Minimal model for powering particle motion from spin imbalance

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Ref.: Phys. Rev. E **95**, 062143 (2017)

Motivation

Quantum mechanics needed for an accurate description description of nanoscale (thermal) machines

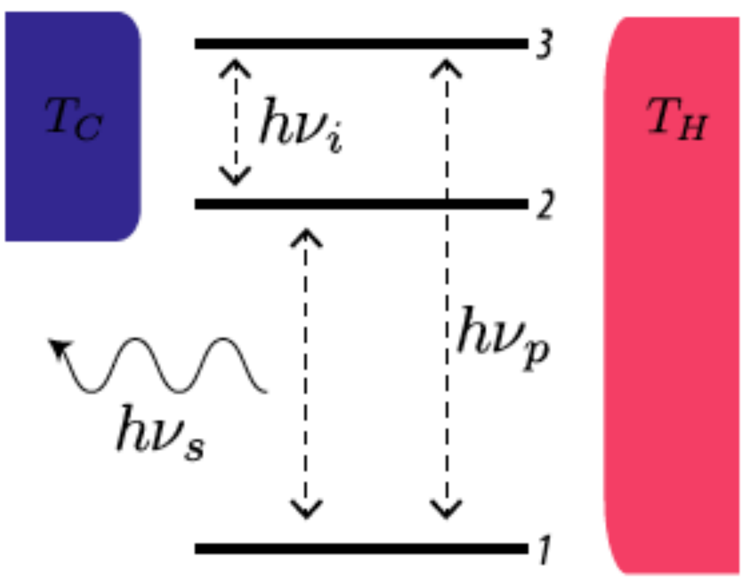
Some questions of *quantum thermodynamics:*

- role of coherence and entanglement
- quantum measurements
- minimum temperature in small quantum chillers
- relevance of quantum statistics
- quantum fluctuations
- feedback effects
- engineered nonequilibrium distributions for the baths

Minimal models of quantum motors

Useful to uncover and analyze fundamental aspects of energy conversion

A long history and a rich literature



THREE-LEVEL MASERS AS HEAT ENGINES

H. E. D. Scovil and E. O. Schulz-DuBois
Bell Telephone Laboratories,
Murray Hill, New Jersey

(Received January 16, 1959)

[PRL 2, 262 (1959)]

$$\eta_M = \frac{\nu_s}{\nu_p} \leq \eta_C = 1 - \frac{T_C}{T_H}$$

PRL 105, 130401 (2010) PHYSICAL REVIEW LETTERS week ending 24 SEPTEMBER 2010

How Small Can Thermal Machines Be? The Smallest Possible Refrigerator

Noah Linden,[1] Sandu Popescu,[2] and Paul Skrzypczyk[2]

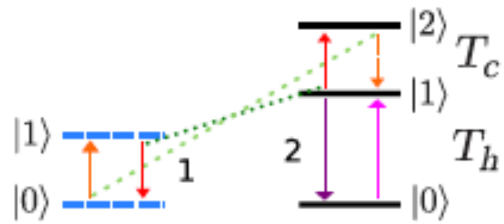
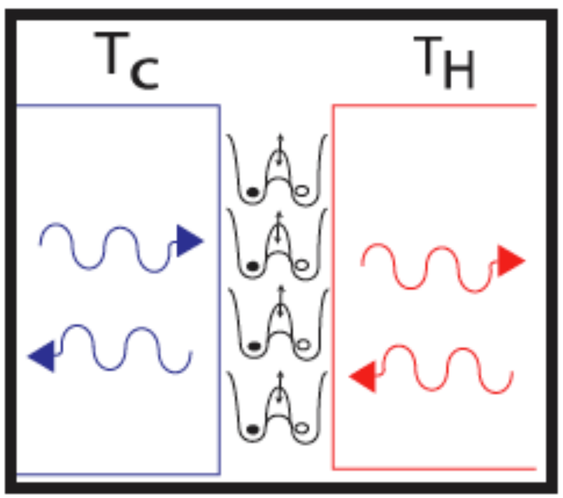


FIG. 5 (color online). Schematic diagram of a fridge consisting of a single qutrit (particle 2) with the object to be cooled (particle 1)

PHYSICAL REVIEW E **87**, 012140 (2013)

Minimal universal quantum heat machine

D. Gelbwaser-Klimovsky,[1] R. Alicki,[1,2] and G. Kurizki[1]



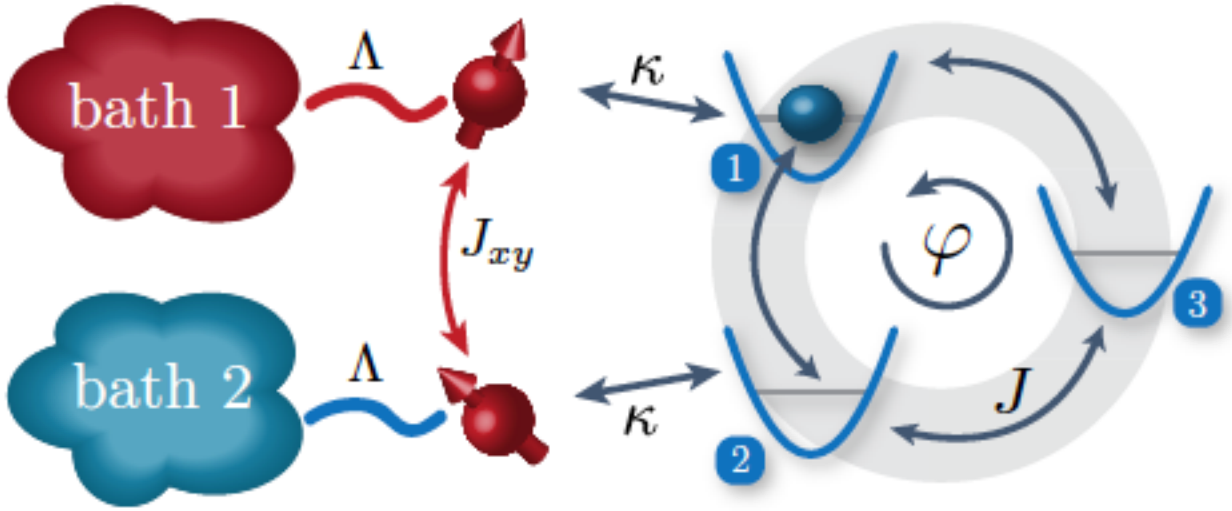
Double-well qubits (with periodically modulated tunneling barrier) between cold and hot baths

A minimal model of coupled flows

For the purposes of energy conversion in nanodevices it is important to consider coupled flows (for instance, phonon and electron transport in thermoelectricity)

Here we introduce a minimal motor for coupled (spin and particle) flows

A dynamical model of an open quantum system



$$\frac{d\hat{\rho}}{dt} = \mathcal{L}(\hat{\rho},t) = -\frac{i}{\hbar}[\hat{H}(t),\hat{\rho}] + \mathcal{D}(\hat{\rho})$$

$$\hat{H} = \hat{H}_\sigma + \hat{H}_a + \hat{H}_{\sigma a},$$

$$\phi(t) = \phi(t+T)$$

$$\hat{H}_\sigma = -J_{xy}(\hat{\sigma}_{x,1}\hat{\sigma}_{x,2} + \hat{\sigma}_{y,1}\hat{\sigma}_{y,2}) + \sum_l h_{z,l}\,\hat{\sigma}_{z,l},$$

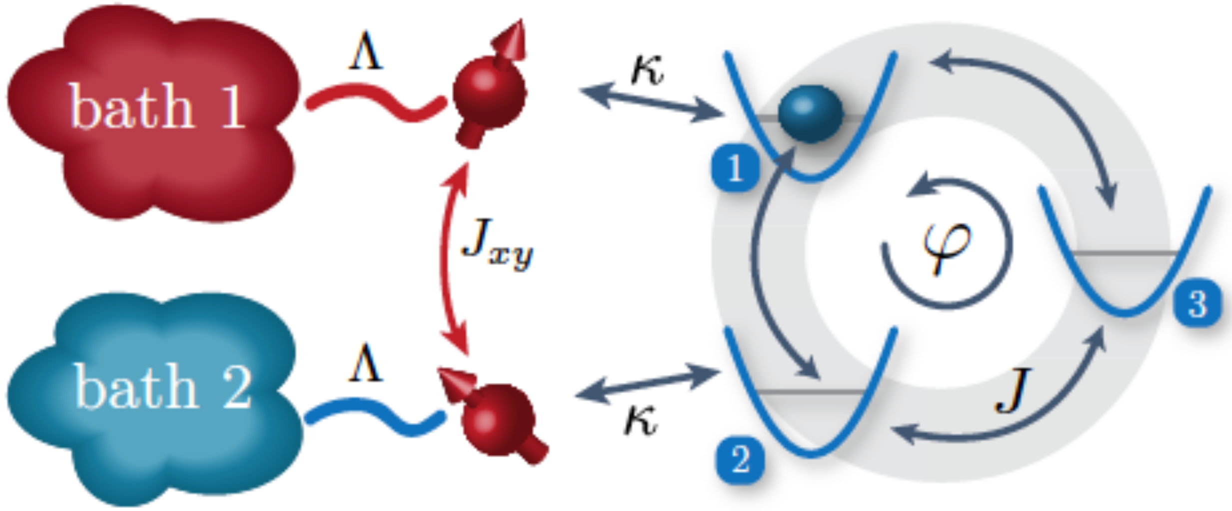
(external periodic driving, e.g. a time-dependent magnetic field)

$$\hat{H}_a = -J\sum_l e^{-i\phi(t)}\hat{a}_l^\dagger \hat{a}_{l+1} + \text{H.c.},$$

$$\hat{H}_{\sigma a} = \sum_{l=1,2} \kappa_l \hat{\sigma}_{z,l}\hat{n}_l$$

(ex: quantum dots, with a capacitative coupling)

Local dissipation (Lindblad form)



$$\frac{d\hat{\rho}}{dt} = \mathcal{L}(\hat{\rho},t) = -\frac{i}{\hbar}[\hat{H}(t),\hat{\rho}] + \mathcal{D}(\hat{\rho})$$

$$\mathcal{D} = \mathcal{D}_{\lambda,1} + \mathcal{D}_{\lambda,2} \qquad \mathcal{D}_{\lambda,l}(\hat{\rho}) = [\lambda_l^+(2\hat{\sigma}_l^+\hat{\rho}\hat{\sigma}_l^- - \hat{\sigma}_l^-\hat{\sigma}_l^+\hat{\rho} - \hat{\rho}\hat{\sigma}_l^-\hat{\sigma}_l^+).$$
$$+ \lambda_l^-(2\hat{\sigma}_l^-\hat{\rho}\hat{\sigma}_l^+ - \hat{\sigma}_l^+\hat{\sigma}_l^-\hat{\rho} - \hat{\rho}\hat{\sigma}_l^+\hat{\sigma}_l^-)]$$

$\Lambda_l = \lambda_l^+ + \lambda_l^-, \quad p_l = \lambda_l^+/\Lambda_l$ relative pumping rate into the upper state

Local Lindblad operators: *out-of-equilibrium magnetization baths* (in general temperature cannot be defined, except for $J_{xy} = \kappa_1 = \kappa_2 = 0$, when $\hat{\rho}_{\text{spin},1} = p_l|\uparrow\rangle_l\langle\uparrow| + (1-p_l)|\downarrow\rangle_l\langle\downarrow|$)

Working of the quantum motor

Diagonalize the Floquet-Lindblad operator to find the periodic steady state:

$$\mathbb{L}_{t_0} = \mathcal{T} e^{\int_{t_0}^{t_0+T} \mathcal{L}(t) dt} \Rightarrow \hat{\rho}_{\rm ps}(t_0)$$

Quantum definition of work and heat currents:

$$E(t) = \langle H(t) \rangle = \mathrm{tr}\{\hat{H}(t)\hat{\rho}_{\rm ps}(t)\}$$

$$dE/dt = \dot{Q} - \dot{W}, \qquad \dot{W}(t) = -\mathrm{tr}\{\frac{\partial H(t)}{\partial t}\hat{\rho}_{\rm ps}(t)\}$$

$$\dot{Q}(t) = \mathrm{tr}\{\hat{H}(t)\frac{\partial \hat{\rho}_{\rm ps}(t)}{\partial t}\}$$

$\mathcal{P}(t) = \dot{W}(t)$ output power

$$\dot{Q}(t) = \sum_{i=1,2} \dot{Q}_{\lambda,i}(t), \quad \dot{Q}_{\lambda,i}(t) = \mathrm{tr}\{\hat{H}(t)\mathcal{D}_{\lambda,i}[\hat{\rho}_{\rm ps}(t)]\}$$

(energy exchanged with the *i*-th bath)

Currents and efficiency

Particle and spin currents averaged over one period:

$$\mathcal{J}_T^n = \frac{1}{T}\int_{t_0}^{t_0+T} \mathrm{tr}[\hat{\rho}_{\mathrm{ps}}(t)\ \hat{j}_l]dt,\ \hat{j}_l = i[e^{i\phi(t)}\hat{a}_{l+1}^\dagger \hat{a}_l - e^{-i\phi(t)}\hat{a}_l^\dagger \hat{a}_{l+1}]$$

$$\mathcal{J}_T^\sigma = \frac{1}{T}\int_{t_0}^{t_0+T} \mathrm{tr}[\hat{\rho}_{\mathrm{ps}}(t)\ \hat{j}^\sigma]dt,$$

$$\hat{j}^\sigma = (2J_{xy})(\hat{\sigma}_{y,1}\hat{\sigma}_{x,2} - \hat{\sigma}_{y,2}\hat{\sigma}_{x,1})$$

Efficiency of the motor:

$$\boxed{\eta = \frac{T\,\mathcal{P}_T}{Q_{\mathrm{abs}}}}$$

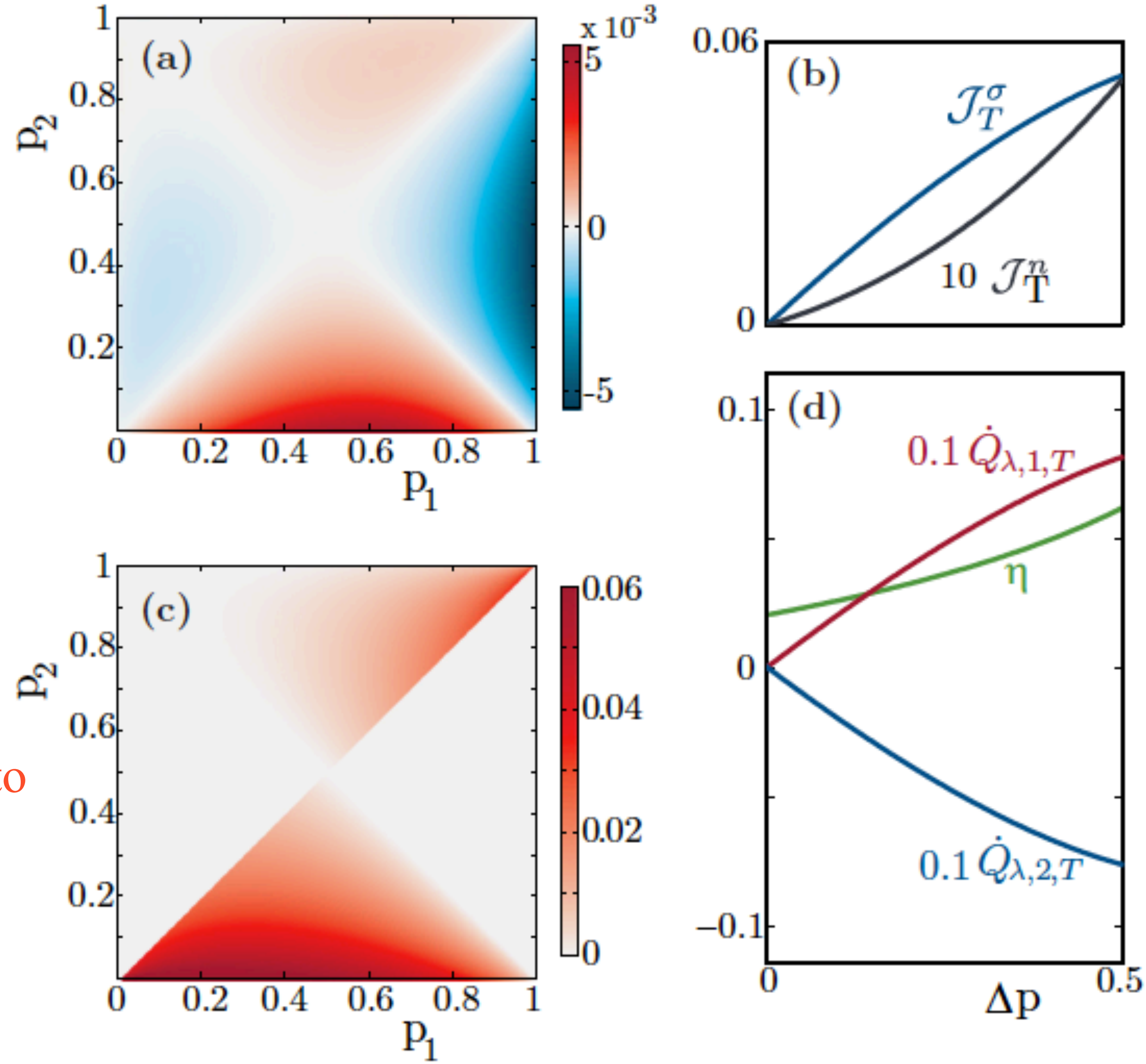
$$\mathcal{P}_T = \dot{Q}_T \propto \frac{\mathcal{J}_T^{\mathrm{n}}}{T}$$

$$Q_{\mathrm{abs}} = \sum_{i=1,2}\int_{t_0}^{t_0+T} \dot{Q}_{\lambda,i}(t)\,\Theta[\dot{Q}_{\lambda,i}(t)]dt$$

Numerical results

particle current (proportional to power)

efficiency
(cannot compare to Carnot efficiency)

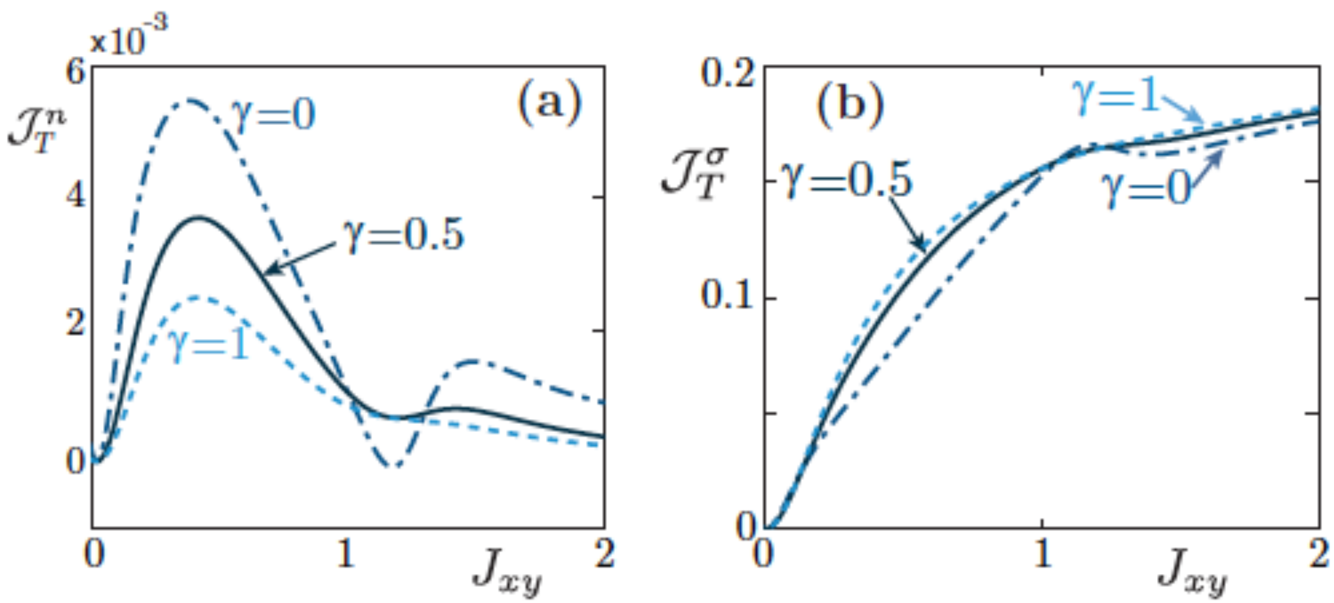


Robustness to dephasing

$$\mathcal{D} = \mathcal{D}_{\lambda,1} + \mathcal{D}_{\lambda,2} + \mathcal{D}_{\gamma}$$

$$\mathcal{D}_{\gamma} = \gamma\left(2\hat{n}_3\hat{\rho}\hat{n}_3 - \hat{n}_3^2\hat{\rho} - \hat{\rho}\hat{n}_3^2\right)$$

$$\dot{Q}_{\gamma}(t) = \mathrm{tr}\{\hat{H}(t)\,\mathcal{D}_{\gamma}[\hat{\rho}_{\mathrm{ps}}(t)]\}$$



Necessity of spin coupling

The particle current (and hence the power) vanishes when either $J_{xy} = 0$ or $\Delta p = 0$

In this case the steady state is time-independent and separable:

$$\hat{\rho}_0 = \hat{\rho}_0^{\text{spin},1} \otimes \hat{\rho}_0^{\text{spin},2} \otimes \hat{\rho}_0^{\text{particle}}$$

$$\hat{\rho}_0^{\text{particle}} = [\hat{\mathbb{1}}/3], \qquad \mathcal{D}_\gamma(\hat{\rho}_0^{\text{particle}}) = 0$$

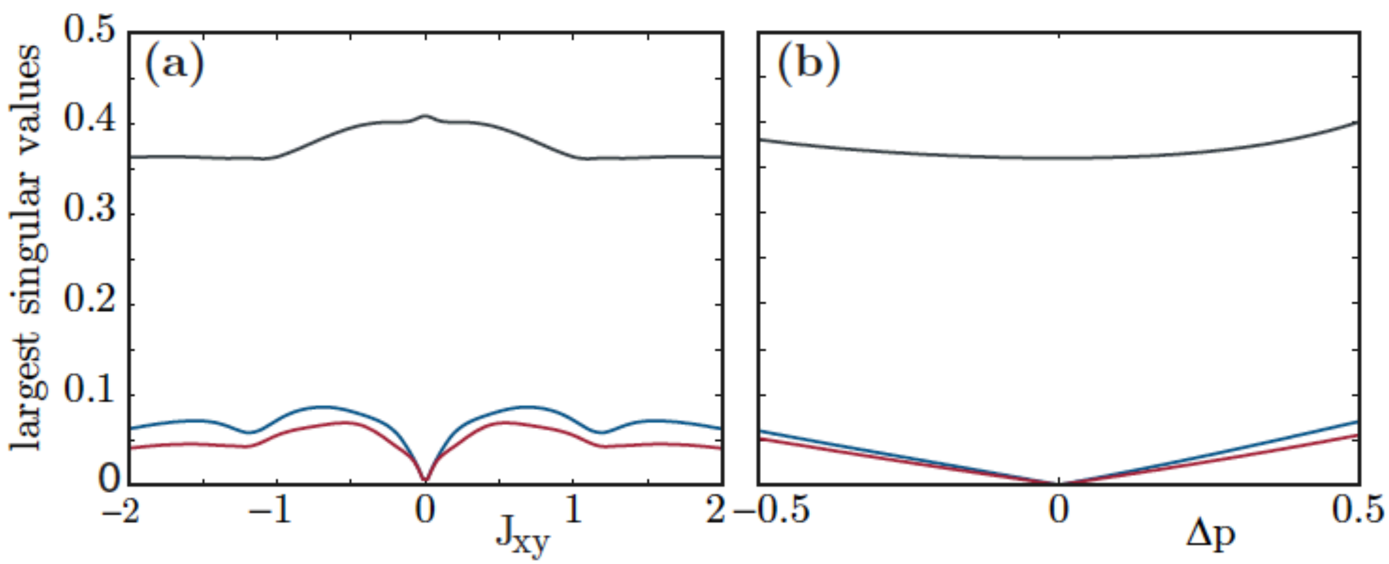
$$\rho_0^{\text{spin},1} = p_l |\uparrow\rangle_l \langle\uparrow| + (1 - p_l)|\downarrow\rangle_l \langle\downarrow|$$

Nonseparability of the periodic steady state

Singular value decomposition:

$$\hat{\rho}_{\mathrm{ps}}(t) = \sum_j s_j(t)\ \hat{B}_j^{\mathrm{spins}}(t) \otimes \hat{B}_j^{\mathrm{particle}}(t)$$

time-averaged largest singular values:



Conclusions

Minimal motor to perform work against a periodic driving from an out-of-equilibrium energy flow

Coupled spin magnetization/particle transport

Non-equilibrium magnetization baths

Possible extensions: use thermal baths, analyze the effects of system size, interactions, particle statistics, and the role of measurement on the motor's performance

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Physics Reports

Volume 694, 9 June 2017, Pages 1-124

Fundamental aspects of steady-state conversion of heat to work at the nanoscale

Giuliano Benenti [a, b], Giulio Casati [a, c], Keiji Saito [d], Robert S. Whitney [e]

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