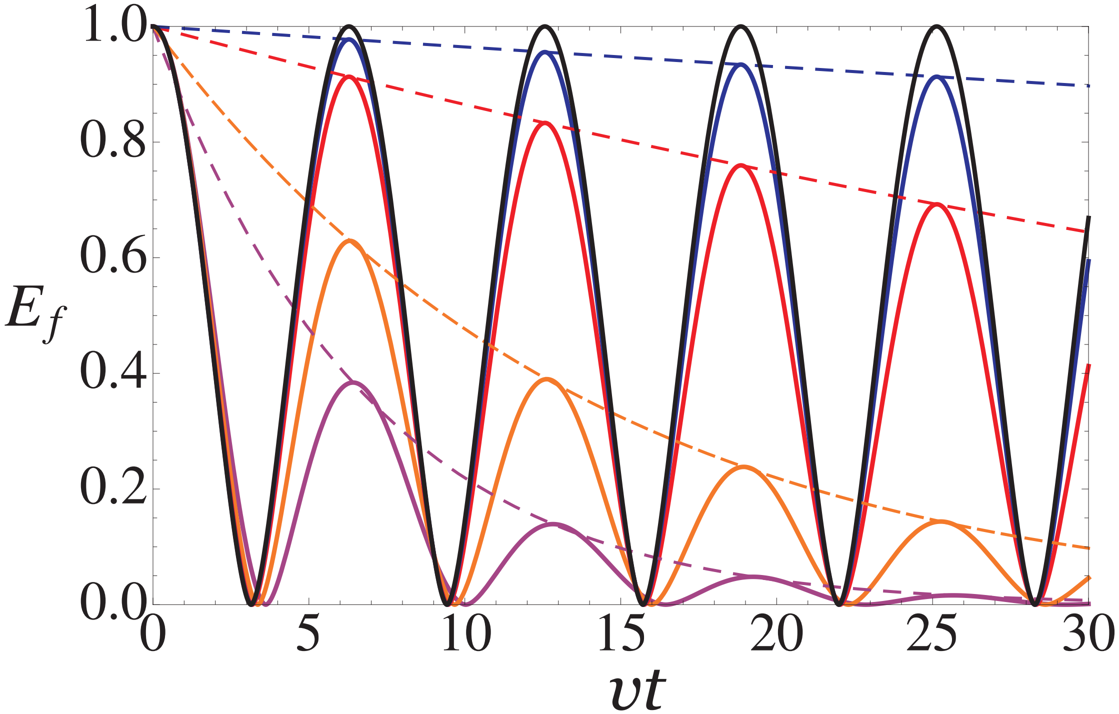
$$\mathcal{H}_0 = -\frac{\Omega_A}{2}\sigma_{z_1} - \frac{\Omega_B}{2}\sigma_{z_2}, \qquad \delta\mathcal{H} = -\frac{\xi(t)}{2}\sigma_{z_1} \qquad \xi(t) \in \{0, v\}$$

switching rate γ

At γ=0: $\mathcal{A} = \left\{ \left(p_0, |\varphi_0(t)\rangle\right), \left(p_v, |\varphi_v(t)\rangle\right) \right\}$

$$|\varphi_0(t)\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + e^{-i(\Omega_A+\Omega_B)t}|11\rangle\right)$$

$$|\varphi_v(t)\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + e^{-ivt}e^{-i(\Omega_A+\Omega_B)t}|11\rangle\right)$$



Entanglement revivals at

$$t_n = 2n\pi/v$$

(for $v/\gamma > 1$)

Peak values drop as

$$f(C) = f(e^{-\gamma t/2})$$

Stochastic low-frequency noise

Stochastic low-frequency noise

$$\mathcal{H}_A = -\frac{\Omega_A}{2}\sigma_z - \frac{\varepsilon(t)}{2}\sigma_z$$

Static noise

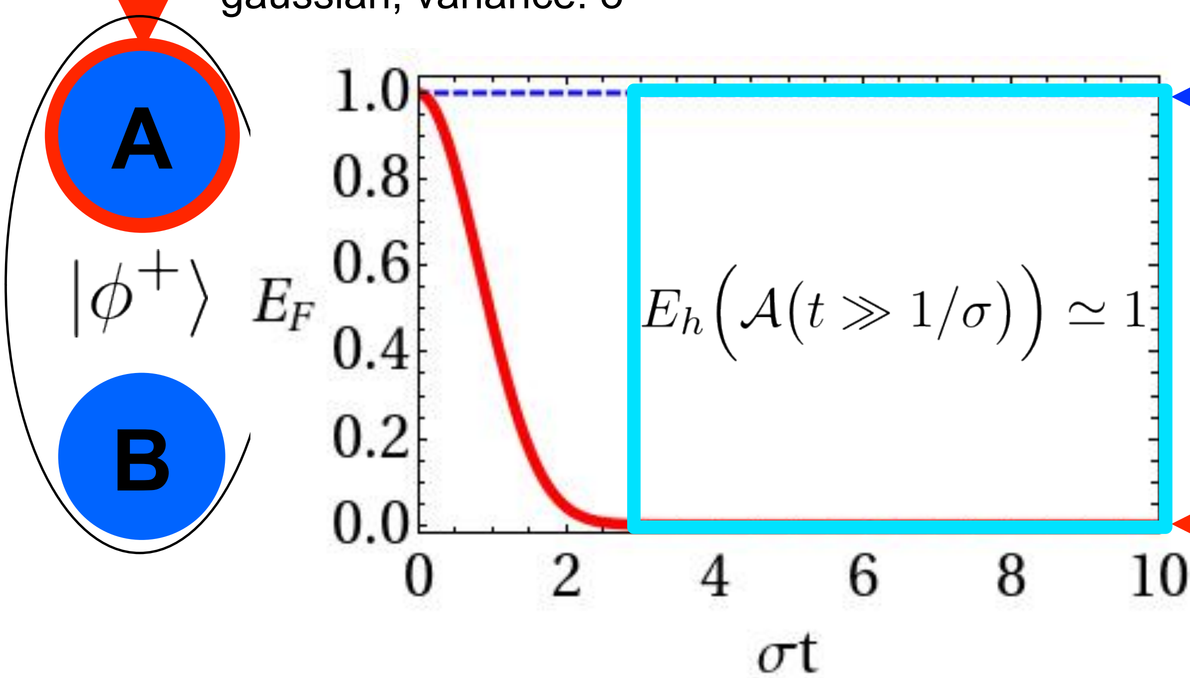
$$\varepsilon(t) \simeq \varepsilon \qquad C(\rho(t)) = e^{-\frac{1}{2}\sigma^2 t^2}$$

gaussian, variance: σ^2

For each realization of $\varepsilon(t)$

$$|\psi_\varepsilon(t)\rangle = e^{-i\frac{\Omega_A+\varepsilon}{2}\sigma_z t}|\phi_+\rangle$$

$$\mathcal{A}(t) = \{p(\varepsilon)d\varepsilon,\ |\psi_\varepsilon(t)\rangle\}$$



$$E_{av}\left(\mathcal{A}(t)\right) = 1$$

$$\rho(t) = \int d\varepsilon p(\varepsilon)|\psi_\varepsilon(t)\rangle\langle\psi_\varepsilon(t)|$$

$$E_F\left(\rho(t \gg 1/\sigma)\right) \simeq 0$$

Recovering entanglement by local pulses

Stochastic low-frequency noise

$$\mathcal{H}_A = -\frac{\Omega_A}{2}\sigma_z - \frac{\varepsilon(t)}{2}\sigma_z + \mathcal{V}(t,\bar{t})\sigma_x$$

$$\mathrm{e}^{-i\sigma_x \pi/2} = -i\sigma_x$$

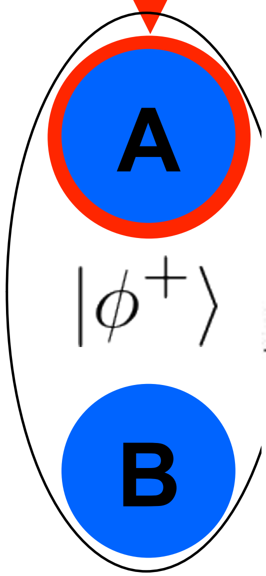
π -pulse around x

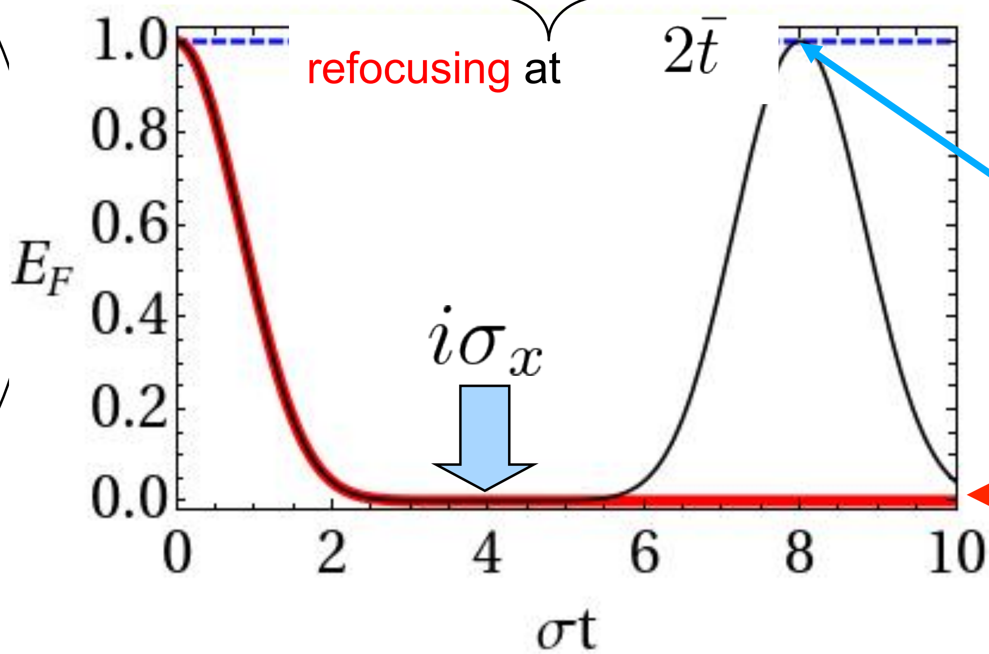
$$i\sigma_x$$

$$|\psi_\varepsilon(t)\rangle = e^{-i\frac{\Omega_A+\varepsilon}{2}\sigma_z(t-\bar{t})}\sigma_x e^{-i\frac{\Omega_A+\varepsilon}{2}\sigma_z\bar{t}}|\psi(0)\rangle$$

$$= \underbrace{e^{-i\frac{\Omega_A+\varepsilon}{2}\sigma_z(t-\bar{t})}e^{+i\frac{\Omega_A+\varepsilon}{2}\sigma_z\bar{t}}}\sigma_x|\psi(0)\rangle$$

refocusing at $2\bar{t}$





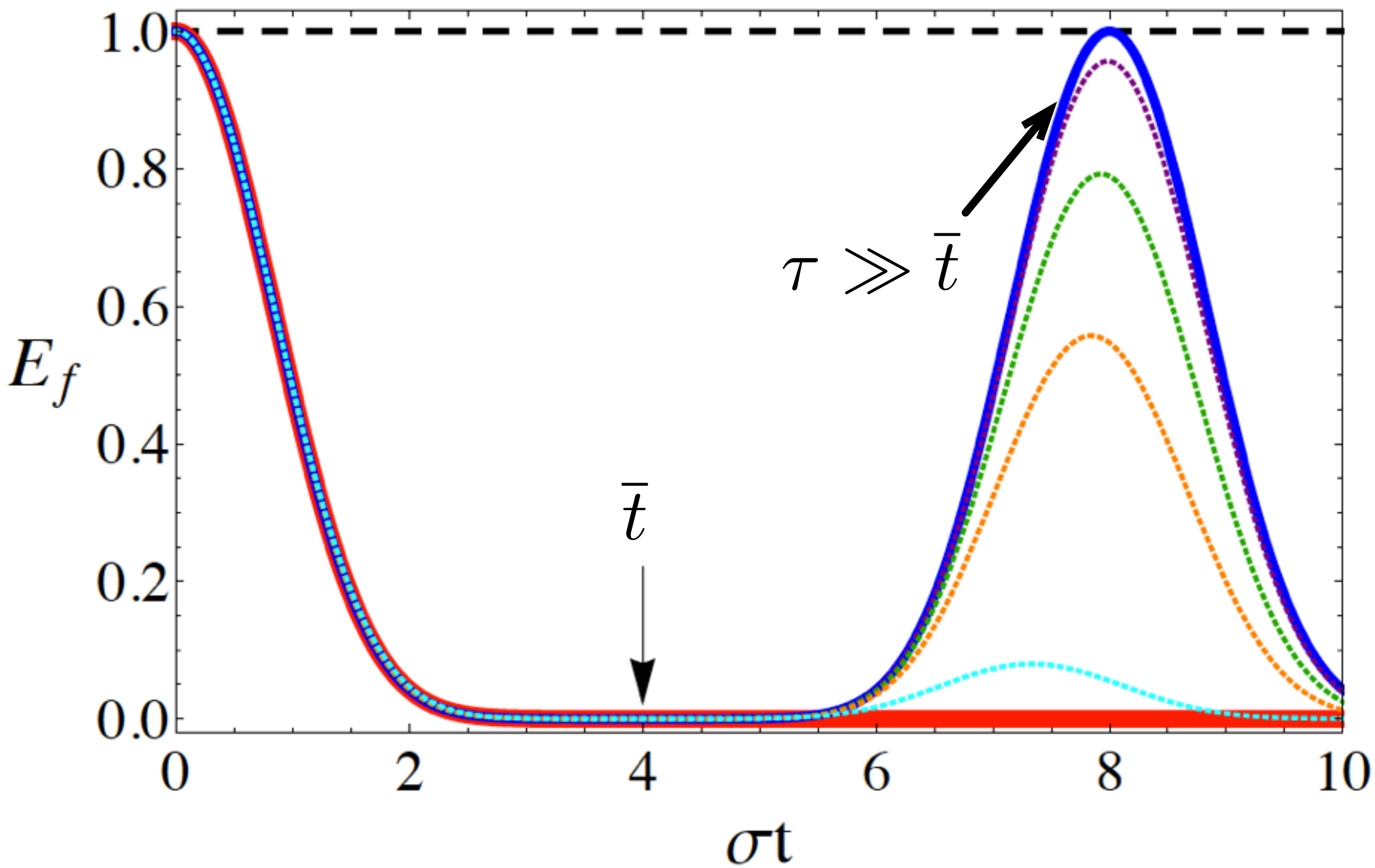
$$E_{av}\big(\mathcal{A}(t)\big) = 1$$

Complete recovery of Entenglement (for static noise)

$$E_F\big(\rho(t \gg 1/\sigma)\big) \simeq 0$$

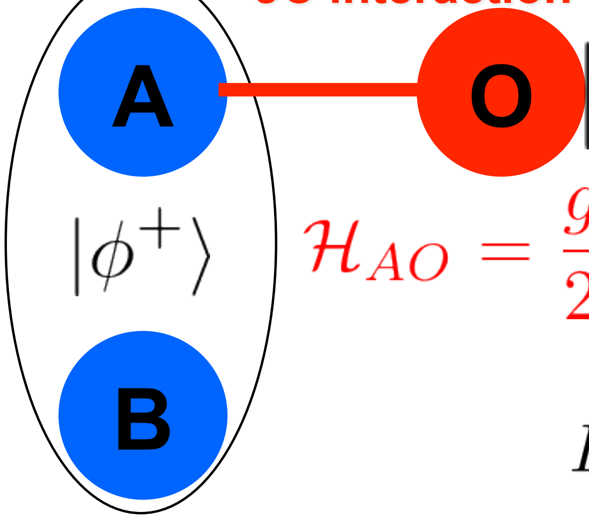
Partial recovery with **finite noise correlation time scale**

$$\langle \varepsilon(t)\varepsilon(0)\rangle = \sigma^2 e^{-|t|/\tau}$$



Non-Markonian dynamics and entanglement revivals

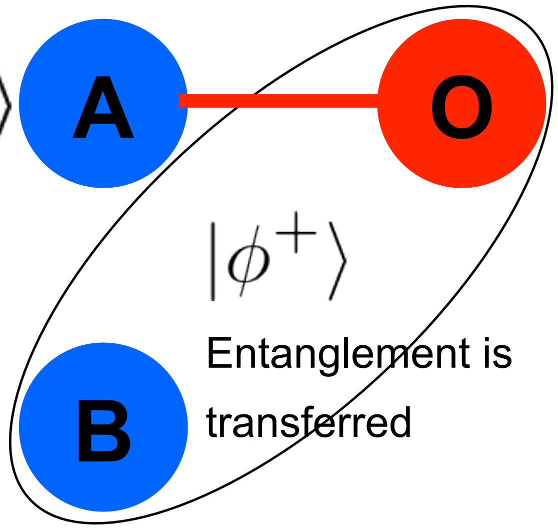
JC interaction with O



$|0\rangle$

$|\phi^+\rangle$

$$\mathcal{H}_{AO} = \frac{g}{2}\left(\sigma_+ a + \sigma_- a^\dagger\right)$$

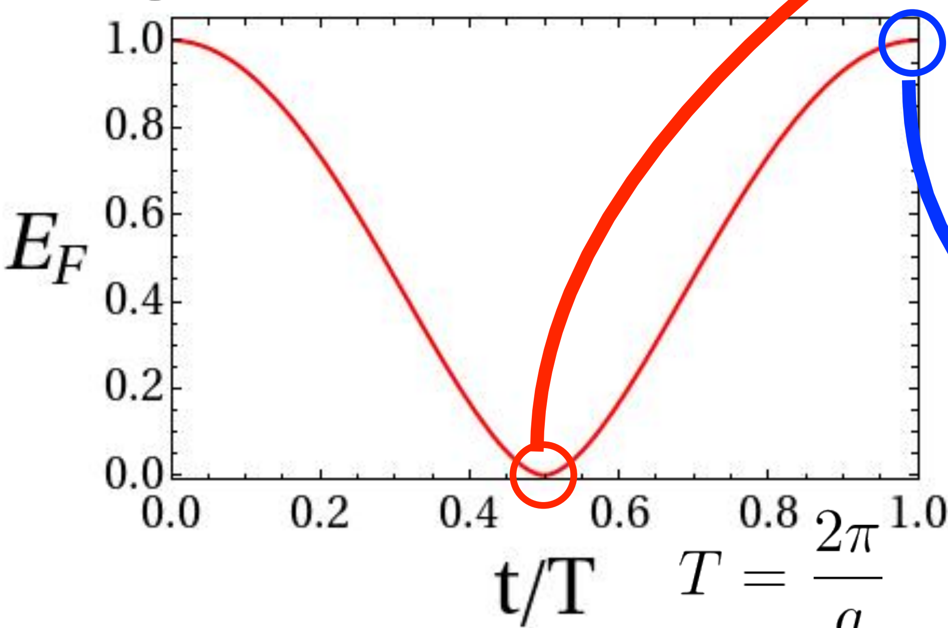


$|0\rangle$

$|\phi^+\rangle$

Entanglement is transferred

$$E_F\big(\rho(T/2)\big) = 0$$



$$T = \frac{2\pi}{g}$$

$$\mathcal{A}(T/2) = \{p_x, |0_A\rangle \otimes |x_B\rangle\} \quad E_{av} = 0$$

$$E_h = E_{av} - E_f = 0$$

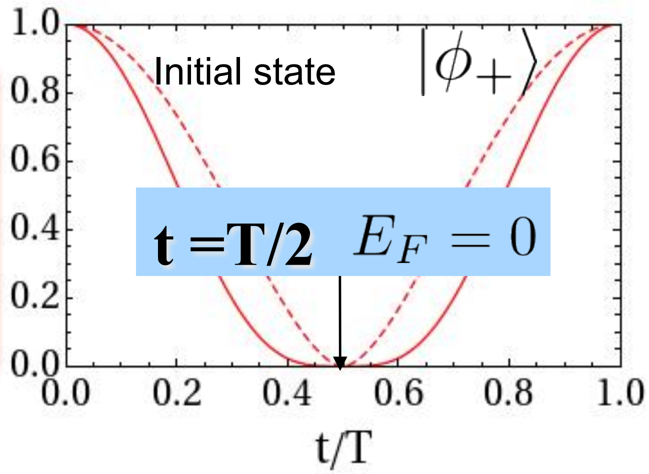
Perfect entanglement back-tranfer from OB to AB

Entanglement revivals in classical vs quantum environments

Random Local Field

$$p_z = \frac{1}{2}, \quad U_z = -i\sigma_z$$

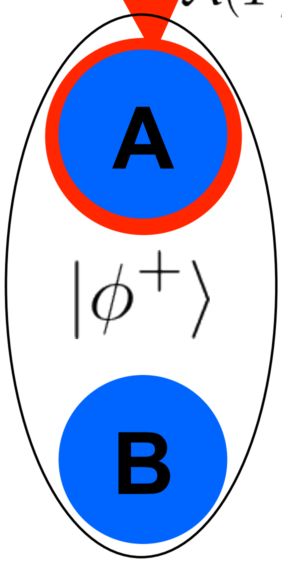
$$p_x = \frac{1}{2}, \quad U_x = -i\sigma_x$$



JC interaction with O

$$\mathcal{H}_{AO} = \frac{g}{2}(\sigma_+ a + \sigma_- a^\dagger)$$

SWAP(A,O)



$$\mathcal{A}(T/2) = \left\{\left(\frac{1}{2}, |\phi_-\rangle\right), \left(\frac{1}{2}, |\psi_+\rangle\right)\right\}$$

$$E_{av} = 1$$

$$E_h = 1$$

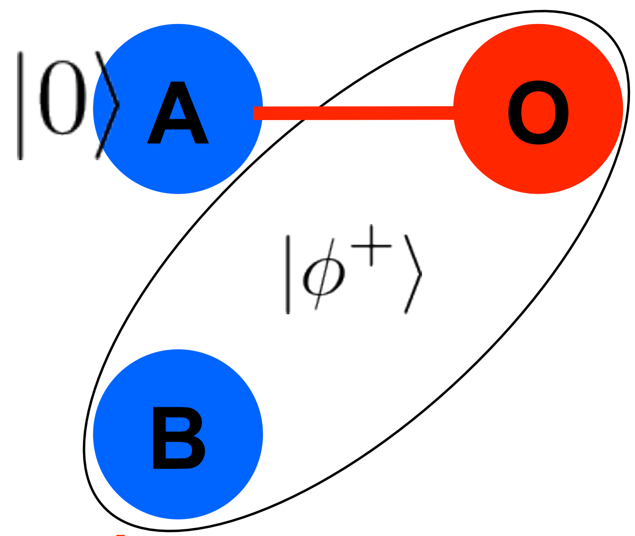
classical information about which random unitaries AB underwent

Recovery!

$$\mathcal{A}(T/2) = \{p_x, |0_A\rangle \otimes |x_B\rangle\}$$

$$E_{av} = 0$$

$$E_h = 0$$



No recovery!

HE in systems with damping

Assume that one qubit of a Bell pair is subject to damping (quantum environment) modeled by a Lindblad master equation

$$\dot{\rho} = -\frac{i}{\hbar}[H, \rho] - \frac{\Gamma}{2}(\sigma_-\sigma_+\rho + \rho\sigma_-\sigma_+) + \Gamma\sigma_+\rho\sigma_-$$

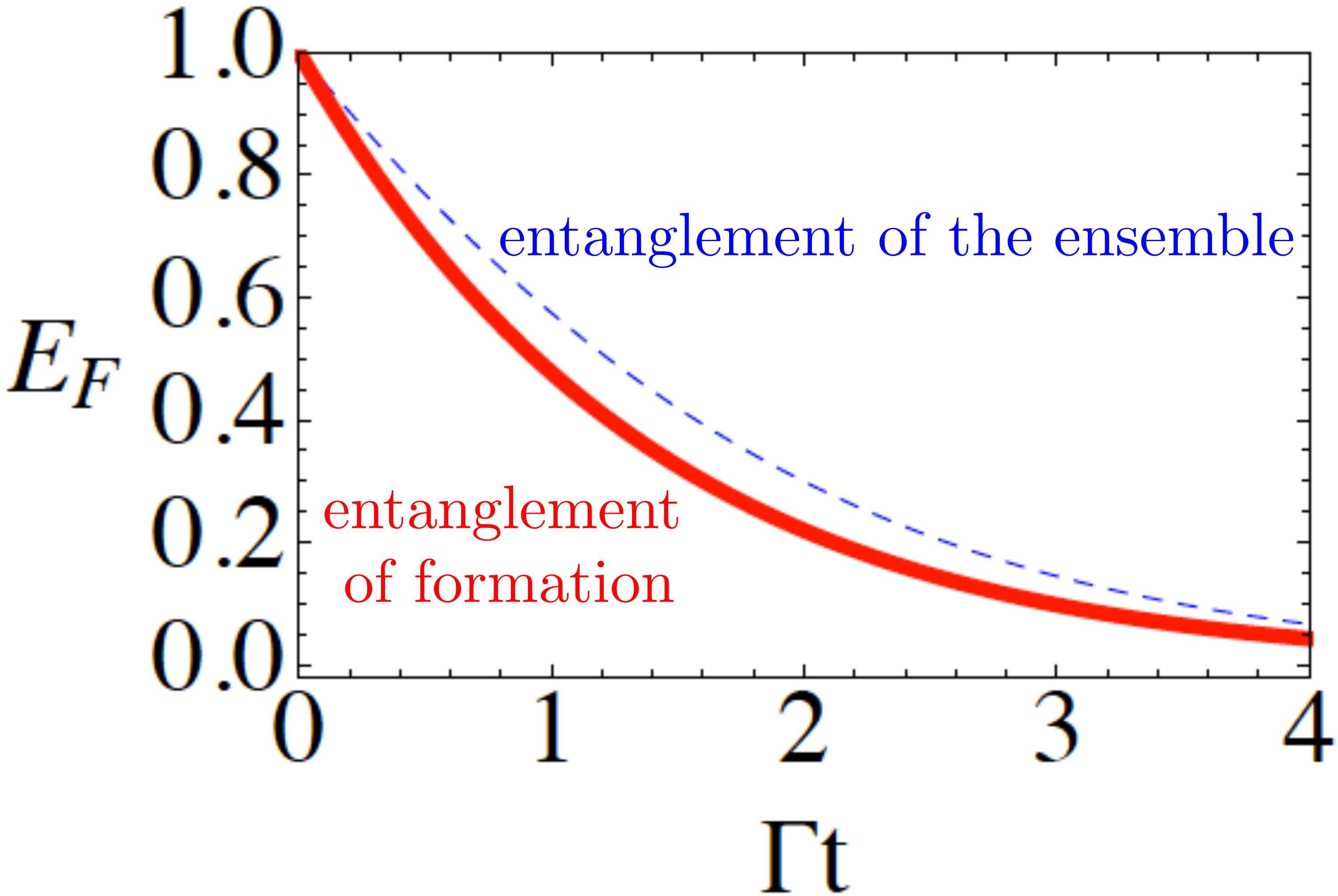
Relevant quantum ensemble when the emitted photons are detected

$$\mathcal{A}(t) = \{(p_0(t), |\varphi_0(t)\rangle), (p_1(t), |\varphi_1(t)\rangle)\}$$

$$|\varphi_0(t)\rangle = |10\rangle,$$

$$|\varphi_1(t)\rangle = \frac{1}{\sqrt{1+e^{-\Gamma t}}}\left(e^{i\frac{\Omega_A+\Omega_B}{2}t}|00\rangle + e^{-i\frac{\Omega_A+\Omega_B}{2}t}e^{-\frac{\Gamma}{2}t}|11\rangle\right)$$

$$p_0(t) = \frac{1}{2}(1-e^{-\Gamma t}), \quad p_1(t) = \frac{1}{2}(1+e^{-\Gamma t})$$



To recover the hidden entanglement it is sufficient to discard the AB systems each time a photon is detected (classical information)

Summary

Defined **“Hidden” Entanglement (HE)** on the basis of the ensemble description of the system dynamics

HE>0: a **recovery** of **entanglement** by **local operation** and **classical communication** is possible, even in realistic cases in which it is not possible to control the environment (HE conceptually different from the entanglement of assistance)

Practical relevance in **solid state quantum computing**: **HE** indicates the amount of entangelment recoverable by local pulses (**dynamical decoupling**: echo, bang-bang....)

Non-Markovian dynamics:
classical vs **quantum** environments