# Inverse currents in coupled transport

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Phys. Rev. E **106**, 044104 (2022)

# Outline

*Coupled flows (e.g., charge and heat): a dynamical system's perspective on a fundamental problem of statistical physics (with practical interest: thermoelectricity,…)*

*Counterintuitive transport phenomena:* negative differential conductivity, *Absolute Negative Mobility (ANM)* (current against a bias)

*Inverse Coupled Currents* (ICC) around a thermal equilibrium state in coupled transport

*Autonomous circular heat engine* based on ICC

# Absolute negative mobility

ANM: permanent average motion against a static force, as illustrated by the donkey, moving in the direction opposite to the one which is required to it

Impossible at thermal equilibrium, a single heat bath would perform work against the force (perpetuum mobile)

Investigated in non-equilibrium setups

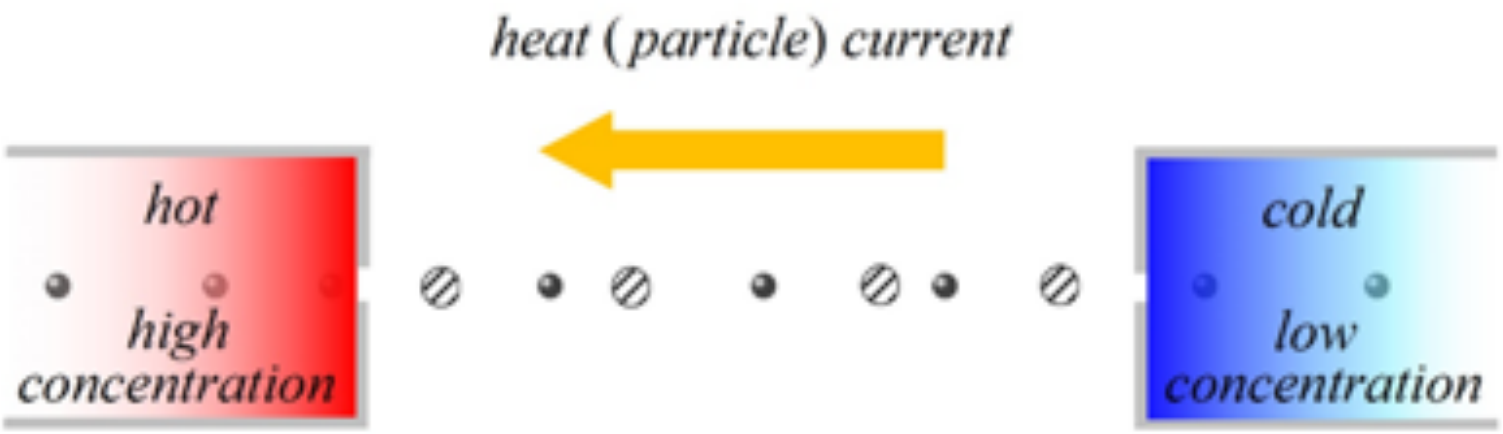
[Cleuren, Van den Broeck, EPL 54, 1 (2001); Eichorn, Reimann, Hanggi, PRL 88, 190601 (2002); Nagel et al., PRL 100, 217001 (2008),…]

ANM in equilibrium, for stochastic dynamics of a tracer particle subject to two driving forces

[Cividini, Mukamel, Posch, J. Phys. A **51**, 085001 (2018)]

# Inverse Currents in Coupled transport (ICC) not forbidden by thermodynamics

For coupled flows it is allowed by thermodynamics to have a current opposite to both thermodynamic forces




Entropy production rate

$$\dot{S} = J_1\mathcal{F}_1 + J_2\mathcal{F}_2$$

$$\mathcal{F}_i > 0 \quad (i = 1,\ 2)$$

one current can be negative,
with overall positive entropy production

# Classical version of Lieb-Liniger model

$$H = \sum_i \frac{p_i^2}{2m_i} + \sum_{i<j} V(x_i - x_j)$$

$$V(x) = h \ \text{ for } \ x \leq |r|$$

$$V(x) = 0 \ \text{ otherwise}$$

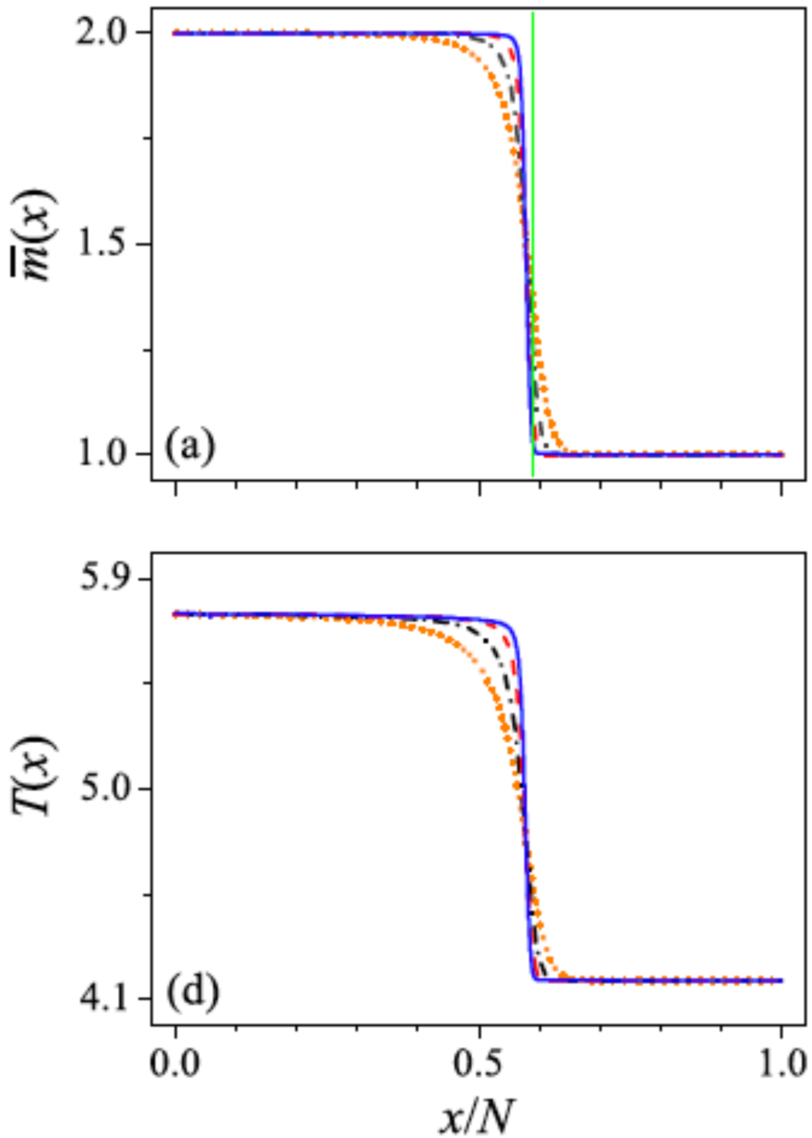
limit case $r \to 0$

Diatomic gas of particles $\quad m_i \in \{\mathcal{M}_1, \mathcal{M}_2\}$

Easier for two colliding particle to overcome when the light particle come from the hot end (higher relative velocity)

# Self-organisation (phase separation) in the far from equilibrium regime

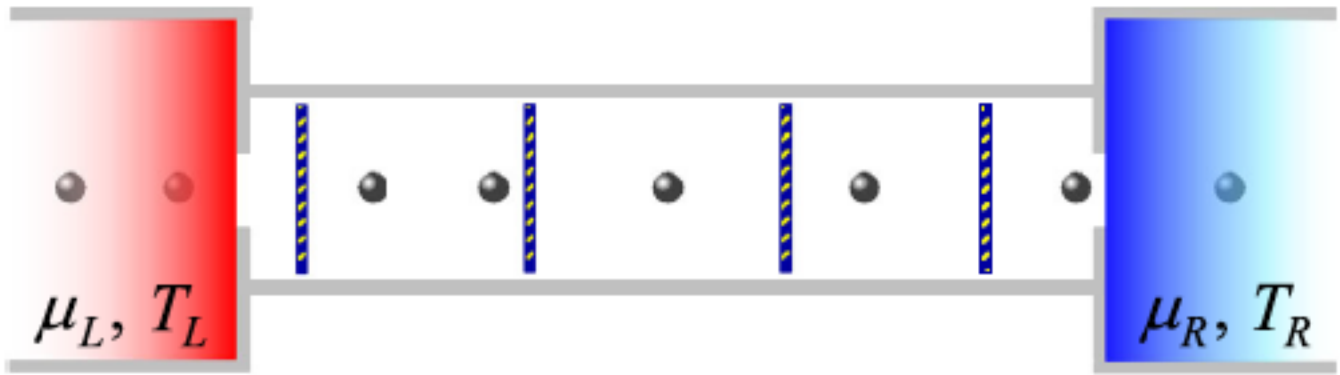
(in a 1D Hamiltonian system)




Strong temperature difference between reservoirs

[J. Wang, G. Casati, PRL **118**, 040601 (2017)]

# Coupled transport in a diatomic gas




Visualized as "bullet" and "rod" particles: rods exchange energy with reservoirs, bullets both energy and particles

Injection rates and energy distribution of injected particles determined by temperature and (electro)chemical potential

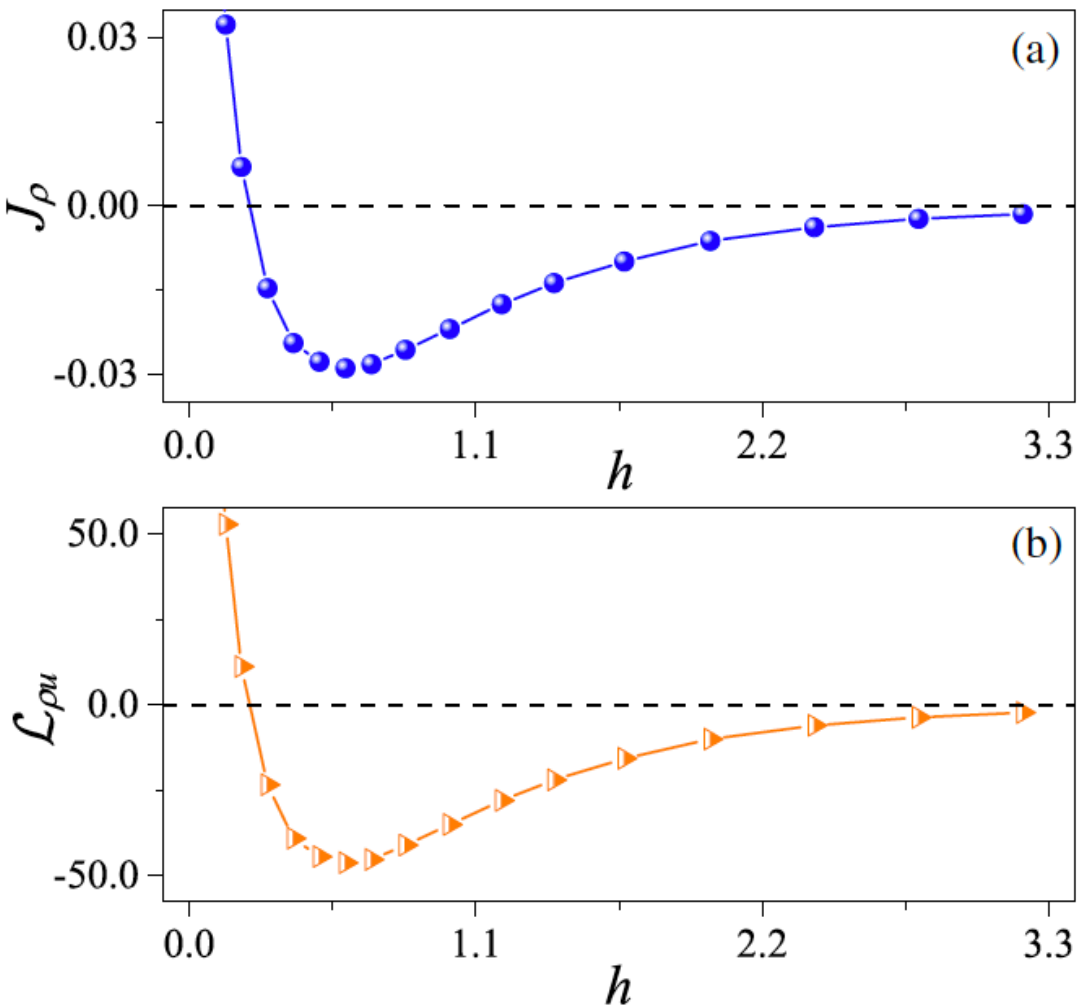
[J. Wang, G. Casati, GB, PRL **124**, 110607 (2020)]

# Negative cross-coefficient (Seebeck)

$$\begin{pmatrix} J_\rho \\ J_u \end{pmatrix} = \begin{pmatrix} \mathcal{L}_{\rho\rho} & \mathcal{L}_{\rho u} \\ \mathcal{L}_{u\rho} & \mathcal{L}_{uu} \end{pmatrix} \begin{pmatrix} \mathcal{F}_\rho / L \\ \mathcal{F}_u / L \end{pmatrix}$$

$$\mathcal{F}_\rho = \mu_L \beta_L - \mu_R \beta_R$$

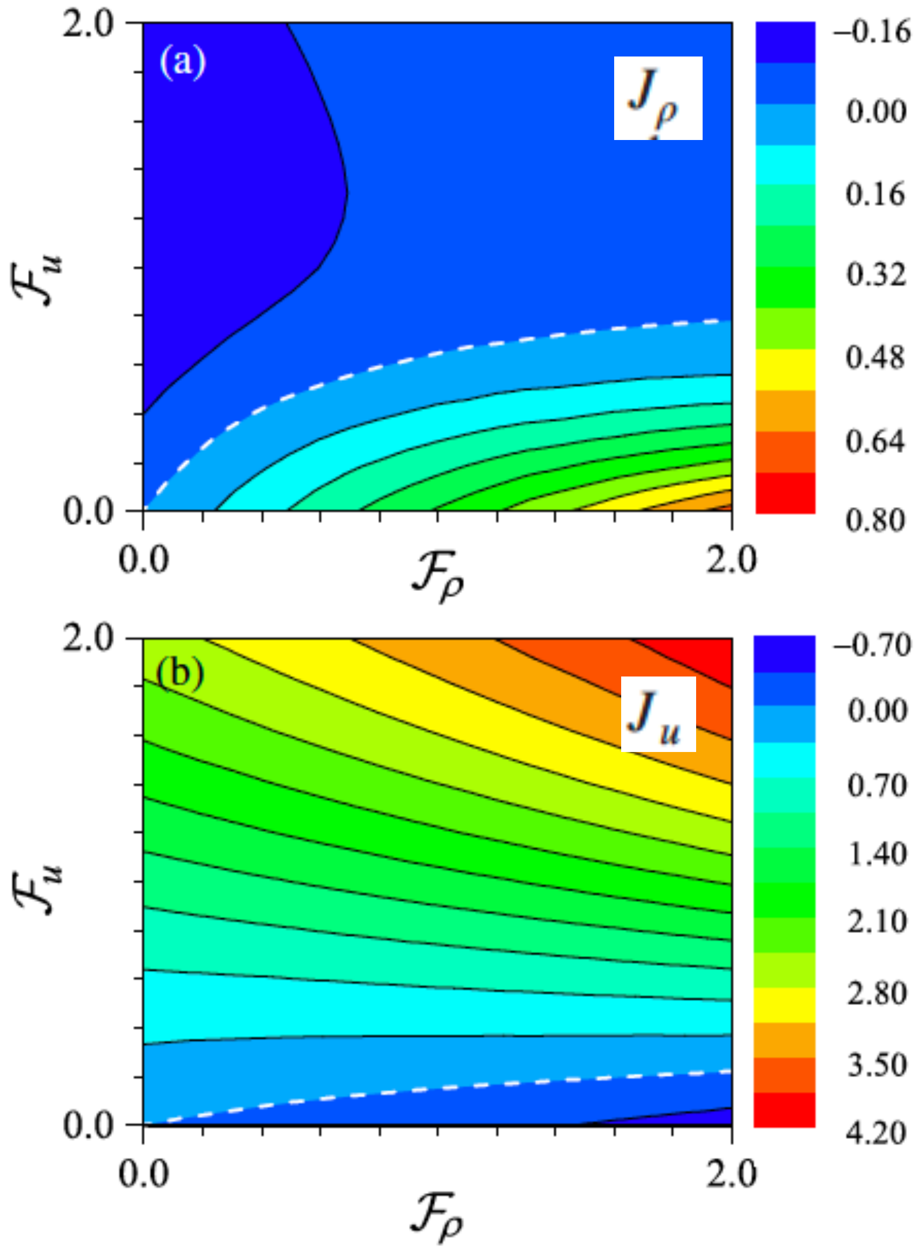
$$\mathcal{F}_u = \beta_R - \beta_L$$




[J. Wang, G. Casati, GB, PRL **124**, 110607 (2020)]

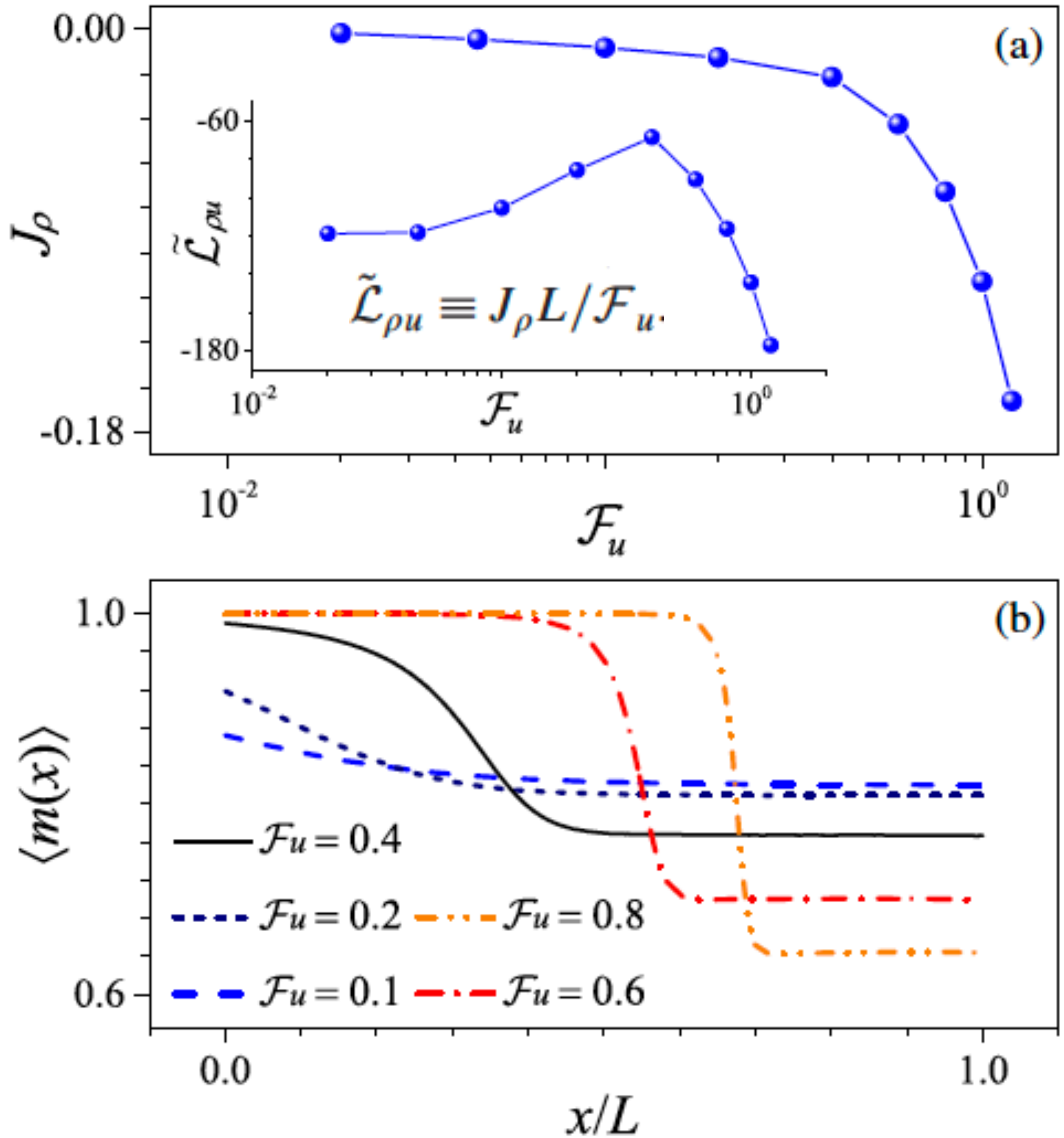
$\mathcal{F}_\rho = 0$ and $\mathcal{F}_u = 0.1$

# Inverse currents in coupled transport (ICC)

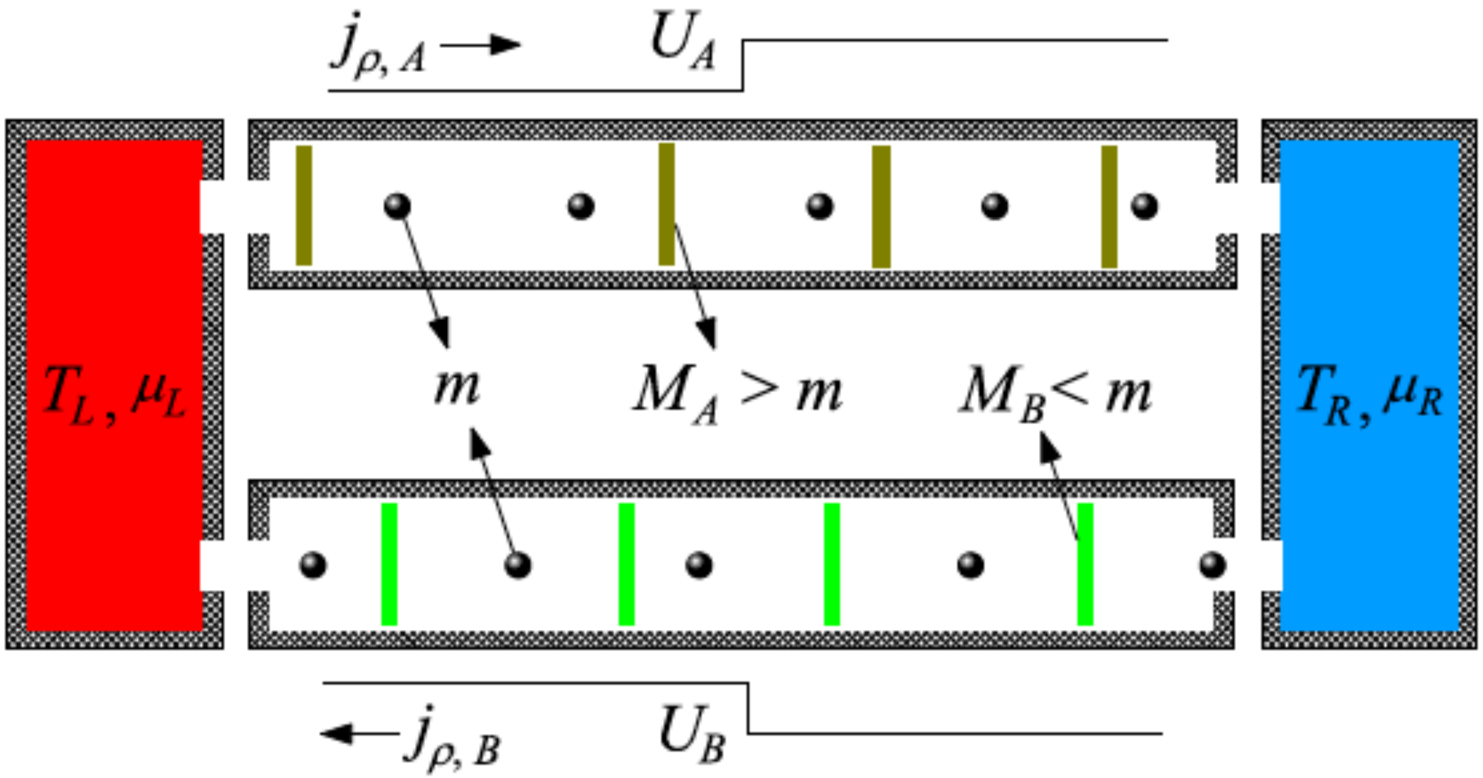



ICC already exists in the linear response regime and is enhanced in the far-from-equilibrium regime with phase separation

# ICC enhanced by phase separation



# Autonomous circular heat engine based on ICC




Chemical potentials in (large but finite) reservoirs autonomously adjust ⇒ Steady-state particle current sustained by temperature difference, also against bias $U_A$, $U_B$

[GB, G. Casati, F. Marchesoni, J. Wang, PRE **106**, 044104 (2022)]

# Linear response analysis

Maximum efficiency and efficiency at maximum power monotonously growing functions of the dimensionless "figure of merit" *YT*

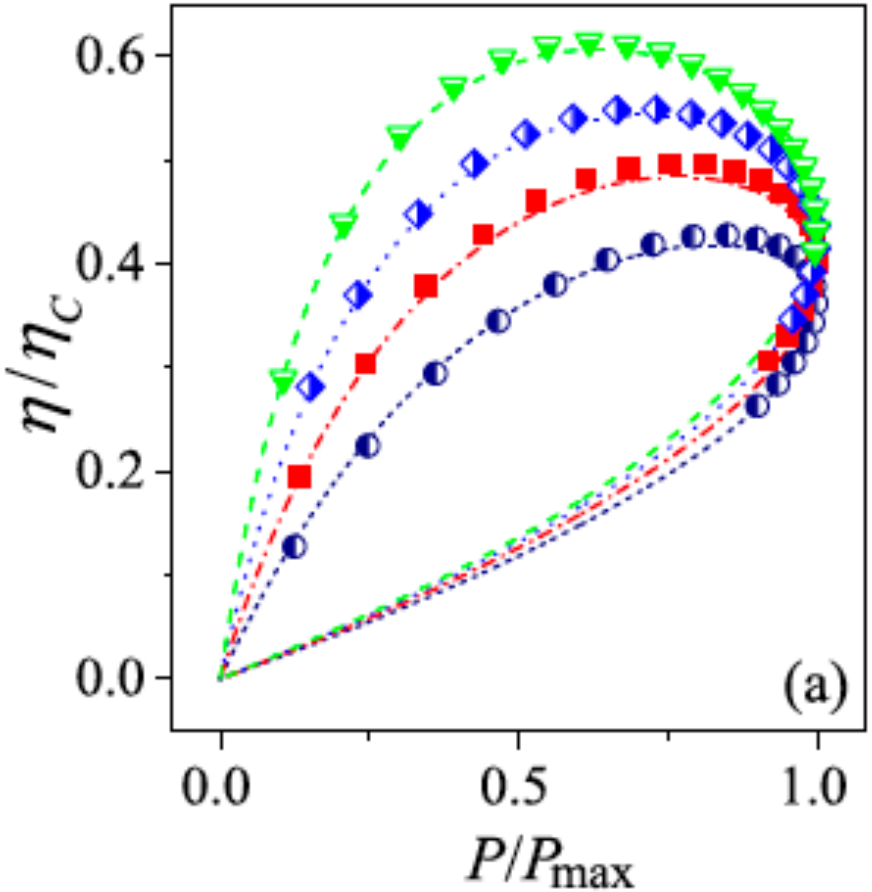
$$YT = \frac{(\sigma_A/L_A)(\sigma_B/L_B)(S_A - S_B)^2}{(\sigma_A/L_A + \sigma_B/L_B)(\kappa_A/L_A + \kappa_B/L_B)} T$$

$$\eta_{\max} = \eta_C \frac{\sqrt{YT+1}-1}{\sqrt{YT+1}+1}, \quad \eta(P_{\max}) = \frac{\eta_C}{2} \frac{YT}{YT+2},$$

$$P_{\max} = \frac{1}{4} \frac{\sigma_A \sigma_B}{\sigma_A L_B + \sigma_B L_A} (S_A - S_B)^2 (\Delta T)^2$$

# Power-efficiency trade-off

$$\frac{\eta}{\eta_C} = \frac{P/P_{\max}}{2[1 + 2/(YT) \mp \sqrt{1 - P/P_{\max}}\,]}$$

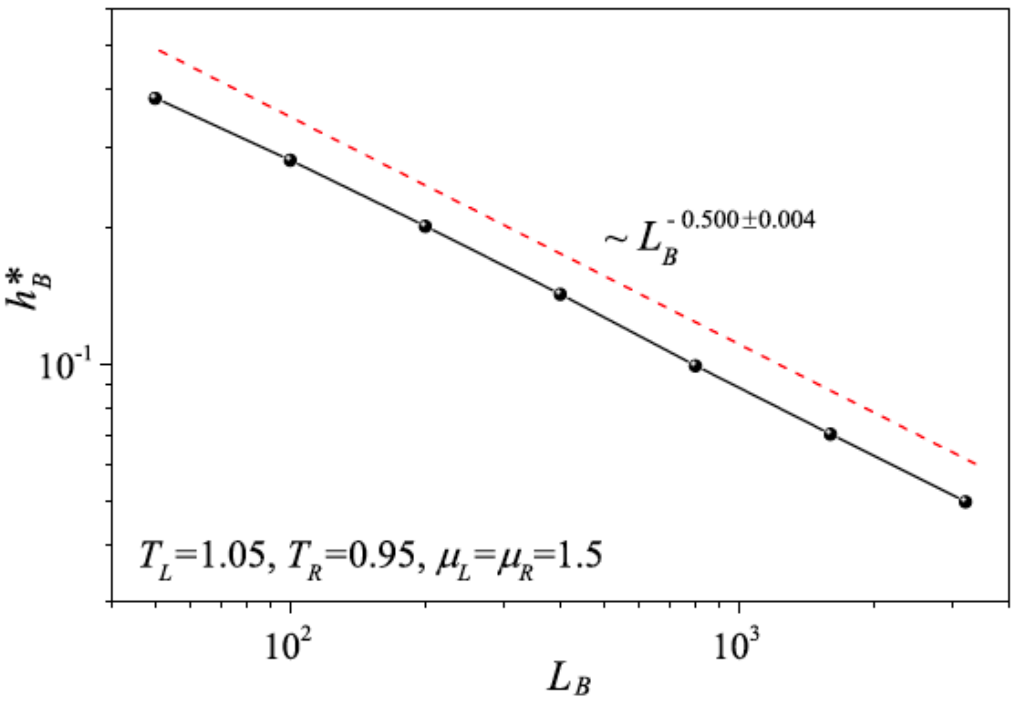


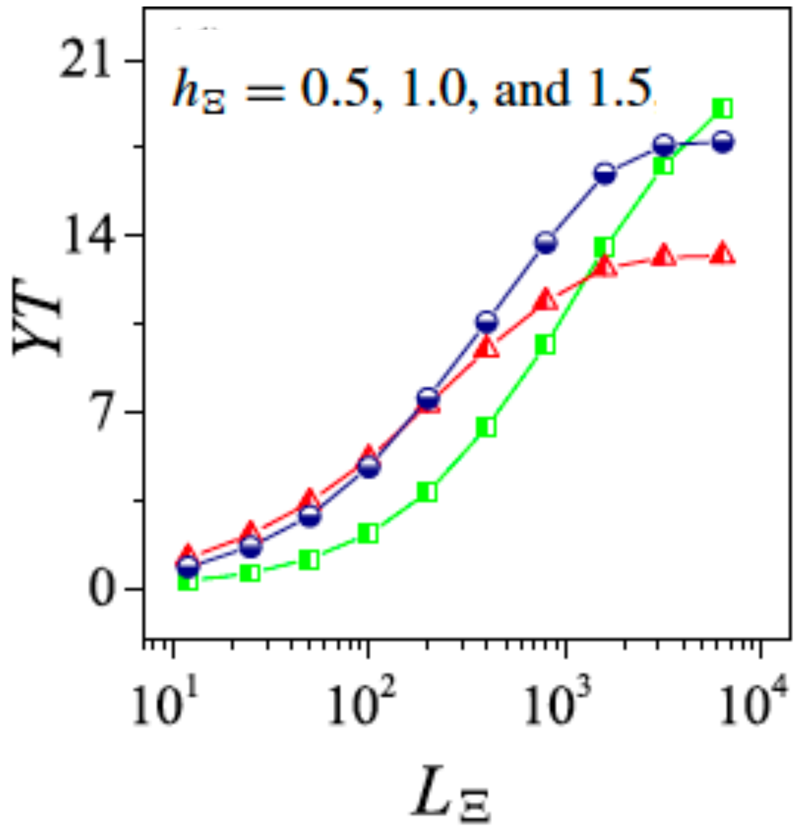

Very efficient heat engine

# Achieving Carnot efficiency at the thermodynamic limit?

Critical barrier height for inverse particle current: $h_B^* \sim L_B^{-0.5}$

Conjecture: diverging *YT* possible adapting barrier height with system size



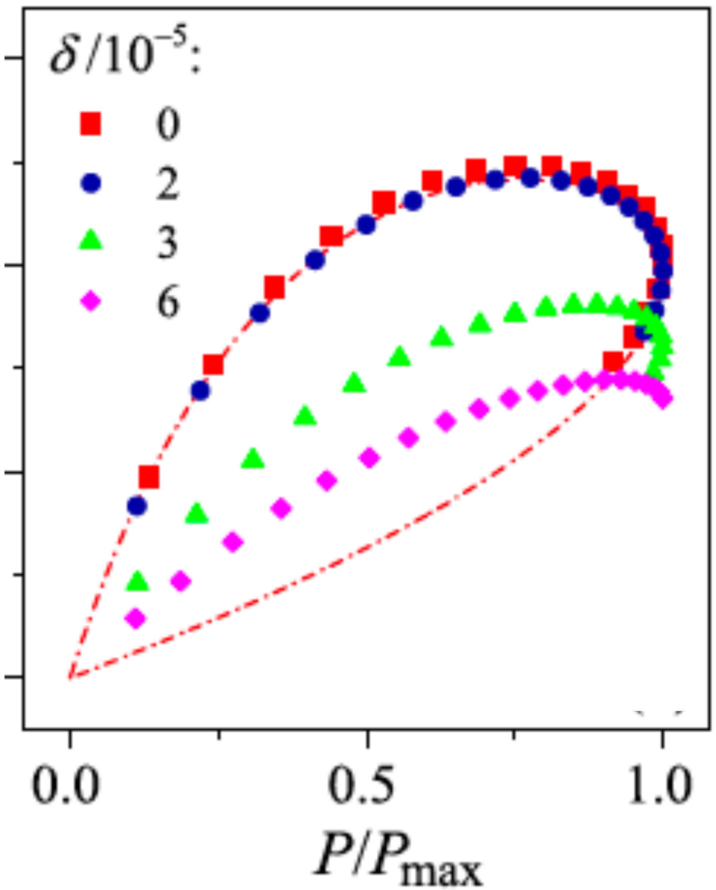



# Introducing dissipation

Model inspired by single-file diffusion in narrow channels: Two particles in a single file do squeeze their way past each other when their relative velocity is large enough.

Collisions may involve the loss of a fraction of the pair kinetic energy

$\delta$ dissipation parameter



# Final remarks

ICC, a most counterintuitive transport phenomenon, possible in dynamical systems, and enhanced by phase separation

Possible to build an efficient autonomous circular heat engine exploiting ICC

It would be interesting to observe the phenomenon in simpler models, classical or quantum, suitable for experimental investigation