Classical Physics and Blackbody Radiation

Giuliano Benenti

Center for Nonlinear and Complex Systems
Univ. Insubria, Como, Italy

In collaboration with:
Jiao Wang (Xiamen)
Giulio Casati (Como)

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OUTLINE

Why to re-examine a very old problem?

A dynamical system's approach: an infinite dimensional system, what is ergodicity there?

Question: following the Newton-Maxwell equations of motion will equipartition (and consequently the ultraviolet catastrophe) be achieved eventually?

(i) *Quasi-stationary state,*
(ii) *Consistency with Stefan-Boltzmann law,*
(iii) *High-frequency cutoff (no ultraviolet catastrophe)*

The ultraviolet catastrophe

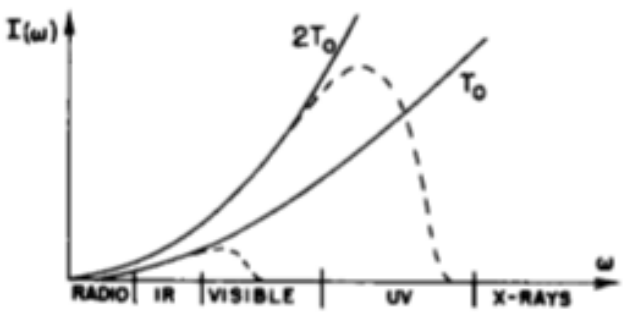


Fig. 41–4. The blackbody intensity distribution at two temperatures, according to classical physics (solid curves). The dashed curves show the actual distribution.

The amount of intensity that there is in our box, per unit frequency range, goes, as we see, as the square of the frequency, which means that if we have a box at any temperature at all, and if we look at the x-rays that are coming out, there will be a lot of them!

Of course we know this is false. When we open the furnace and take a look at it, we do not burn our eyes out from x-rays at all. It is completely false. Furthermore, the *total energy* in the box, the total of all this intensity summed over all frequencies, would be the area under this infinite curve. Therefore, something is fundamentally, powerfully, and absolutely wrong.

Thus was the classical theory *absolutely incapable* of correctly describing the distribution of light from a blackbody, just as it was incapable of correctly describing the specific heats of gases. Physicists went back and forth over this derivation from many different points of view, and there is no escape. This *is* the prediction of classical physics. Equation (41.13) is called *Rayleigh's law*, and it is the prediction of classical physics, and is obviously absurd.

$$I(\omega) = \frac{\omega^2 kT}{\pi^2 c^2}$$

Rayleigh-Jeans law
(from the equipartition theorem, average energy per mode kT)

[Feynman's Lectures on Physics, Vol. I, Ch. 41]

Planck's formula and the development of quantum theory

$$B(\nu, T) = \frac{2h\nu^3}{c^2} \frac{1}{\exp\left(\frac{h\nu}{k_B T}\right) - 1}$$

(quantized modes)

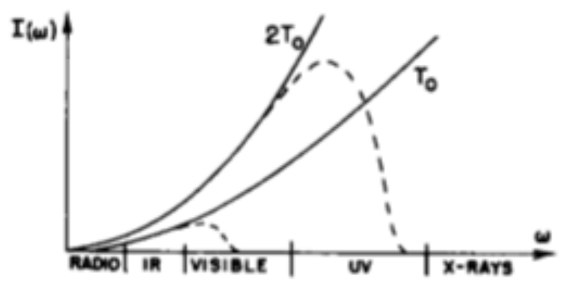


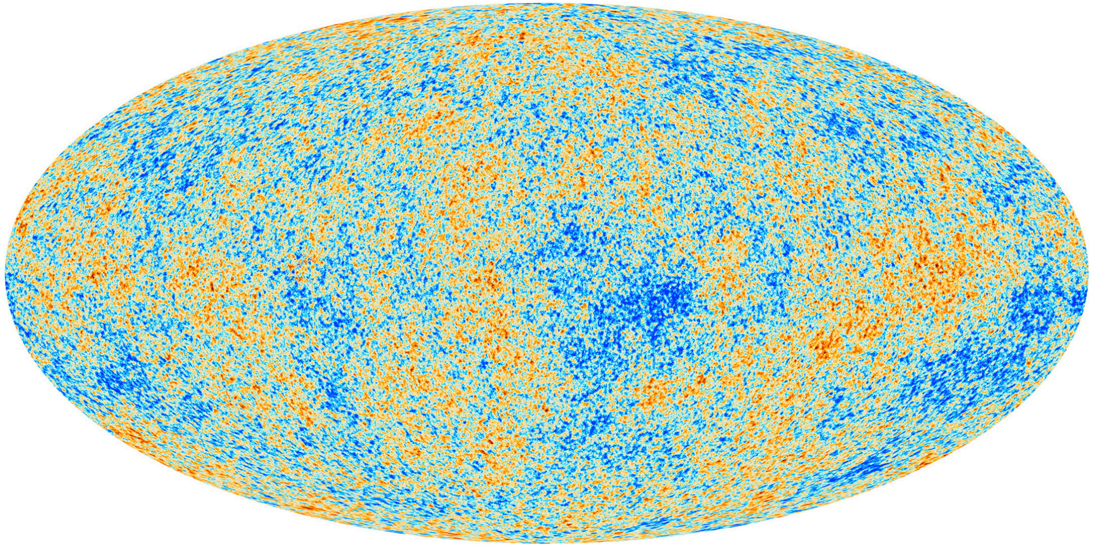
Fig. 41–4. The blackbody intensity distribution at two temperatures, according to classical physics (solid curves). The dashed curves show the actual distribution.

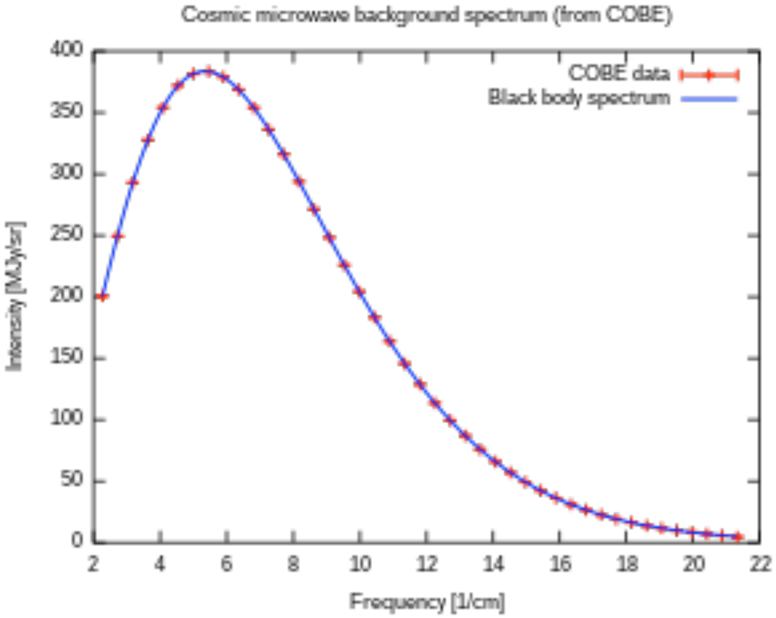
It was unfortunate that the first problem dealt with in the systematic development of quantum theory was that of the energy quantization of harmonic electromagnetic vibrations. There can be no doubt that much intellectual effort could have been saved on the part of the early quantum theoretists—as well as on the part of the reader of the present text—if a conceptually less involved issue had initiated the development of the theory.

This raises the question: Was it possible that other problems at issue at the time, if worked out consistently, could have brought about the same conceptual reorientation as did the problem of black-body radiation? And perhaps in a logically less complicated way? Consider, for example, the well-known irreconcilability with classical physics of the specific heat of solids at low temperatures, a problem whose solution was obtained *de facto* in terms of concepts formed in solving the black-body problem. One may conjecture how an independent and consistent solution of this problem on specific heat would have influenced the progress of theoretical physics. It seems highly probable that in this hypothetical case energy quantization of material systems (atoms or molecules) would have preceded that of waves and the approach to quantum theory would have been conceptually less difficult.

[Max Jammer, *The conceptual development of quantum mechanics*]

Ubiquitous distribution



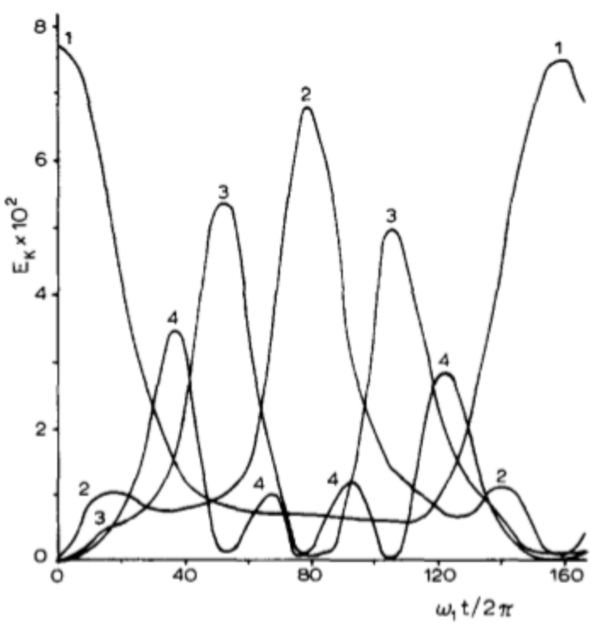


https://www.itp.uni-hannover.de/fileadmin/itp/emeritus/zawischa/static_html/blackbody.html

The Fermi-Pasta-Ulam-Tsingou model

$$H = \sum_{k=1}^{N-1} \tfrac{1}{2} P_k^2 + \frac{1}{2} \sum_{k=0}^{N-1} (Q_{k+1} - Q_k)^2 + \frac{\alpha}{3} \sum_{k=0}^{N-1} (Q_{k+1} - Q_k)^3 ,$$

Integrated numerically the equations of motion of a nonlinear chain. Equipartition of energy between the harmonic normal modes?



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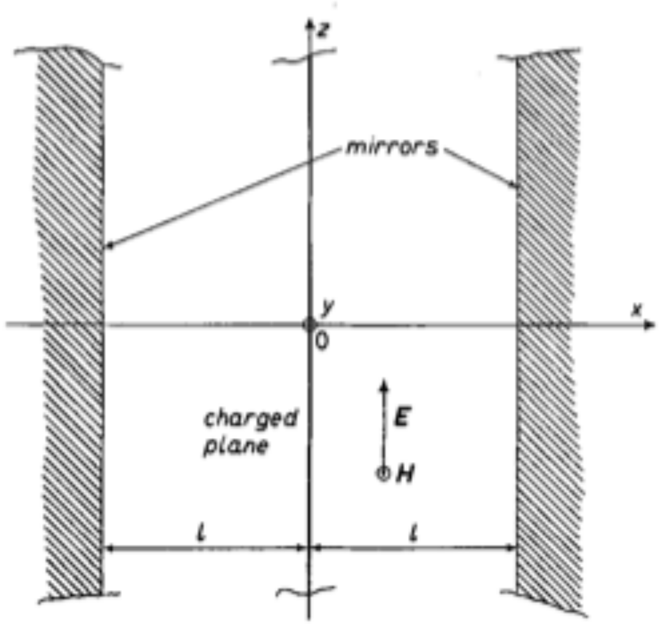
PHYSICS REPORTS

THE FERMI–PASTA–ULAM PROBLEM: PARADOX TURNS DISCOVERY*

Joseph FORD

School of Physics, Georgia Institute of Technology, Atlanta, GA 30332, USA

A dynamical model of a classical radiant cavity



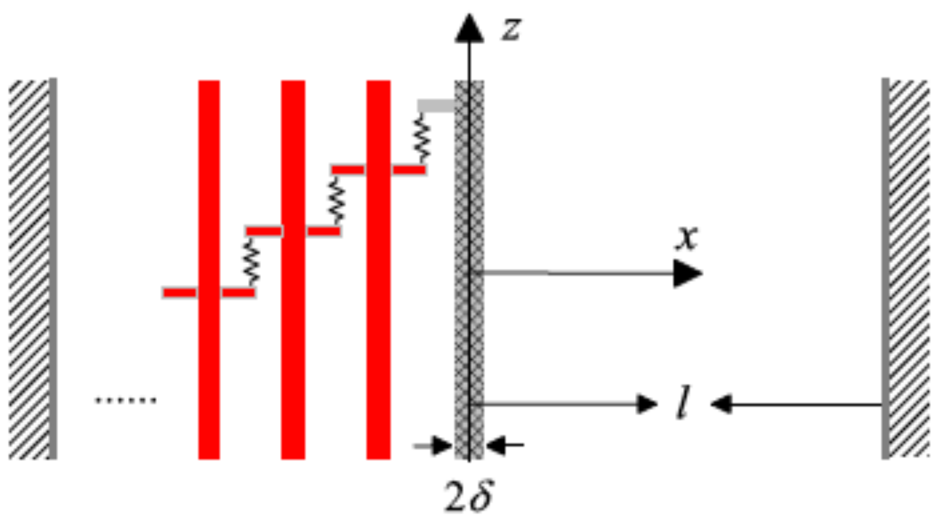
$$\begin{cases} \Box A_x = \Box A_y = 0\,, \\ \Box A_z = -\mu_0 \sigma \delta(x)\, \dot{z}\,, \\ \nabla \cdot \boldsymbol{A} = 0\,, \\ \nabla^2 \Phi = \dfrac{\sigma}{\varepsilon_0}\, \delta(x)\,, \end{cases}$$

[Bocchieri, Crotti, Loinger, Lett. Nuovo Cimento **4**, 741 (1972)]

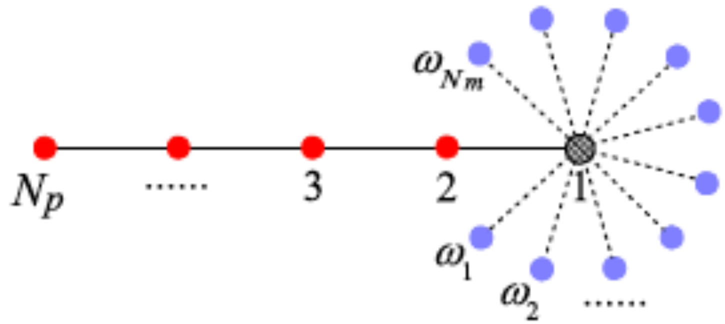
Linear model solvable, no equipartition.
What about the case with nonlinearity added to matter?

[some Refs.: Casati, Guarneri, Valz Gris, Phys. Rev. A **16**, 1237 (1977), Benettin and Galgani, J. Stat. Phys **27**, 153 (1982), Livi, Pettini. Ruffo, Vulpiani, J. Phys. A **20**, 577 (1987), Alabiso, Casartelli, Sello, J. Stat. Phys. **54**, 361 (1989), GB, Casati, Guarneri, EPL **46**, 307 (1999)]

Our model



Schematic drawing



Topology

The Hamiltonian

$$H = \sum_{j=2}^{N_p} \left[\frac{P_j^2}{2m} + \tilde{V}(z_j) + V(z_{j-1}, z_j) \right] + \tilde{V}(z_1)$$

$$+ \frac{1}{2m} \left(P_1 - \varepsilon \sum_{k=1}^{\infty} a_k q_k \right)^2 + \frac{1}{2} \sum_{k=1}^{\infty} (p_k^2 + \omega_k^2 q_k^2)$$

ϕ^4 model $\quad \tilde{V}(z_j) = \frac{1}{4}\gamma z_j^4, \qquad V(z_{j-1}, z_j) = \frac{1}{2}\kappa(z_j - z_{j-1})^2$

(for the considered energies, chaotic for Np>4 , with Np positive Lyapunov exponents)

$\varepsilon = 2\sigma\sqrt{\pi/l}$ matter-field coupling parameter

$a_k = \int_{-\delta}^{\delta} f(x) \cos(\omega_k x/c) dx \qquad \omega_k = (\pi c/2l)(2k-1)$

f(x) normalized (transverse) charge distribution

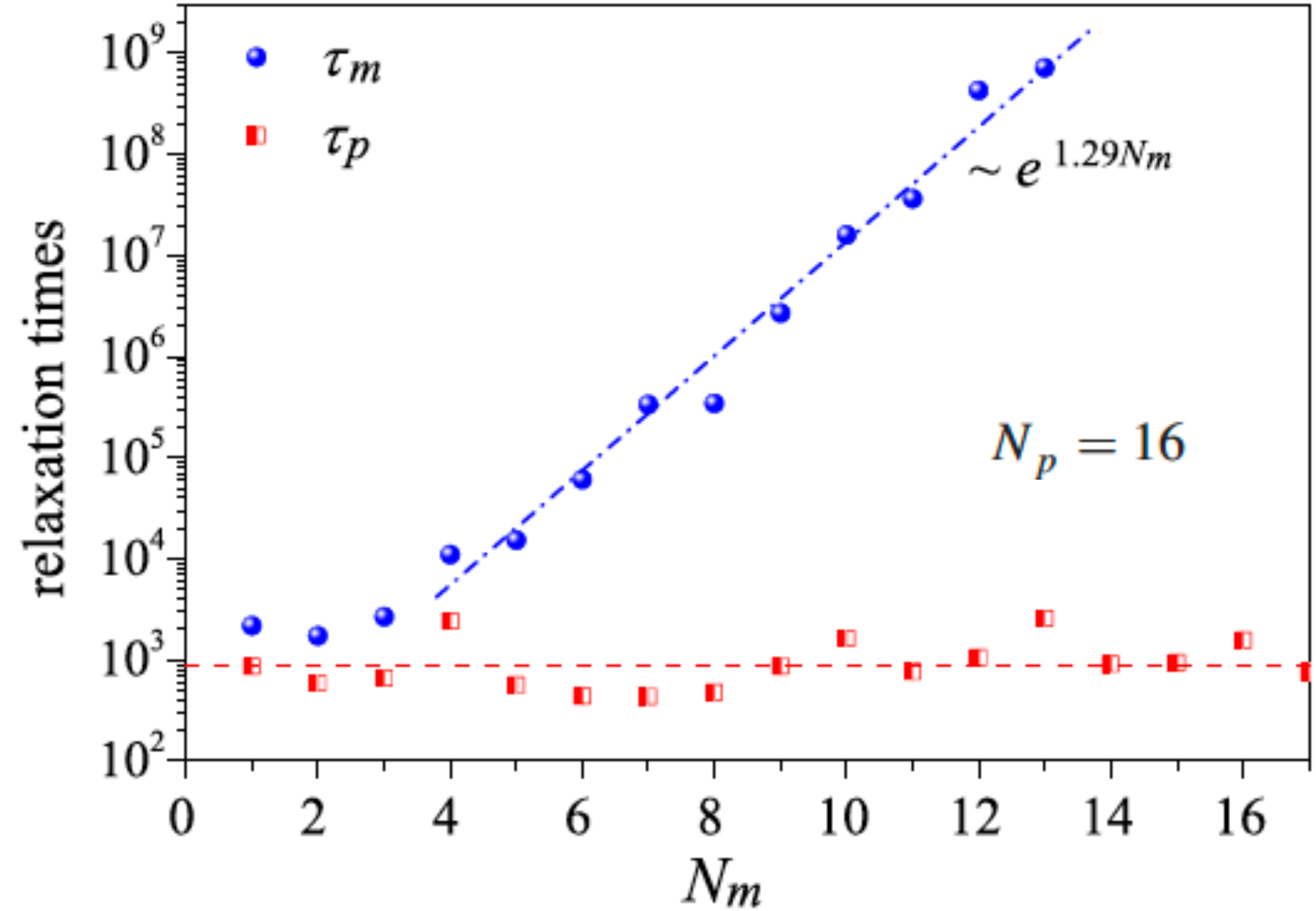
Essential features of the model (and of field-matter interaction)

The field modes can exchange energy only via interaction with matter

The electromagnetic field in the cavity is linear and nonlinearity is provided by matter

Frequencies are not upper bounded for the field

Relaxation times

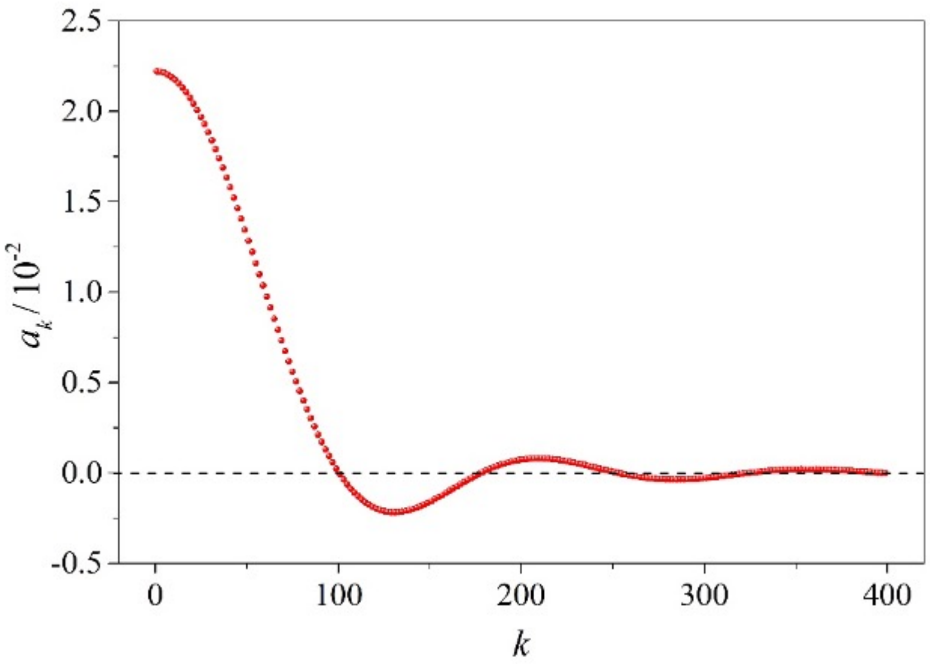


$$S_p(t) = \sum_{j=1}^{N_p} \overline{E}_{p,j}(t) \ln \overline{E}_{p,j}(t) \qquad S_m(t) = \sum_{k=1}^{N_m} \overline{E}_{m,k}(t) \ln \overline{E}_{m,k}(t)$$

Equipartition time when these quantities reach 90% of Np or of Nm

Charge distribution

$$f(x) = A\exp(\delta^2/(x^2-\delta^2))\ (|x|<\delta)$$



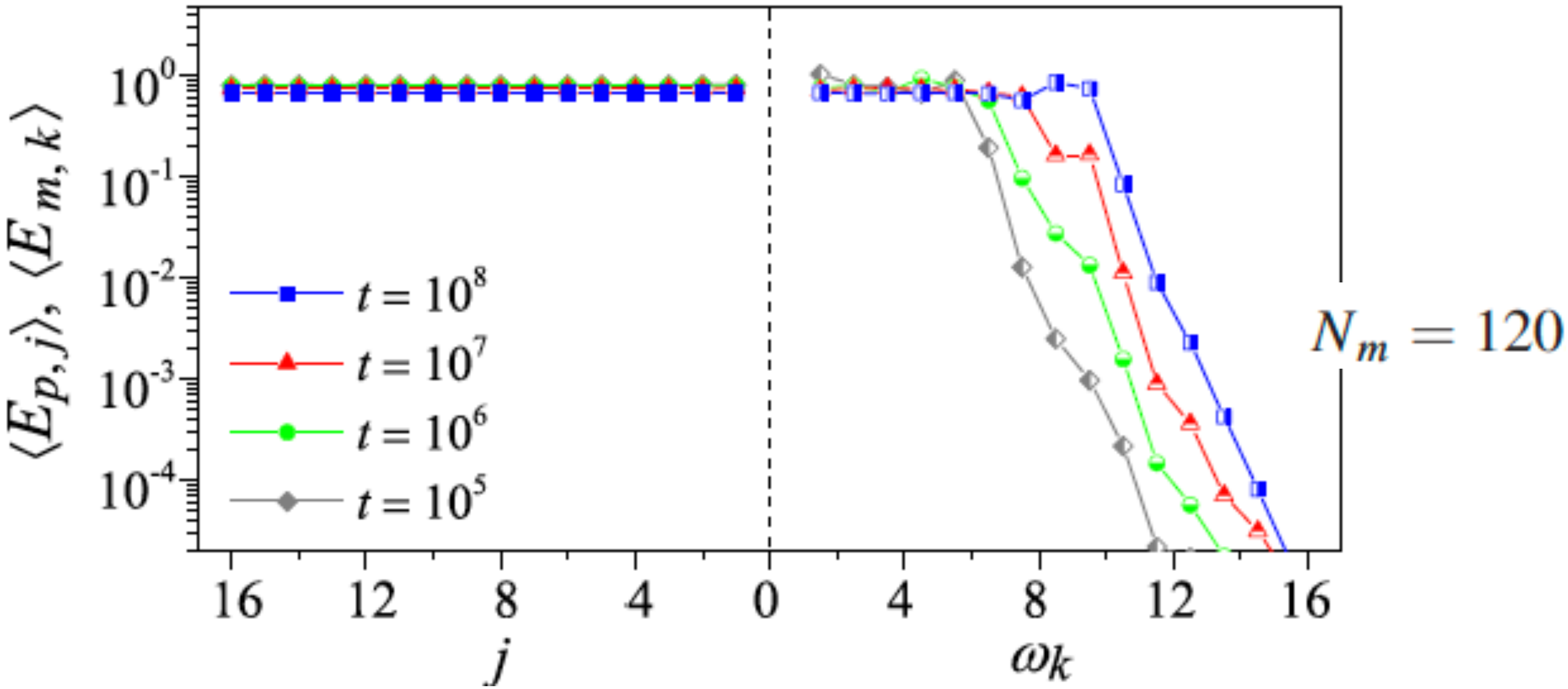
$$a_k = \int_{-\delta}^{\delta} f(x)\cos\frac{\omega_k x}{c}\,dx$$

This choice is not crucial: very similar results obtained also for other distribution function, also for the

$$f_{delta}(x) = \delta(x), \qquad a_k = 1$$

Quasi-stationary state

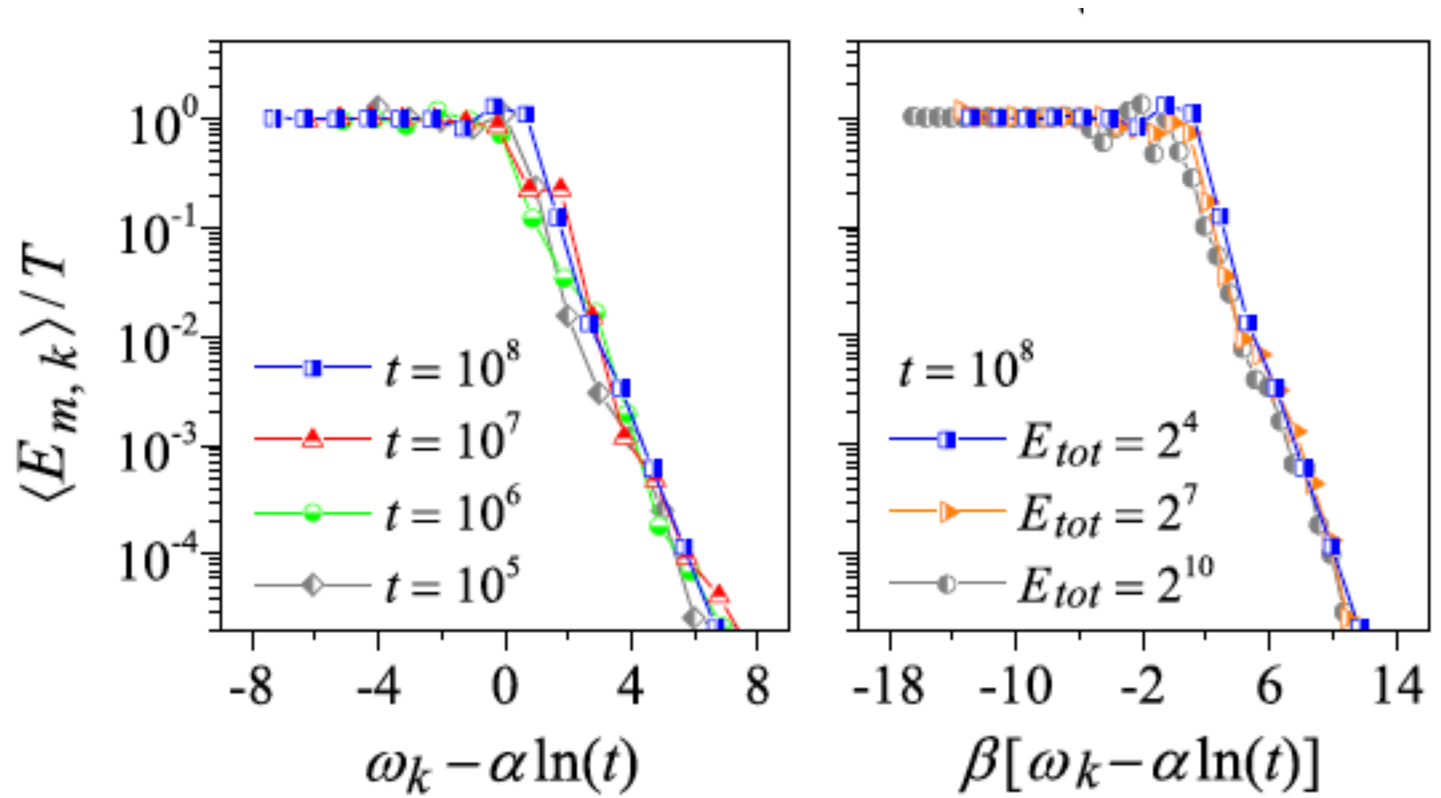
Plateau followed by exponential decay



Field modes in the plateau thermalised with matter

Logarithmically slow evolution of the distribution
(no ultraviolet catastrophe)

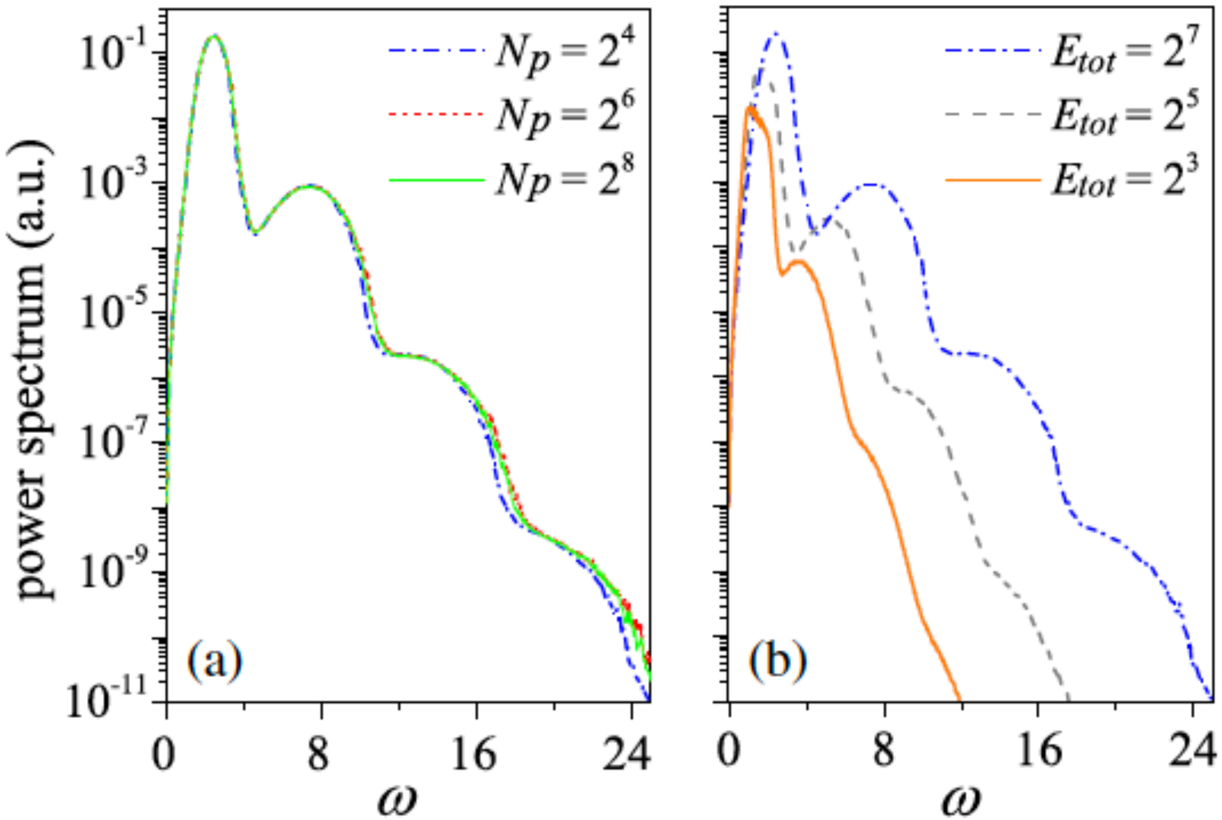
Scaling properties



$\alpha = 0.23 E_{\mathrm{tot}}^{0.24}$ $\qquad$ $\beta = 3.22 E_{\mathrm{tot}}^{-0.22}$

Power spectrum of the charged plate

Why an exponential tail? The power spectrum of the charged plate has a finite frequency band, followed by an exponential tail.



The thermalization time decreases increasing total energy

Thermodynamics of the radiant cavity

Most of the energy in the thermalized plateau: It it possible to scribe the quasi-stationary state by equilibrium thermodynamics?

Stefan-Boltzmann law (in 1D)?

Fundamental thermodynamic relation $dU = TdS - pdV$

Use Maxwell's relations $\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial P}{\partial T}\right)_V$

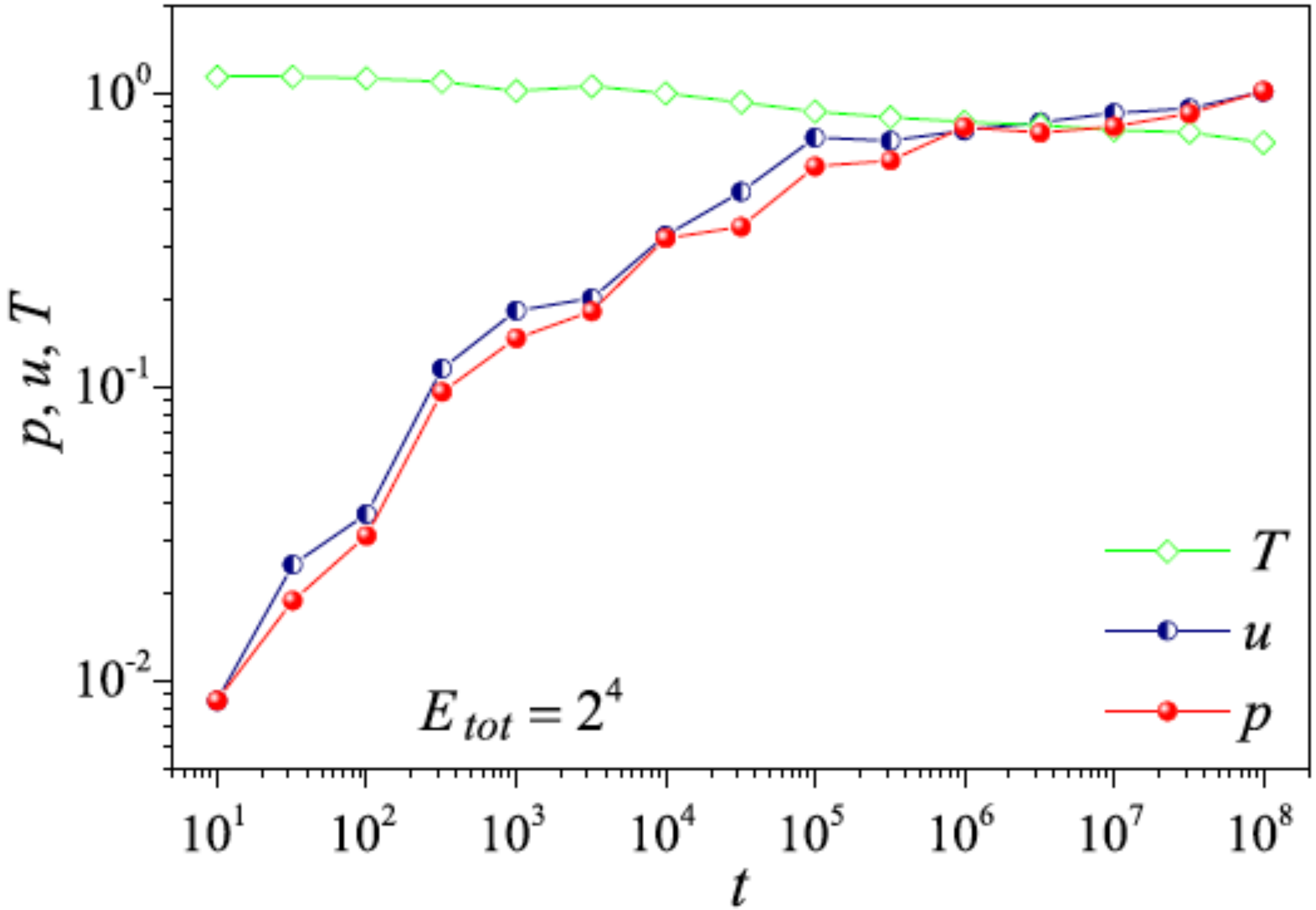
$$\left.\frac{\partial U}{\partial V}\right|_T = -p + T\left.\frac{\partial p}{\partial T}\right|_V$$

Radiation pressure (1D): $p = u, \quad u = U/V$

Stefan-Boltzmann: $u(T) = CT^2$

In contradiction with Rayleigh-Jeans law $E \propto T$

Computing thermodynamic quantities

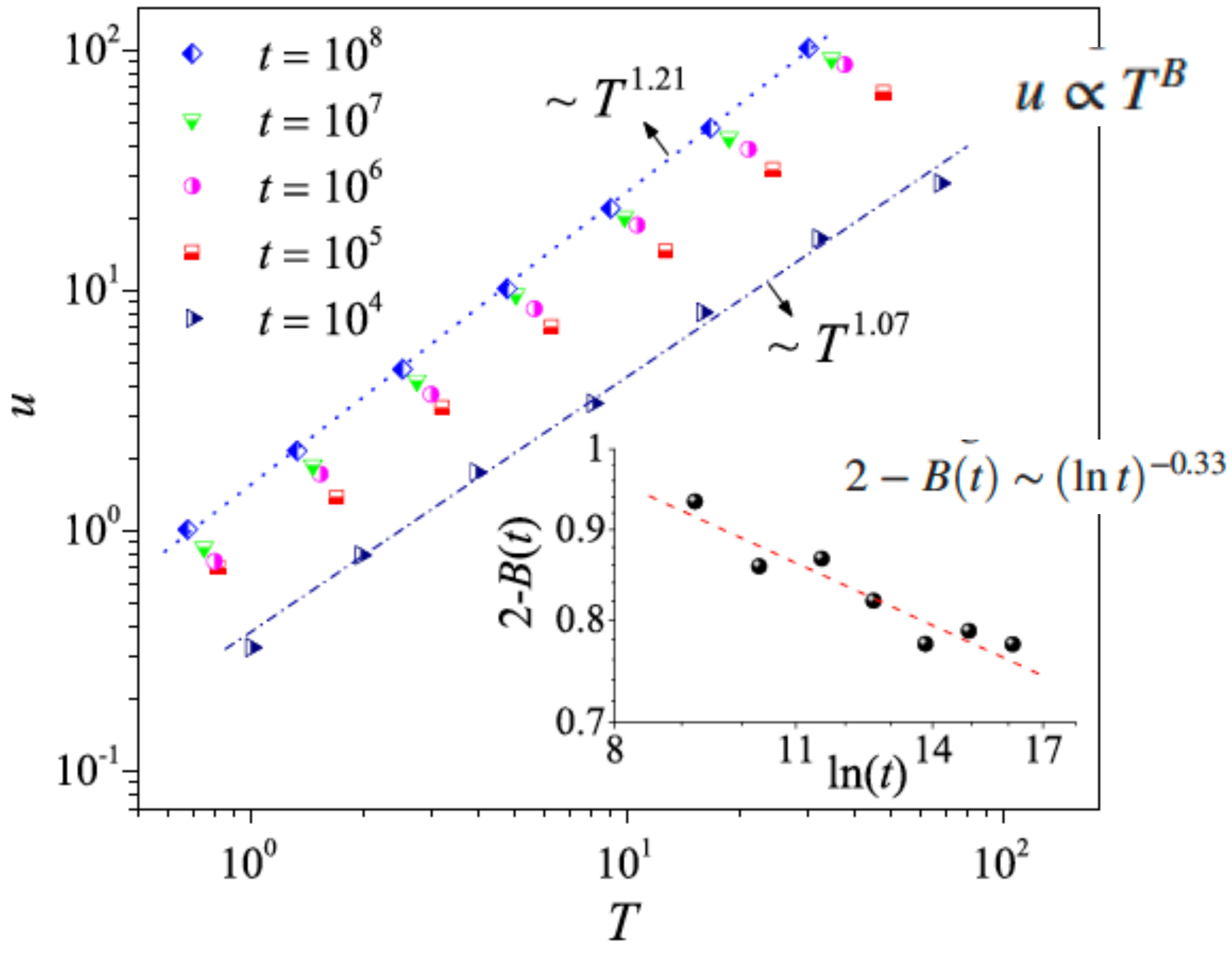


$T = \langle E_{p,j} \rangle$ temperature of matter

$u = U/V$ energy density of the field

To compute pressure perturb the distance between the mirrors and compute the corresponding energy change

Approaching the Stefan-Boltzmann law



Outlook

Thermalization fast within the bandwidth of mechanical motion and slow outside should be a generic feature of classical mechanics of field-matter interaction

Extension to the three-dimensional case would be interesting, to check thermodynamic consistency also in that case. Stefan-Boltzmann law $E \propto T^4$, Wien's displacement law

What about the quantum case? Lack of ergodicity?