# Cyclic solid-state quantum battery: Thermodynamic characterization & quantum hardware simulation

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arXiv:2407.07157 [quant-ph]

# Outline

As with quantum heat engines, it is necessary to develop the thermodynamics of quantum batteries

Cyclic quantum battery model, based on an interacting bipartite system, weakly coupled to a thermal bath

Cycle simulated on (superconducting) IBM quantum hardware

# The second quantum revolution

First quantum revolution: we learnt rules controlling physics at small scales

Many technological applications: transistors, lasers,…

Second quantum revolution: building real quantum machines, exploiting superposition and entanglement to develop new technologies

Quantum advantage? In computation, secure information transmission, sensing,…

Quantum computation and information is a rapidly developing interdisciplinary field. It is not easy to understand its fundamental concepts and central results without facing numerous technical details. This book provides the reader with a useful guide. In particular, the initial chapters offer a simple and self-contained introduction; no previous knowledge of quantum mechanics or classical computation is required.

Various important aspects of quantum computation and information are covered in depth, starting from the foundations (the basic concepts of computational complexity, energy, entropy, and information, quantum superposition and entanglement, elementary quantum gates, the main quantum algorithms, quantum teleportation, and quantum cryptography) up to advanced topics (like entanglement measures, quantum discord, quantum noise, quantum channels, quantum error correction, quantum simulators, and tensor networks).

It can be used as a broad range textbook for a course in quantum information and computation, both for upper-level undergraduate students and for graduate students. It contains a large number of solved exercises, which are an essential complement to the text, as they will help the student to become familiar with the subject. The book may also be useful as general education for readers who want to know the fundamental principles of quantum information and computation.

"Thorough introductions to classical computation and irreversibility, and a primer of quantum theory, lead into the heart of this impressive and substantial book. All the topics – quantum algorithms, quantum error correction, adiabatic quantum computing and decoherence are just a few – are explained carefully and in detail. Particularly attractive are the connections between the conceptual structures and mathematical formalisms, and the different experimental protocols for bringing them to practice. A more wide-ranging, comprehensive, and definitive text is hard to imagine."

— Sir Michael Berry, *University of Bristol, UK*

"This second edition of the textbook is a timely and very comprehensive update in a rapidly developing field, both in theory as well as in the experimental implementation of quantum information processing. The book provides a solid introduction into the field, a deeper insight in the formal description of quantum information as well as a well laid-out overview on several platforms for quantum simulation and quantum computation. All in all, a well-written and commendable textbook, which will prove very valuable both for the novices and the scholars in the fields of quantum computation and information."

— Rainer Blatt, *Universität Innsbruck and IQOQI Innsbruck, Austria*

"The book by Benenti, Casati, Rossini and Strini is an excellent introduction to the fascinating field of quantum information, of great benefit for scientists entering the field and a very useful reference for people already working in it. The second edition of the book is considerably extended with new chapters, as the one on many-body systems, and necessary updates, most notably on the physical implementations."

— Rosario Fazio, *The Abdus Salam International Centre for Theoretical Physics, Trieste, Italy*

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Benenti
Casati
Rossini
Strini

Principles of Quantum Computation and Information

A Comprehensive Textbook

Giuliano Benenti Giulio Casati
Davide Rossini Giuliano Strini

S A T O R
A R E P O
T E N E T
O P E R A
R O T A S

# Principles of Quantum Computation and Information

## A Comprehensive Textbook

**World Scientific**

# Quantum thermodynamics

PRX QUANTUM **3**, 020101 (2022)

Perspective

## Quantum Technologies Need a Quantum Energy Initiative

Alexia Auffèves*

*Université Grenoble Alpes, CNRS, Grenoble INP, Institut Néel, Grenoble 38000, France*

(Received 18 November 2021; revised 11 April 2022; published 1 June 2022)

Quantum technologies are currently the object of high expectations from governments and private companies, as they hold the promise to shape safer and faster ways to extract, exchange, and treat information. However, despite its major potential impact for industry and society, the question of their energetic footprint has remained in a blind spot of current deployment strategies. In this Perspective, I argue that quantum technologies must urgently plan for the creation and structuration of a transverse quantum energy initiative, connecting quantum thermodynamics, quantum information science, quantum physics, and engineering. Such an initiative is the only path towards energy-efficient, sustainable quantum technologies, and to possibly bring out an energetic quantum advantage.

# What is a battery?

A mountain dam is a battery

Water wants to fall but we keep it up

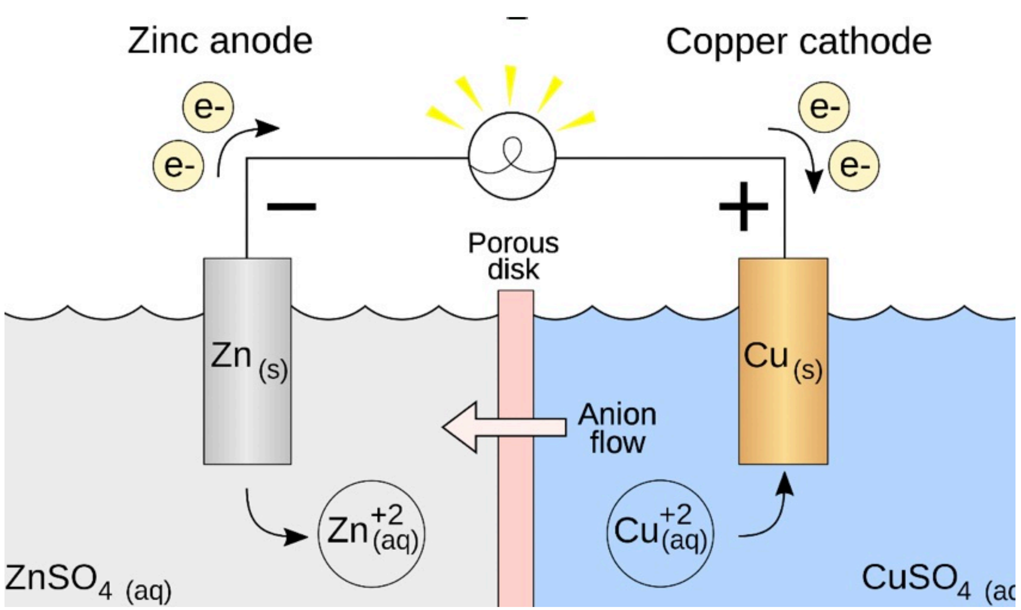
Energy is stored: when water falls we can use it (a turbine will spin, …)

What does charging a battery mean?
An empty dam does not store energy
Bringing water into the dam we charge the battery

# Another example: Galvanic batteries

Same principle but with electrons:

* They like Cu but they are kept on Zn
* Energy stored and ready to be released
* External connection: electrons flow




Recharging the battery:

* Electrons back to Zn
* Energy stored again

# Quantum batteries

PHYSICAL REVIEW E **87**, 042123 (2013)

**Entanglement boost for extractable work from ensembles of quantum batteries**

Robert Alicki*
*Institute of Theoretical Physics and Astrophysics, University of Gdańsk, Poland*

Mark Fannes†
*Instituut voor Theoretische Fysica, KU Leuven, B-3001 Leuven, Belgium*
(Received 20 November 2012; published 25 April 2013)

Because we are dealing with small quantum systems we may wonder whether using processes that entangle two identical copies of a given battery can yield a higher energy extraction. More generally, what happens to a large number of copies?

# Quantum advantage in charging power?

PRL **118,** 150601 (2017) PHYSICAL REVIEW LETTERS week ending 14 APRIL 2017

## Enhancing the Charging Power of Quantum Batteries

Francesco Campaioli,[1,*] Felix A. Pollock,[1] Felix C. Binder,[2] Lucas Céleri,[3] John Goold,[4]
Sai Vinjanampathy,[5,6] and Kavan Modi[1,†]

*"… we demonstrate that quantum mechanics can lead to an enhancement in the amount of work deposited per unit time, i.e., the charging power, when N batteries are charged collectively … We derive analytic upper bounds for the collective quantum advantage …"*

PHYSICAL REVIEW LETTERS 128, 140501 (2022)

Editors' Suggestion | Featured in Physics

## Quantum Charging Advantage Cannot Be Extensive without Global Operations

Ju-Yeon Gyhm,[1,2,*] Dominik Šafránek,[1,†,§] and Dario Rosa[1,‡,§]
[1]*Center for Theoretical Physics of Complex Systems, Institute for Basic Science (IBS), Daejeon 34126, Republic of Korea*
[2]*Department of Physics and Astronomy, Seoul National University, 1 Gwanak-ro, Seoul 08826, Korea*

(Received 13 August 2021; accepted 8 February 2022; published 4 April 2022)

Quantum batteries are devices made from quantum states, which store and release energy in a fast and efficient manner, thus offering numerous possibilities in future technological applications. They offer a significant charging speedup when compared to classical batteries, due to the possibility of using entangling charging operations. We show that the maximal speedup that can be achieved is extensive in the number of cells, thus offering at most quadratic scaling in the charging power over the classically achievable linear scaling. To reach such a scaling, a global charging protocol, charging all the cells collectively, needs to be employed. This concludes the quest on the limits of charging power of quantum batteries and adds to other results in which quantum methods are known to provide at most quadratic scaling over their classical counterparts.

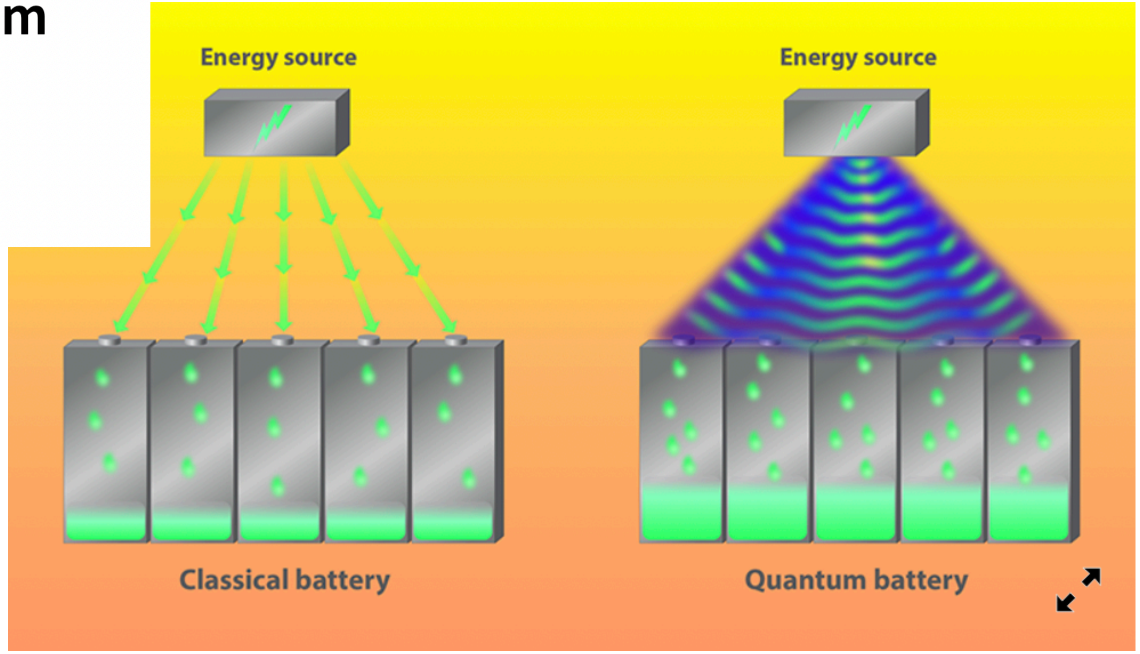
VIEWPOINT

## Sizing Up the Potential of Quantum Batteries

Sourav Bhattacharjee
Indian Institute of Technology, Kanpur, Kanpur-208016, Uttar Pradesh, India

April 4, 2022 • *Physics* 15, 50

A quantum battery consisting of N cells could charge up to N times faster



# First experimental implementations

SCIENCE ADVANCES | RESEARCH ARTICLE

PHYSICS

## Superabsorption in an organic microcavity: Toward a quantum battery

James Q. Quach[1]*, Kirsty E. McGhee[2], Lucia Ganzer[3], Dominic M. Rouse[4], Brendon W. Lovett[4], Erik M. Gauger[5], Jonathan Keeling[4], Giulio Cerullo[3], David G. Lidzey[2], Tersilla Virgili[3]*

*"... we implement experimentally a paradigmatic model of a quantum battery, constructed of a microcavity enclosing a molecular dye ... Ultrafast optical spectroscopy allows us to observe charging dynamics at femtosecond resolution ..."*

Organic molecules play the role of two-level systems embedded in a microcavity: super extensive energy charging

# Quantum processors as quantum batteries

*batteries*

*Article*

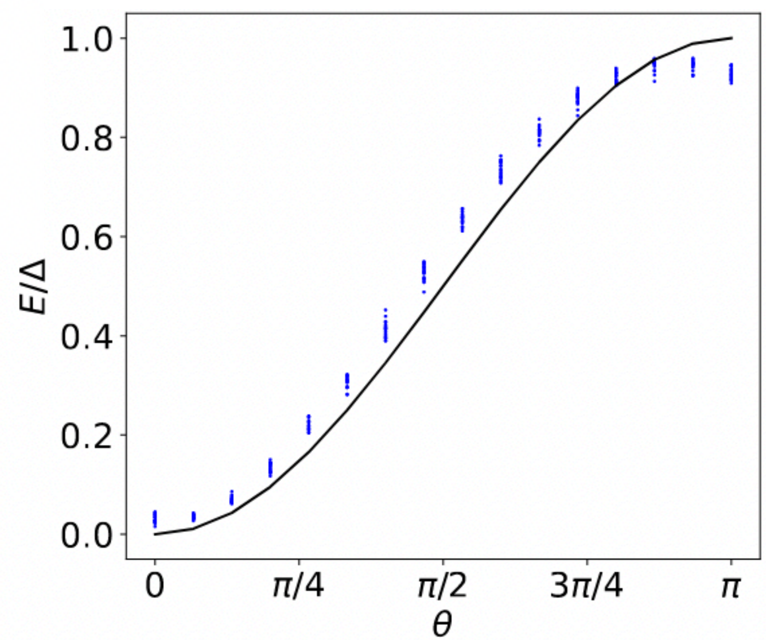
**IBM Quantum Platforms: A Quantum Battery Perspective**

**Giulia Gemme [1], Michele Grossi [2], Dario Ferraro [1,3,*], Sofia Vallecorsa [2] and Maura Sassetti [1,3]**

$$
\begin{aligned}
H &= H_{QB} + H_C \\
&= \frac{\Delta}{2}(1-\sigma_z) + g f(t)\cos(\omega t)\sigma_x
\end{aligned}
$$

$$f(t) = \mathcal{N} e^{-\frac{(t-t_m/2)^2}{2\sigma^2}}$$

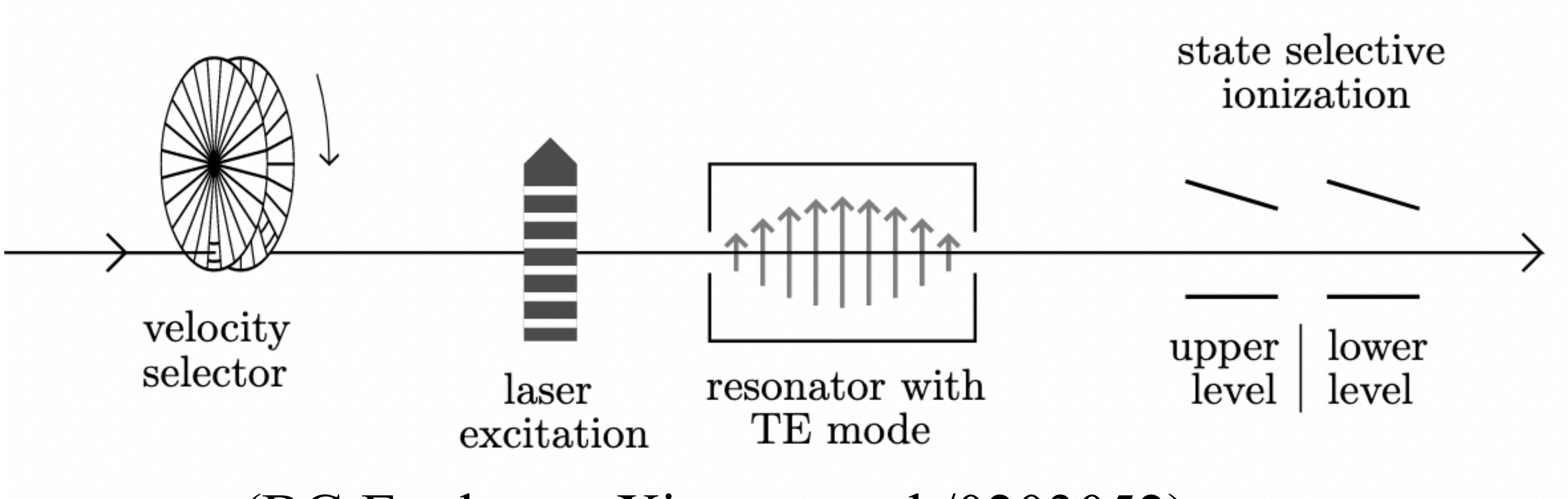
$$\theta(t_m) = g\int_0^{t_m} f(\tau)d\tau$$




Qutrit quantum batteries:

G. Gemme et al., Phys. Rev. Research **6**, 023091(2024)

# Multilevel quantum batteries: Revisiting micromasers




(BG Englert, arXiv:quant-ph/0203052)

*"Even a faint beam of atoms can pump the resonator effectively. In this way, a maser is operated in which single atoms traversing the resonator provide for an efficient pump. One is then dealing with a microscopic maser indeed, a micromaser."*

*Quantum Sci. Technol.* **7** (2022) 04LT01 https://doi.org/10.108

Quantum Science and Technology

**LETTER**

# Micromasers as quantum batteries

Vahid Shaghaghi[1,2,3], Varinder Singh[3], Giuliano Benenti[1,2,4] and Dario Rosa[3,5,*]

INDEPENDENT

## Quantum battery breakthrough paves way for instant recharging

Scientists hope discovery will usher in a new era of ultra-efficient batteries

physicsworld

QUANTUM | RESEARCH UPDATE

Micromasers make a promising platform for quantum batteries

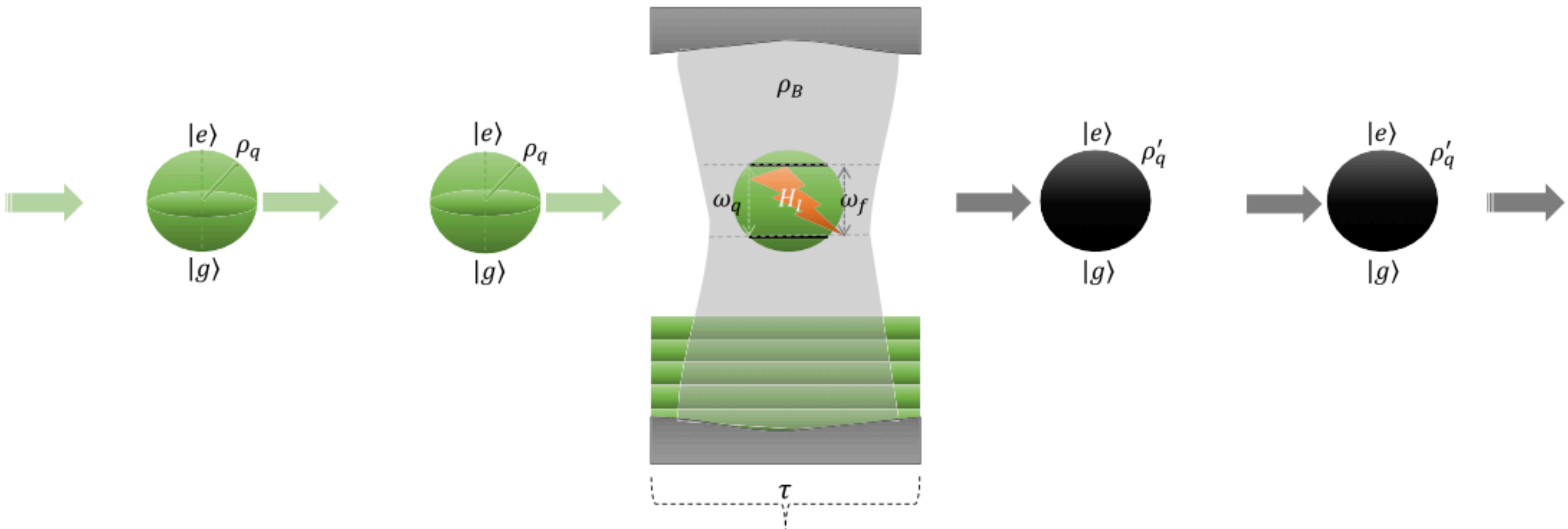



**Figure 1.** A pictorial view of the micromaser quantum battery. The incoming qubits are initially prepared in a superposition of ground and excited states. When entering the cavity they interact with the EM field and they decrease their energy, leaving the cavity in a low energy state (represented with a black colored Bloch sphere). In the figure, $\omega_\mathrm{q}$ and $\omega_\mathrm{f}$ denote the frequencies of the qubits and the fields, respectively, which in the main text have been assumed to be equal and denoted by $\omega$ (resonant condition).

# Ergotropy

Ergotropy: maximum work which can be extracted via cyclic unitaries

[Allahverdyan, Balian, Nieuwenhuizen, Europhysics Letters 67, 565 (2004)]

$$\varrho = \sum_{k=1}^{d} \lambda_k^{\downarrow} |\lambda_k^{\downarrow}\rangle\langle\lambda_k^{\downarrow}| \qquad \text{density matrix} \qquad \lambda_k^{\downarrow} \geq \lambda_{k+1}^{\downarrow}$$

$$H = \sum_{k=1}^{d} \epsilon_k^{\uparrow} |\epsilon_k^{\uparrow}\rangle\langle\epsilon_k^{\uparrow}| \qquad \text{Hamiltonian} \qquad \epsilon_k^{\uparrow} \leq \epsilon_{k+1}^{\uparrow}$$

$$E(\varrho) = \mathrm{Tr}[H\varrho] = \mathcal{E}(\varrho) + E(\pi)$$

Ergotropy

Energy of the corresponding passive state

$$\pi = U(\vec{\theta})\varrho U^\dagger(\vec{\theta}) = \sum_{k=1}^{d} \lambda_k^\downarrow |\epsilon_k^\uparrow\rangle\langle\epsilon_k^\uparrow| \qquad U(\vec{\theta}) = \sum_{k=1}^{d} e^{i\theta_k} |\epsilon_k^\uparrow\rangle\langle\lambda_k^\downarrow|$$

$$\mathcal{E}(\varrho) = E(\varrho) - E(\pi) = \sum_{j,k=1}^{d} \lambda_j^\downarrow \epsilon_k^\uparrow (|\langle\lambda_j^\downarrow|\epsilon_k^\uparrow\rangle|^2 - \delta_{jk})$$

Phases irrelevant for ergotropy but relevant for the efficiency of a cyclic battery

Thermal (Gibbs) state passive

$$\tau_\beta = \frac{e^{-\beta H}}{\mathrm{Tr}[e^{-\beta H}]} = \sum_{k=1}^{d} \frac{e^{-\beta \varepsilon_k^\uparrow}}{\mathrm{Tr}[e^{-\beta H}]} |\epsilon_k^\uparrow\rangle\langle\epsilon_k^\uparrow|$$

# Thermodynamics of a quantum battery

PHYSICAL REVIEW RESEARCH **2**, 033413 (2020)

**Charging assisted by thermalization**

Karen V. Hovhannisyan,[1,2] Felipe Barra,[3] and Alberto Imparato[4]

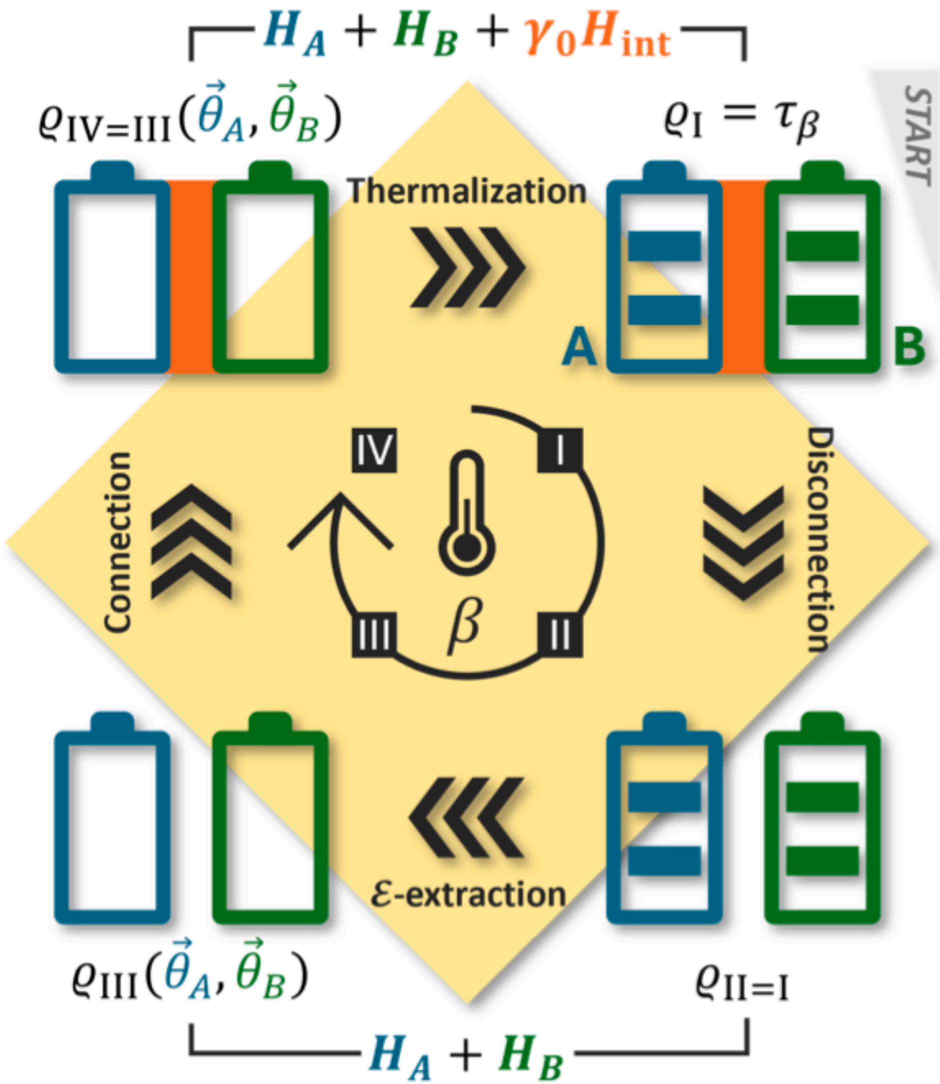
Strong system-bath coupling: the overall thermal state is not a thermal state for the system alone

After disconnecting the system from the bath the state of the system is not passive and work can be extracted

Thermodynamic cycle: connect, let thermalize, disconnect, extract work

Main technical difficulty: a strong system-environment coupling must be switched on and off

# Cyclic bipartite quantum battery




Four-stroke cycle

System always weakly coupled to the environment

[L. Razzoli, G. Gemme, I. Khomchenko, M. Sassetti, H. Ouerdane, D. Ferraro, G. B., preprint arXiv::2407.07157 [quant-ph]]

# Ergotropy extraction protocols

**Protocol (i)** Local extraction from a single subsystem

$$\mathcal{U}_s(\vec{\theta}_A) = U_A(\vec{\theta}_A) \otimes I_B \qquad \mathcal{E}^{(s)} = \mathcal{E}_A$$

**Protocol (ii)** Local extraction from both subsystems

$$\mathcal{U}_l(\vec{\theta}_A, \vec{\theta}_B) = U_A(\vec{\theta}_A) \otimes U_B(\vec{\theta}_B) \qquad \mathcal{E}^{(l)} = \mathcal{E}_A + \mathcal{E}_B$$

**Protocol (iii)** Global extraction from both subsystems, including correlations

$$\pi_g = \mathcal{U}_g \tau_\beta \mathcal{U}_g^\dagger \qquad \mathcal{E}^{(l)} \leq \mathcal{E}^{(g)}$$

Go to the passive state with respect to $H_A + H_B$

# Quantum battery efficiency

Ratio between extracted work (ergotropy) and disconnection/connection energy cost

$$\eta^{(p)}(\theta) = \frac{\mathcal{E}^{(p)}}{E_d + E_c^{(p)}(\theta)}, \qquad p = s, l, g$$

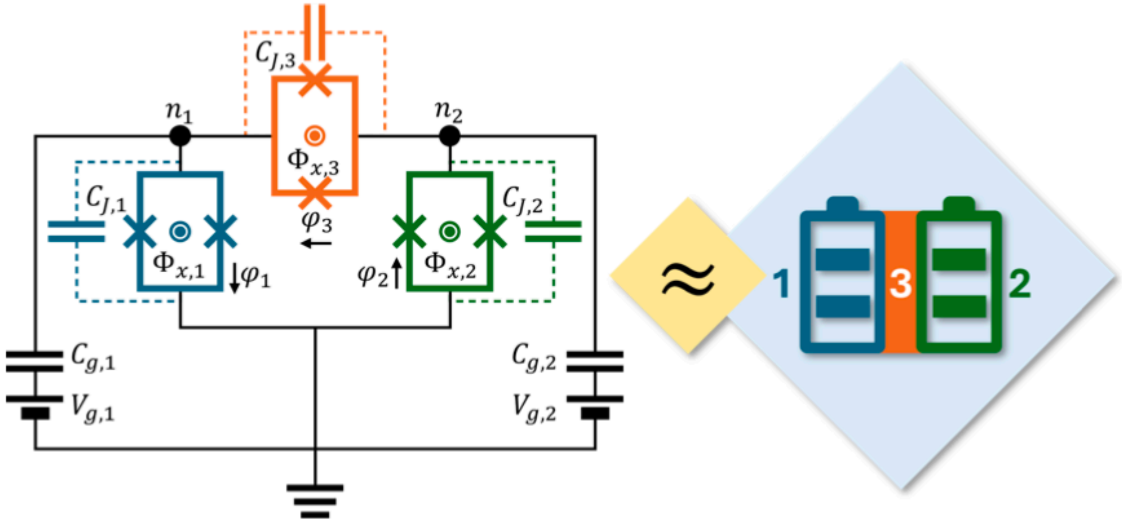
Starting from a passive (thermal) state energy can only increase (work performed on the system): $E_d - \mathcal{E}^{(p)} + E_c^{(p)}(\theta) \geq 0$

Ergotropy positive by definition: $\mathcal{E}^{(p)} \geq 0$

Resulting efficiency bounds:

$$0 \leq \eta^{(p)}(\theta) \leq 1$$

# Josephson quantum battery (circuit model)




An effective two-qubit Hamiltonian can be derived:

$$H = \Omega \sum_{k=1}^{2} \sigma_k^z - E_J \cos\left(\pi \frac{\Phi_x}{\Phi_0}\right)\left[\sum_{k=1}^{2} \sigma_k^x + (\sigma_1^x \sigma_2^x - \sigma_1^y \sigma_2^y)\right]$$

In units of ***Ω***:

$$H(t) = \sigma_A^z + \sigma_B^z - \gamma(t)(\sigma_A^x + \sigma_B^x + \sigma_A^x \sigma_B^x - \sigma_A^y \sigma_B^y)$$

# Energetics of the working cycle

Energy cost for disconnection (always positive)

$$E_d = \gamma_0(2\langle\sigma_A^x\rangle + \langle\sigma_A^x\sigma_B^x\rangle - \langle\sigma_A^y\sigma_B^y\rangle)$$

Unitary transformation for local ergotropy extraction for a qubit

$$U(\theta) = \begin{pmatrix} e^{i\theta}\cos\alpha & e^{i\theta}\sin\alpha \\ -e^{-i\theta}\sin\alpha & e^{-i\theta}\cos\alpha \end{pmatrix} = e^{i\theta\sigma_z}e^{i\alpha\sigma_y}$$

$$\alpha = \arctan[-(r+z)/x] \qquad r = \sqrt{x^2+y^2+z^2}$$

Ergotropy

$$\mathcal{E}^{(s)} = \mathcal{E} \qquad \mathcal{E}^{(l)} = 2\mathcal{E} \qquad \mathcal{E} = z + r = \langle\sigma_A^z\rangle + \sqrt{\langle\sigma_A^x\rangle^2 + \langle\sigma_A^z\rangle^2}$$

$$\mathcal{E}^{(g)} = 2\langle\sigma_A^z\rangle + \frac{2}{Z}\left(e^{-\beta\epsilon_1^\uparrow} - e^{-\beta\epsilon_4^\uparrow}\right) \geq \mathcal{E}^{(l)}$$

# Energy cost for connection (can be negative!)

$$E_c^{(s)}(\theta) = -\gamma_0\Big\{\langle\sigma_A^x\rangle + \cos(2\theta)\Big[\cos(2\alpha)(\langle\sigma_A^x\rangle + \langle\sigma_A^x\sigma_B^x\rangle)$$

$$- \langle\sigma_A^y\sigma_B^y\rangle - \sin(2\alpha)(\langle\sigma_A^z\rangle + \langle\sigma_A^z\sigma_B^x\rangle)\Big]\Big\}$$

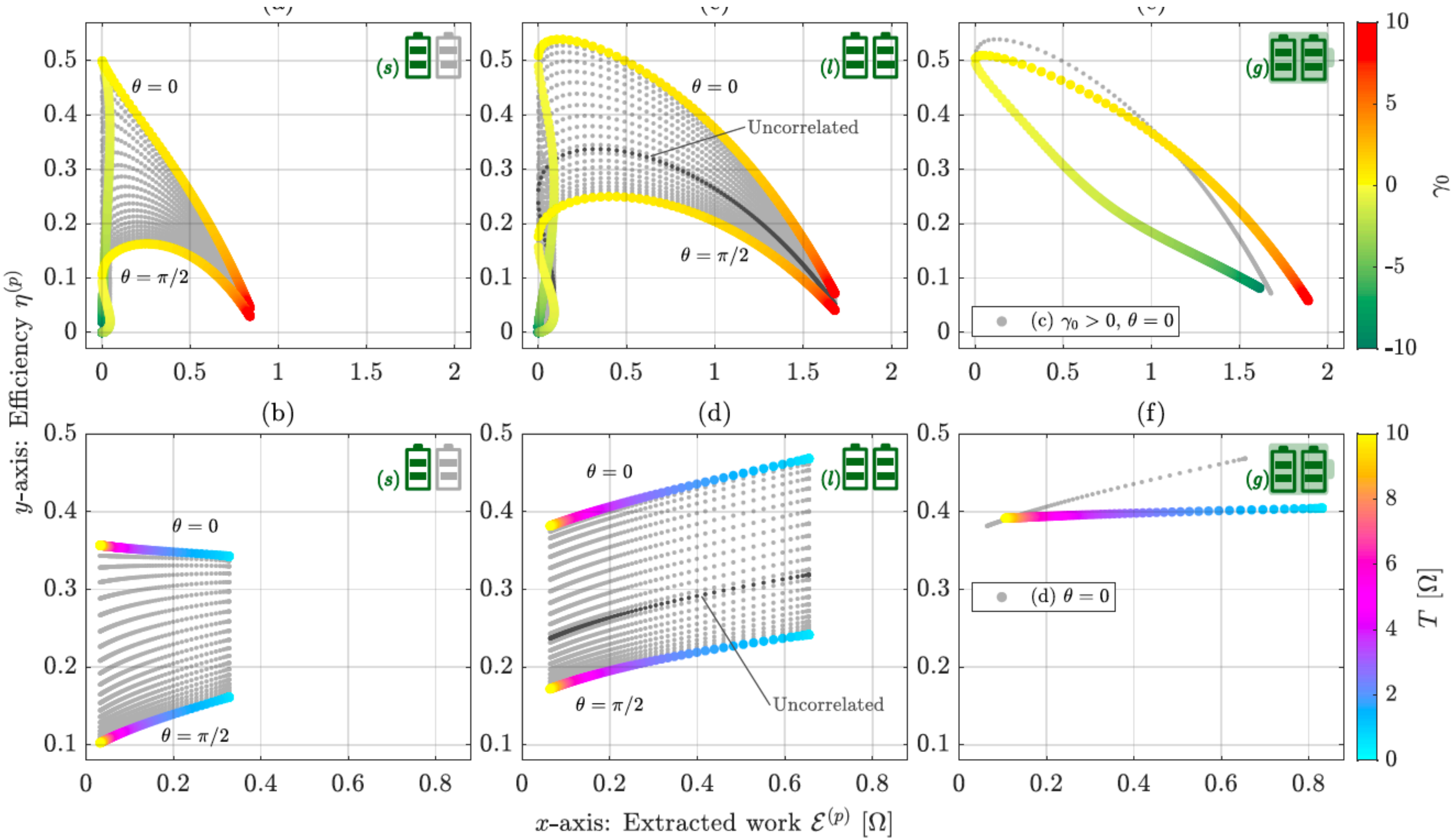
$$E_c^{(l)}(\theta) = -\gamma_0\cos(2\theta)\Big(\cos^2(2\alpha)\langle\sigma_A^x\sigma_B^x\rangle - \langle\sigma_A^y\sigma_B^y\rangle$$

$$+ \sin^2(2\alpha)\langle\sigma_A^z\sigma_B^z\rangle - \sin(4\alpha)\langle\sigma_A^x\sigma_B^z\rangle\Big)$$

$$\theta \equiv \theta_A + \theta_B$$

$$E_c^{(g)} \equiv 0$$

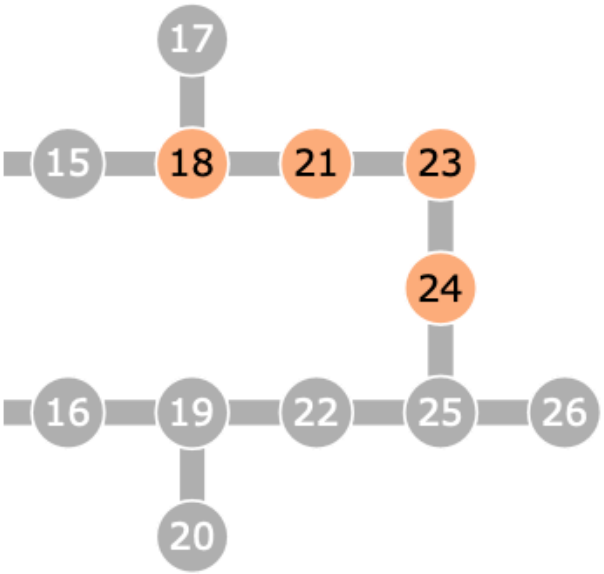
# Numerical results



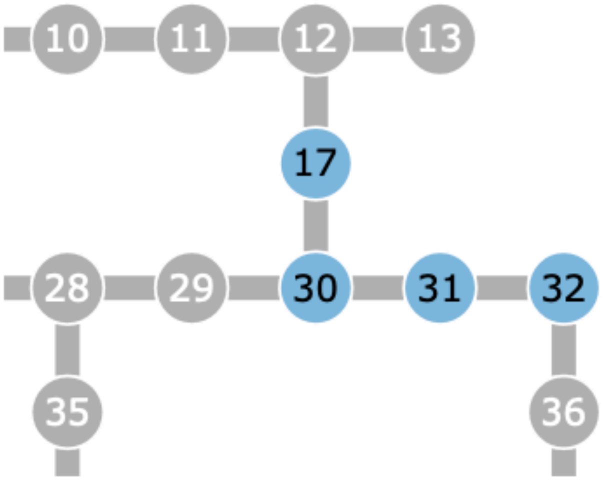

Pareto front for $\theta = 0 \quad \gamma_0 > 0$

Efficiency can be higher (for a given ergotropy) in the local extraction setup

# Simulations on superconducting quantum hardware




(a) ibm cairo




(b) ibm brisbane

# Preparation of a Gibbs state

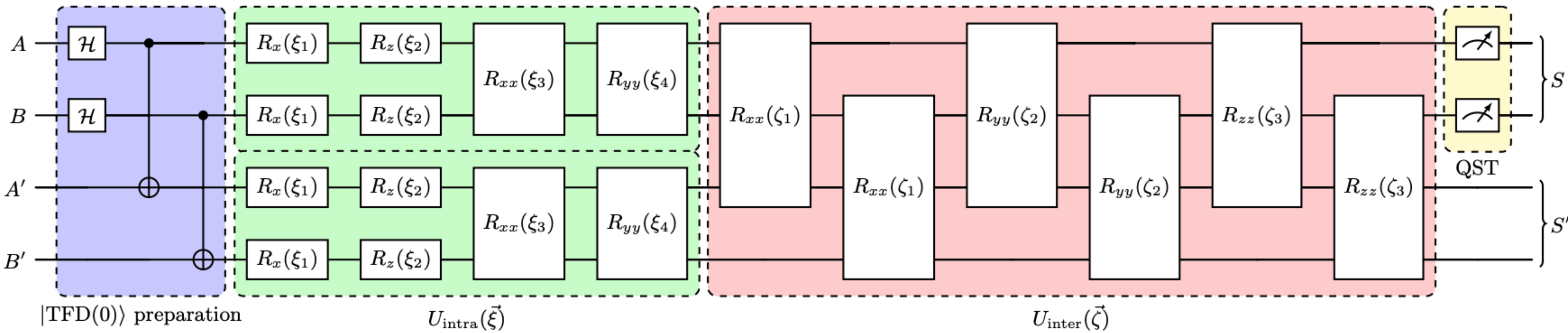
Obtained tracing over a pure thermofield double (TFD) state

$$|\mathrm{TFD}(\beta)\rangle = \sum_{k=1}^{d} \frac{e^{-\frac{\beta \varepsilon_k^{\uparrow}}{2}}}{\sqrt{\mathrm{Tr}[e^{-\beta H}]}} |\epsilon_k^{\uparrow}\rangle_S |\epsilon_k'^{\uparrow}\rangle_{S'}$$

Gibbs state for S obtained after tracing over the ancillary system S'

# Variational ansatz

## Quantum approximate optimization algorithm (QAOA)




$$U_{\text{TFD}}(\vec{\xi},\vec{\zeta}) = U_{\text{inter}}(\vec{\zeta})U_{\text{intra}}(\vec{\xi})$$

$$\begin{aligned}U_{\text{intra}}(\vec{\xi}) =& e^{-i\frac{\xi_4}{2}(\sigma_A^y\otimes\sigma_B^y+\sigma_{A'}^y\otimes\sigma_{B'}^y)}\\ &\times e^{-i\frac{\xi_3}{2}(\sigma_A^x\otimes\sigma_B^x+\sigma_{A'}^x\otimes\sigma_{B'}^x)}\\ &\times e^{-i\frac{\xi_2}{2}(\sigma_A^z+\sigma_B^z+\sigma_{A'}^z+\sigma_{B'}^z)}\\ &\times e^{-i\frac{\xi_1}{2}(\sigma_A^x+\sigma_B^x+\sigma_{A'}^x+\sigma_{B'}^x)}\end{aligned}$$

$$\begin{aligned}U_{\text{inter}}(\vec{\zeta}) =& e^{-i\frac{\zeta_3}{2}(\sigma_A^z\otimes\sigma_{A'}^z+\sigma_B^z\otimes\sigma_{B'}^z)}\\ &\times e^{-i\frac{\zeta_2}{2}(\sigma_A^y\otimes\sigma_{A'}^y+\sigma_B^y\otimes\sigma_{B'}^y)}\\ &\times e^{-i\frac{\zeta_1}{2}(\sigma_A^x\otimes\sigma_{A'}^x+\sigma_B^x\otimes\sigma_{B'}^x)}\end{aligned}$$

# Infidelity cost function

$$1 - F(\tilde{\tau}(\vec{\xi}, \vec{\zeta}), \tau_\beta)$$

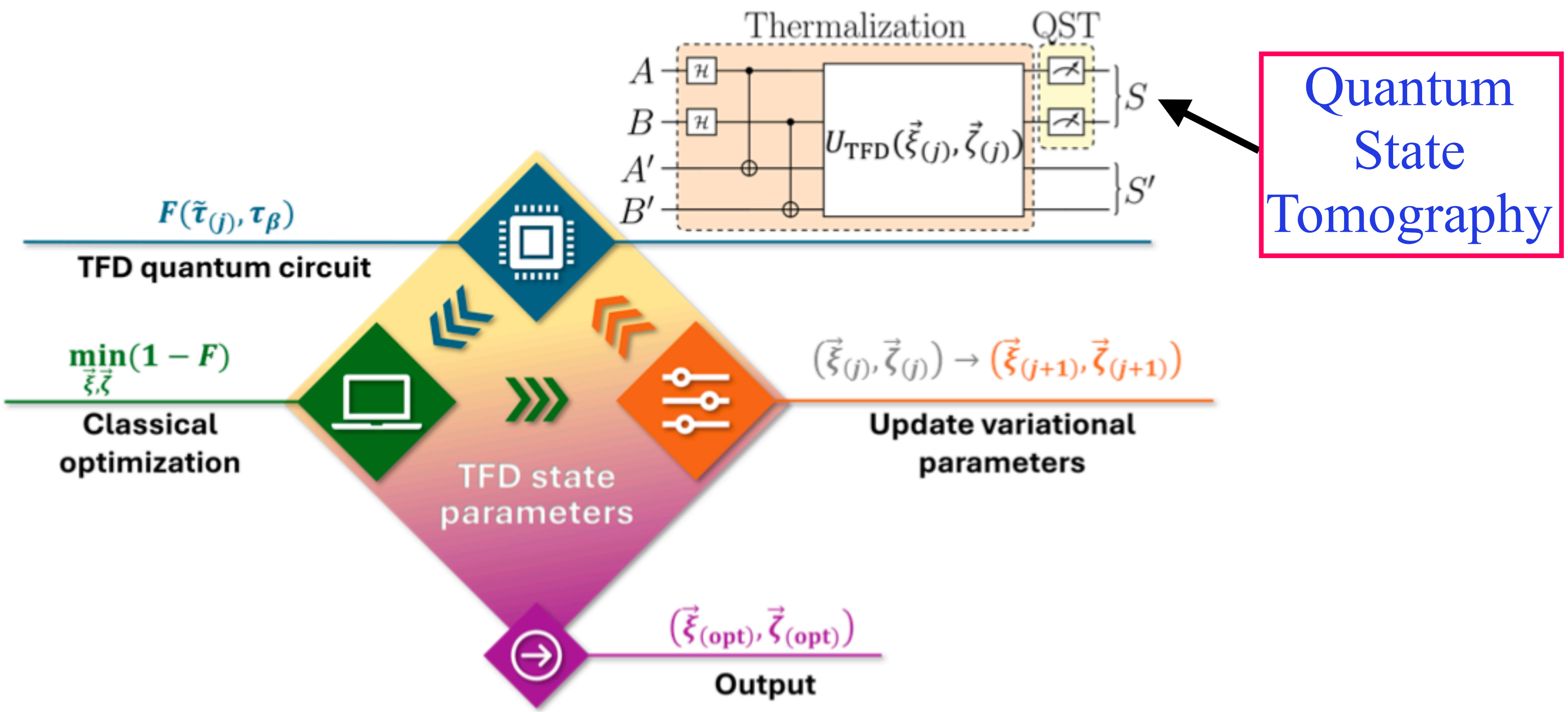
Gibbs state

$$\tilde{\tau}(\vec{\xi}, \vec{\zeta}) = \mathrm{Tr}_{S'}[U_{\mathrm{TFD}}(\vec{\xi}, \vec{\zeta}) \, |\mathrm{TFD}(0)\rangle \langle \mathrm{TFD}(0)| \, U^\dagger_{\mathrm{TFD}}(\vec{\xi}, \vec{\zeta})]$$

$$F(\rho, \sigma) = \mathrm{Tr}\left[\sqrt{\sqrt{\rho}\sigma\sqrt{\rho}}\right]$$

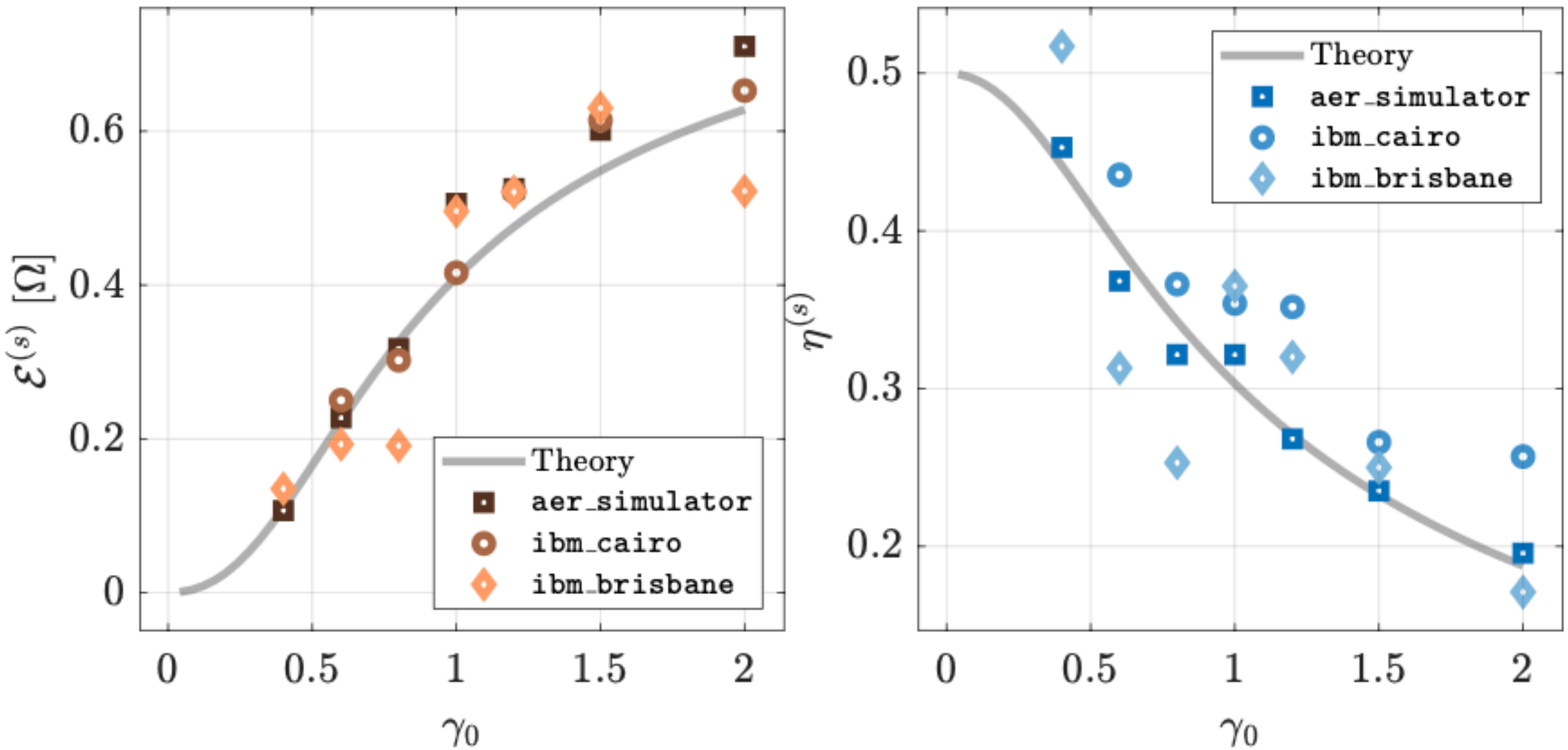
fidelity between two density matrices

# Variational quantum algorithm




Average fidelity 0.97 (simulator) and 0.91 (real quantum hardware) after about 500 iterations

# Performance of the simulated quantum battery




Ergotropy extraction time ~ 100 ns

Longitudinal and transverse relaxation times ~ 100 $\mu$s

Power of the order of $10^{-2}$ fW

# Outlook

Weakly coupled thermal bath used as a battery charger

Non-trivial correlations between qubits can be used to improve performance of the quantum battery

Local unitary operations can (for a given ergotropy) significantly enhance the efficiency of the energy extraction cycle compared to global operations.

Towards solid-state quantum batteries: circuit model and quantum hardware simulations

Future prospects: entangling quantum gates could be used to exploit the quantum advantage in the charging time of quantum batteries