Recovering Entanglement by Local Operations

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Outline

Problem: entanglement revivals without non-local operations

Quantification of entanglement based on the density operator formalism vs ensemble description

Definition of hidden entanglement

Examples: Random local fields, solid state system affected by low frequency noise, quantum information scheme, etc.

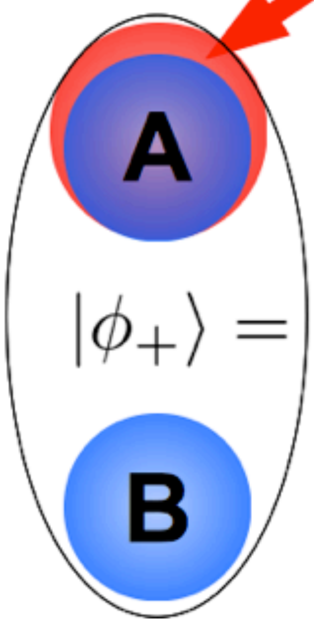
Non-Markovian dynamics and non-monotonous behavior of entanglement

Example: random local field

Random Local Field

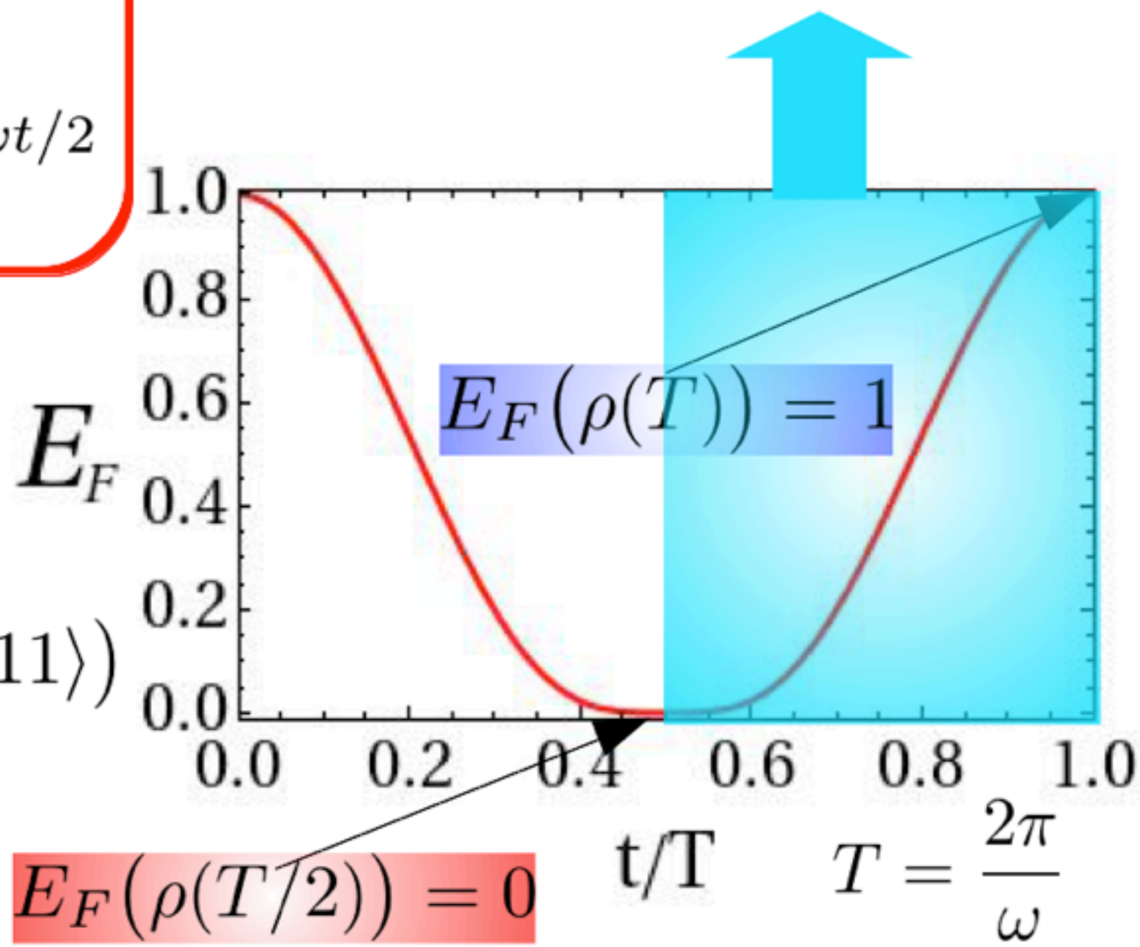
$$p_x = \frac{1}{2}, \quad U_x = e^{-i\sigma_x \omega t/2}$$

$$p_z = \frac{1}{2}, \quad U_z = e^{-i\sigma_z \omega t/2}$$



$$|\phi_+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

entanglement increases
without **non-local operations**



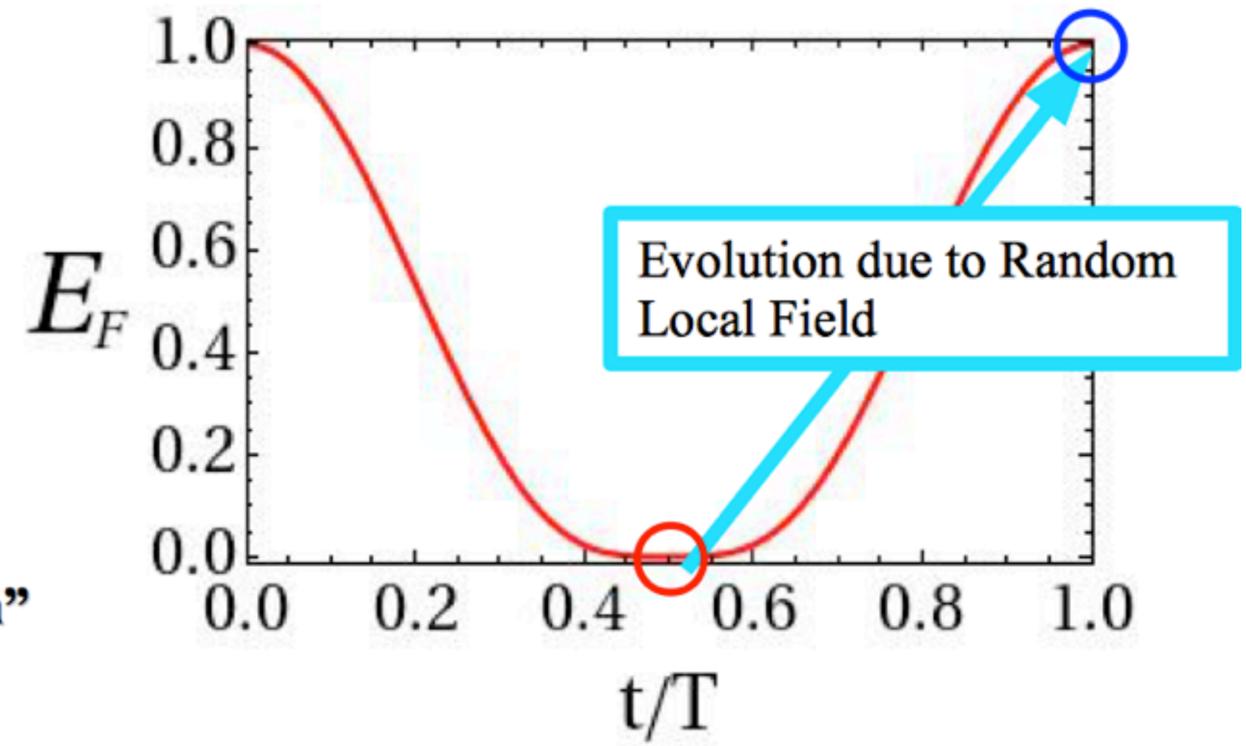
$$T = \frac{2\pi}{\omega}$$

$E_F\left(\rho(T/2)\right) = 0$ $\xrightarrow{]0.5T,\ T]}$ $E_F\left(\rho(T)\right) = 1$

Manifestation of **quantum correlations** that must **be present before**

Density operator formalism does not capture these **quantum correlations**

quantum correlations are *"hidden"* at T/2



Can we quantify these "hidden" correlations?

Quantum ensemble description and hidden entanglement

Quantum trajectories:

$$p_x, \quad |\psi_x(t)\rangle = U_x|\phi_+\rangle$$

$$p_z, \quad |\psi_z(t)\rangle = U_z|\phi_+\rangle$$

$$\mathcal{A}(t) = \{p_i, \, |\psi_i(t)\rangle\} \Rightarrow E_{av}(\mathcal{A}) = \sum_i p_i \, E(|\psi_i(t)\rangle)$$

Quantum ensemble

Density operator:

$$\rho(t) = \sum_i p_i |\psi_i(t)\rangle\langle\psi_i(t)| \Rightarrow E(\rho)$$

mixture

We can define **"hidden" entanglement:**

$$E_h(\mathcal{A}) \equiv E_{av}(\mathcal{A}) - E(\rho)$$

$$= \sum_i p_i E(|\psi_i(t)\rangle) - E\Big(\sum_i p_i |\psi_i(t)\rangle\langle\psi_i(t)|\Big)$$

E is any convex entanglement measure

E_h: Entanglement not exploitable, lack of classical information

It may be **recovered without non-local operations**

Example: random local field

$$\boxed{E_{av}(\mathcal{A})} = \sum_i p_i\, E\big(|\psi_i(t)\rangle\big) = \sum_i p_i\, E\big(U_i|\phi_+\rangle\big) = \boxed{1 \quad \forall\, t}$$

$T/2 \qquad \rho(T/2)$ separable

$$U_x(T/2) = -i\sigma_x \quad U_z(T/2) = -i\sigma_z$$

$$\mathcal{A}(T/2) = \left\{ \left(\frac{1}{2}, |\phi_-\rangle\right), \left(\frac{1}{2}, |\psi_+\rangle\right) \right\}$$

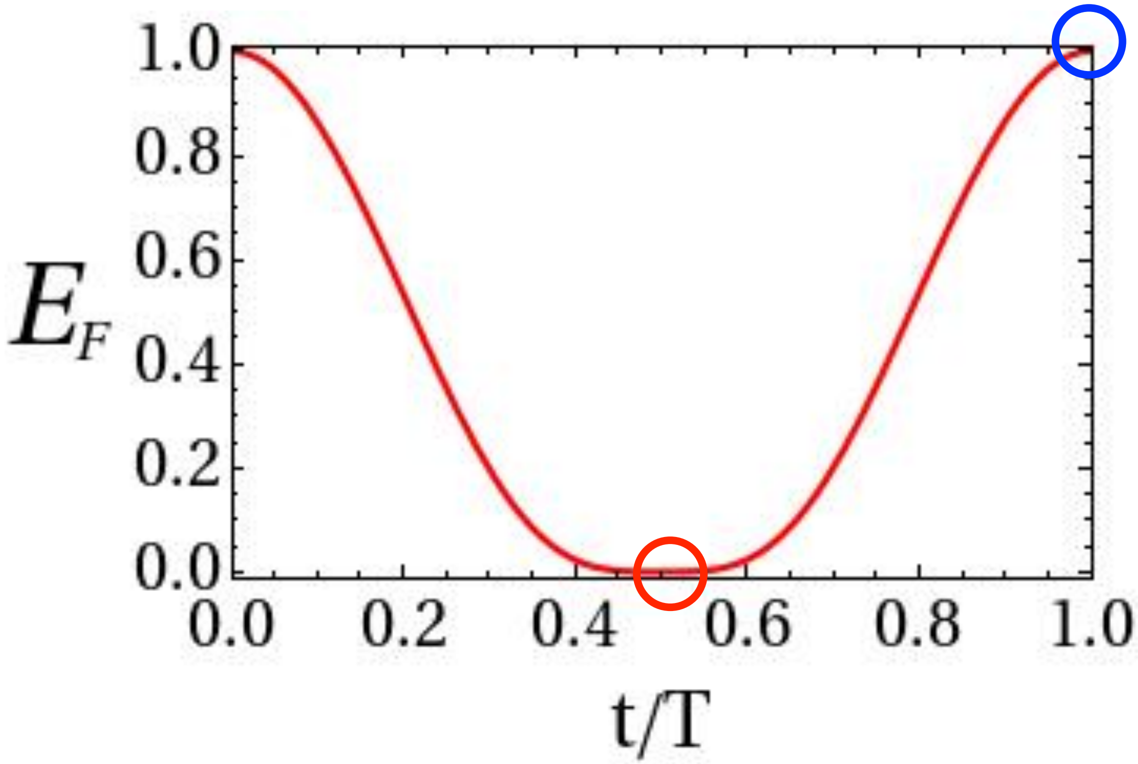
$$E_h(T/2) = 1$$

$T \qquad \rho(T)$ entangled

$$U_x(T) = U_z(T) = I_d$$

$$\mathcal{A}(T) = \{1, |\phi_+\rangle\}$$

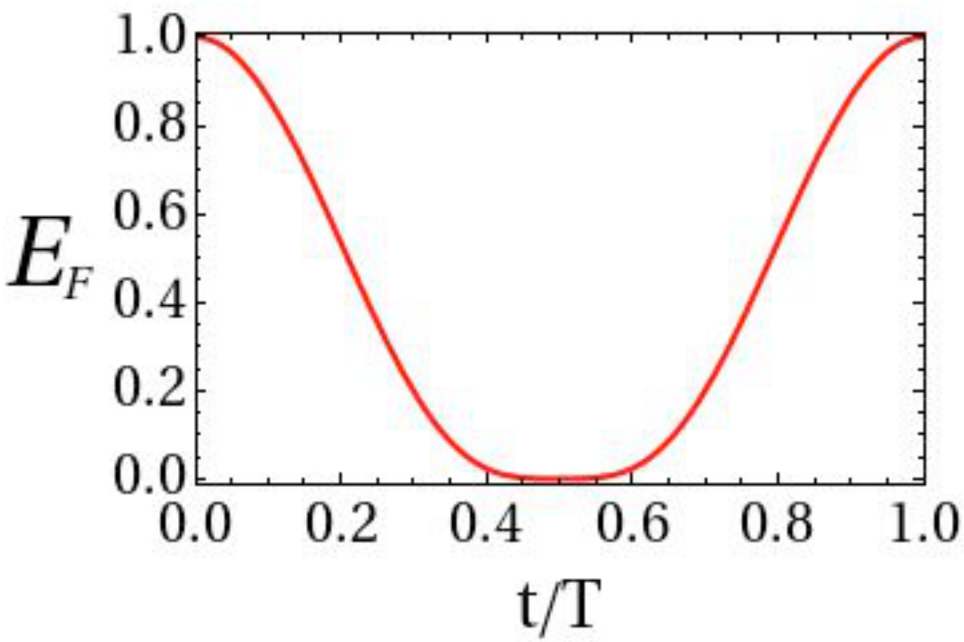
$$E_h(T) = 0$$



Violation of the monotonicity axiom?

Monotonicity axiom:
Entanglement cannot increase under LOCC

The entanglement recovery is induced by local operations that are not LOCC (the evolution of the density operator from T/2 to T is not a CPT map)



Purely dephasing random telegraph noise

$$\mathcal{H} = \mathcal{H}_0 + \delta\mathcal{H},$$

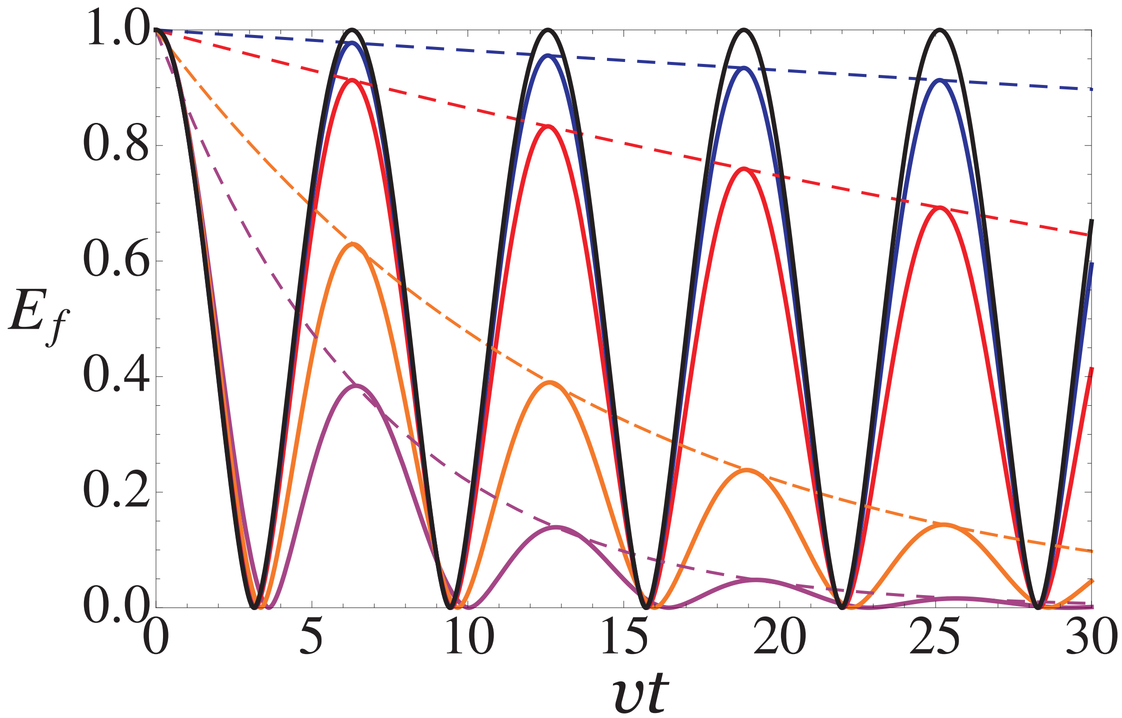
$$\mathcal{H}_0 = -\frac{\Omega_A}{2}\sigma_{z_1} - \frac{\Omega_B}{2}\sigma_{z_2}, \qquad \delta\mathcal{H} = -\frac{\xi(t)}{2}\sigma_{z_1} \quad \xi(t) \in \{0, v\}$$

switching rate γ

At γ=0: $\mathcal{A} = \left\{ \left(p_0, |\varphi_0(t)\rangle\right), \left(p_v, |\varphi_v(t)\rangle\right) \right\}$

$$|\varphi_0(t)\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + e^{-i(\Omega_A+\Omega_B)t}|11\rangle\right)$$

$$|\varphi_v(t)\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + e^{-ivt}e^{-i(\Omega_A+\Omega_B)t}|11\rangle\right)$$



Entanglement revivals at

$$t_n = 2n\pi/v$$

(for $v/\gamma > 1$)

Peak values drop as

$$f(C) = f(e^{-\gamma t/2})$$

Stochastic low-frequency noise

Stochastic low-frequency noise

$$\mathcal{H}_A = -\frac{\Omega_A}{2}\sigma_z - \frac{\varepsilon(t)}{2}\sigma_z$$

Static noise

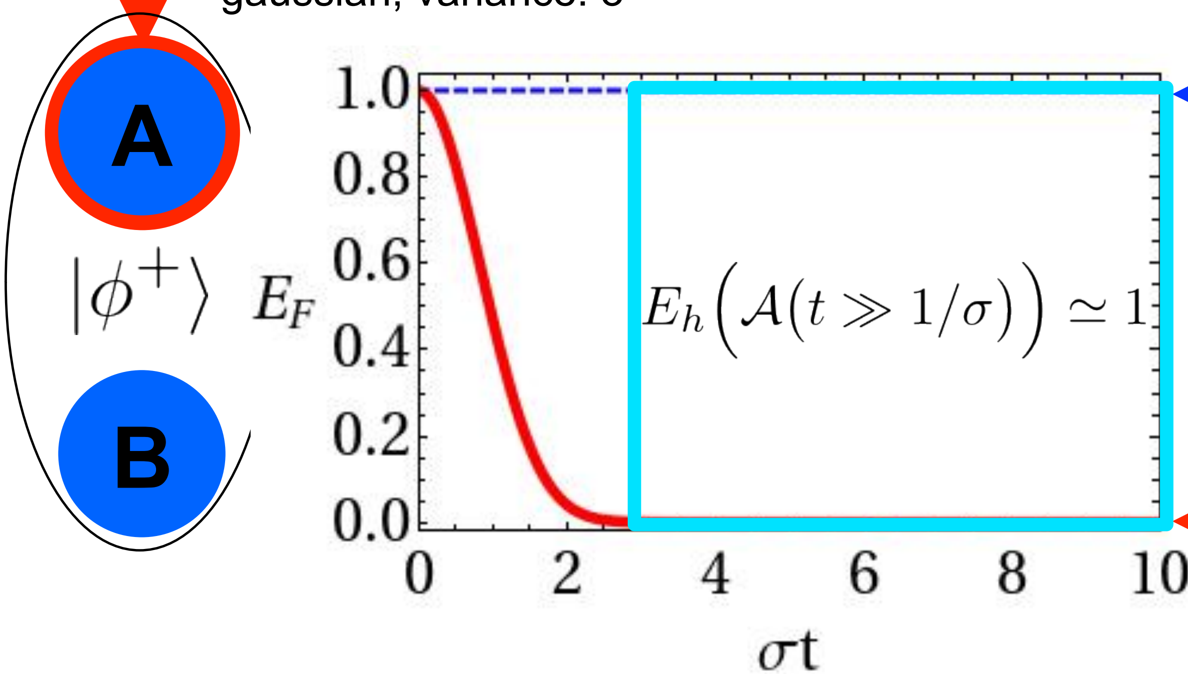
$$\varepsilon(t) \simeq \varepsilon \qquad C(\rho(t)) = e^{-\frac{1}{2}\sigma^2 t^2}$$

gaussian, variance: σ^2

Quantum trajectories:

$$|\psi_\varepsilon(t)\rangle = e^{-i\frac{\Omega_A+\varepsilon}{2}\sigma_z t}|\phi_+\rangle$$

$$\mathcal{A}(t) = \{p(\varepsilon)d\varepsilon,\ |\psi_\varepsilon(t)\rangle\}$$



$$E_{av}\big(\mathcal{A}(t)\big) = 1$$

$$\rho(t) = \int d\varepsilon p(\varepsilon)|\psi_\varepsilon(t)\rangle\langle\psi_\varepsilon(t)|$$

$$E_F\big(\rho(t \gg 1/\sigma)\big) \simeq 0$$

Recovering entanglement by local pulses

Stochastic low-frequency noise

$$\mathcal{H}_A = -\frac{\Omega_A}{2}\sigma_z - \frac{\varepsilon(t)}{2}\sigma_z + \mathcal{V}(t,\bar{t})\sigma_x$$

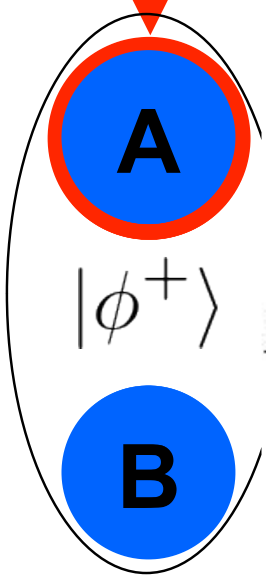
$$\mathrm{e}^{-i\sigma_x \pi/2} = -i\sigma_x$$

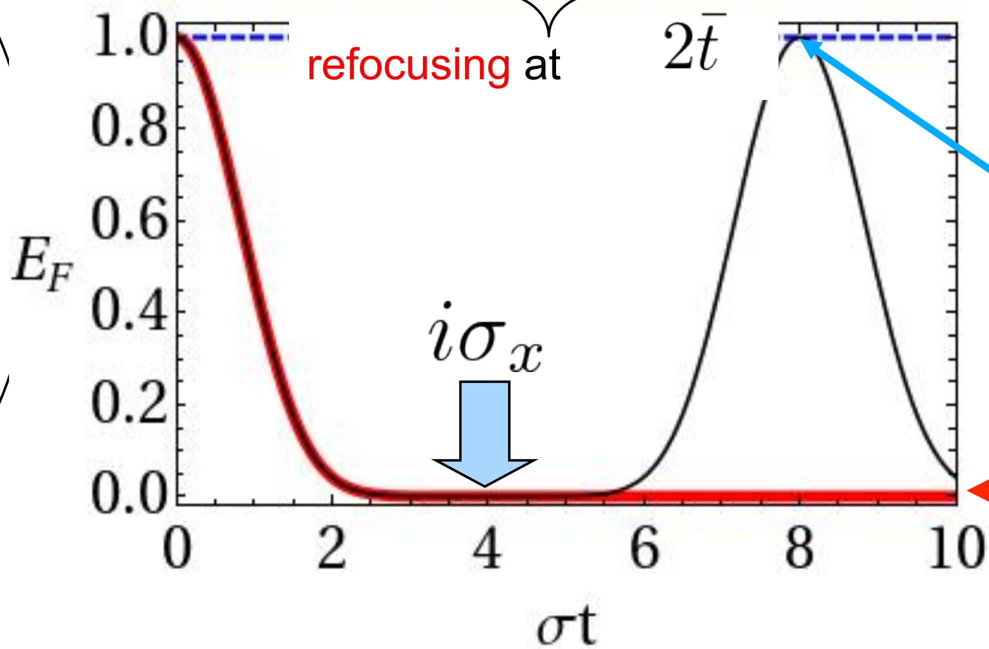
π -pulse around x

$$i\sigma_x$$

$$|\psi_\varepsilon(t)\rangle = e^{-i\frac{\Omega_A+\varepsilon}{2}\sigma_z(t-\bar{t})}\sigma_x e^{-i\frac{\Omega_A+\varepsilon}{2}\sigma_z\bar{t}}|\psi(0)\rangle$$

$$= \underbrace{e^{-i\frac{\Omega_A+\varepsilon}{2}\sigma_z(t-\bar{t})}e^{+i\frac{\Omega_A+\varepsilon}{2}\sigma_z\bar{t}}}_{\text{refocusing at } 2\bar{t}}\sigma_x|\psi(0)\rangle$$





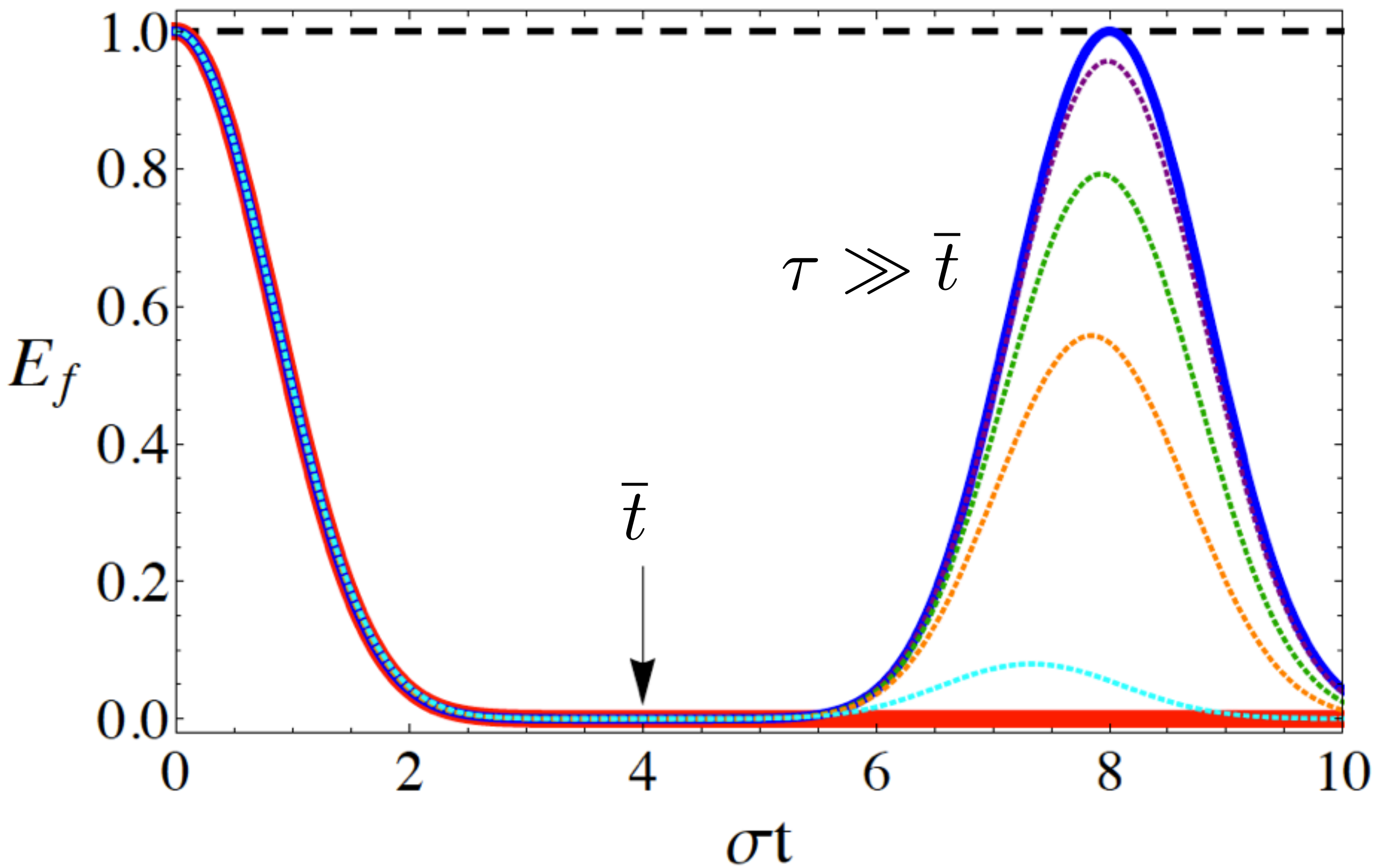
$$E_{av}\big(\mathcal{A}(t)\big) = 1$$

Complete recovery of Entenglement (for static noise)

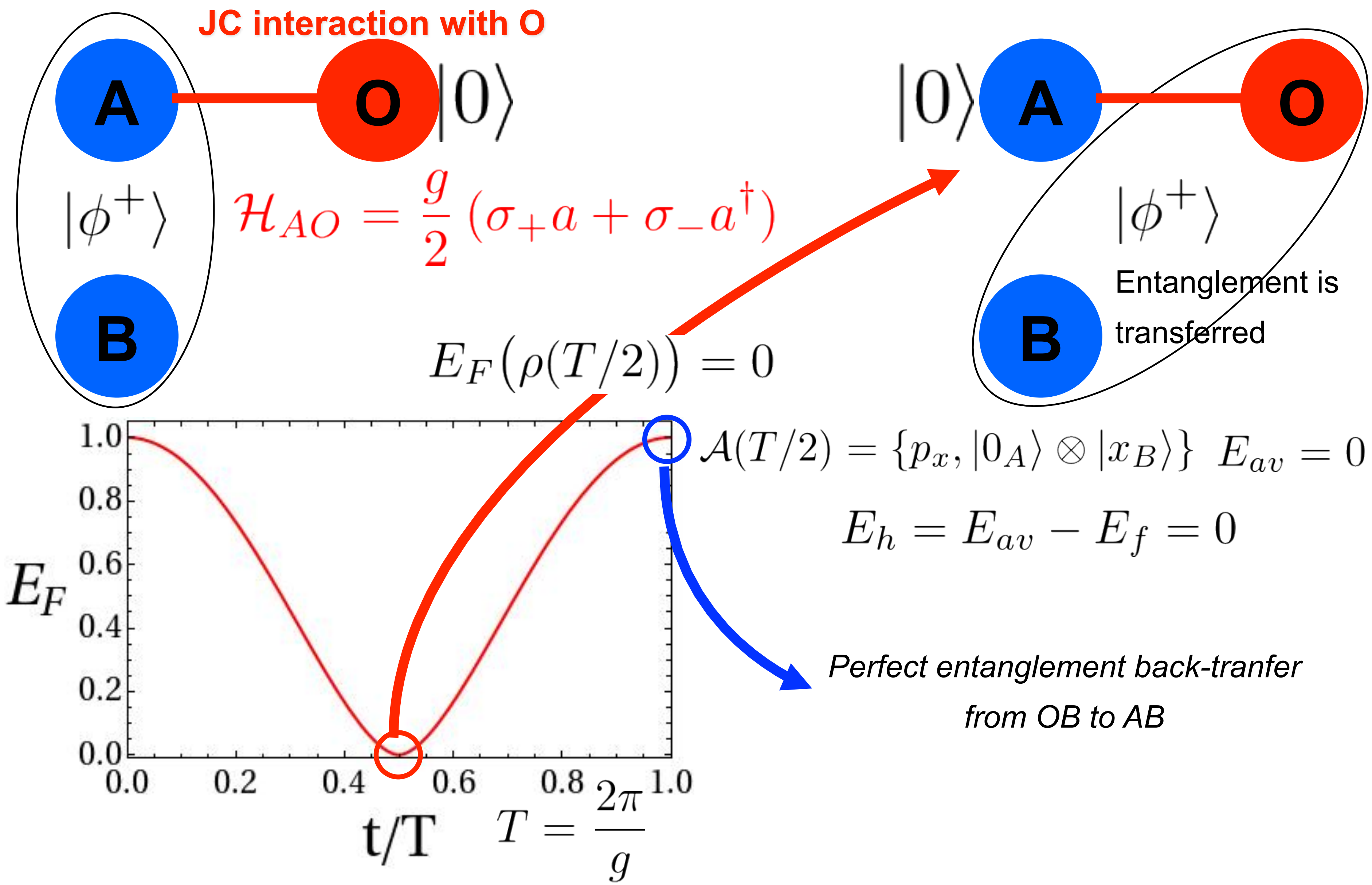
$$E_F\Big(\rho(t \gg 1/\sigma)\Big) \simeq 0$$

Partial recovery with **finite noise correlation time scale**

$$\langle \varepsilon(t)\varepsilon(0)\rangle = \sigma^2 e^{-|t|/\tau}$$



Non-Markonian dynamics and entanglement revivals

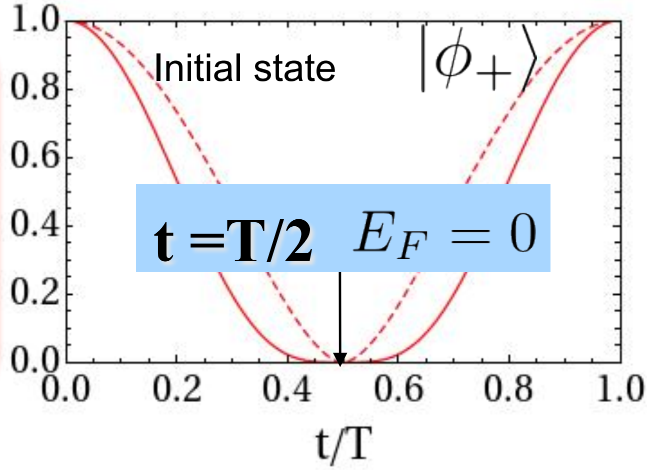


Entanglement revivals in classical vs quantum environments

Random Local Field

$$p_z = \frac{1}{2}, \quad U_z = -i\sigma_z$$

$$p_x = \frac{1}{2}, \quad U_x = -i\sigma_x$$



JC interaction with O

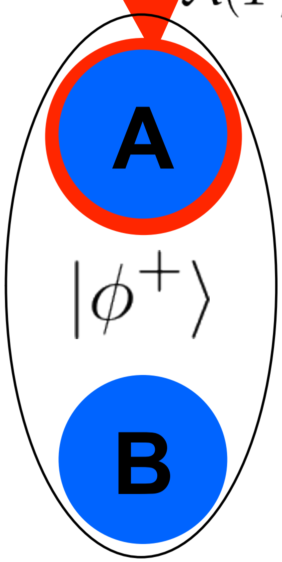
$$\mathcal{H}_{AO} = \frac{g}{2}(\sigma_+ a + \sigma_- a^\dagger)$$

SWAP(A,O)

$$\mathcal{A}(T/2) = \left\{ \left(\frac{1}{2}, |\phi_-\rangle\right), \left(\frac{1}{2}, |\psi_+\rangle\right) \right\}$$

$$E_{av} = 1$$

$$E_h = 1$$



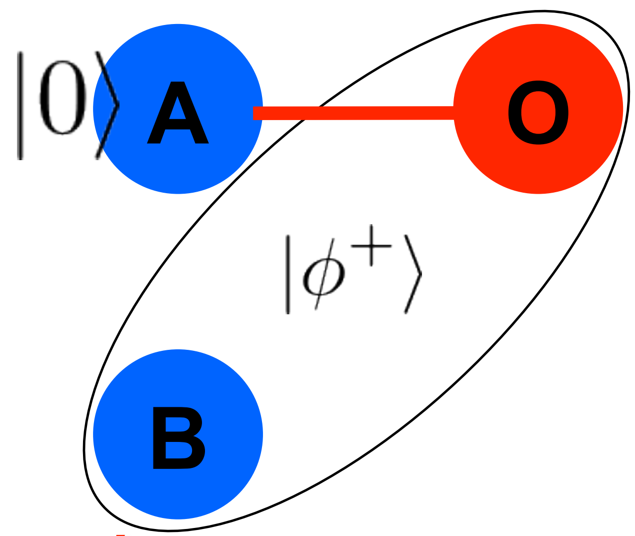
classical information about which random unitaries AB underwent

Recovery!

$$\mathcal{A}(T/2) = \{p_x, |0_A\rangle \otimes |x_B\rangle\}$$

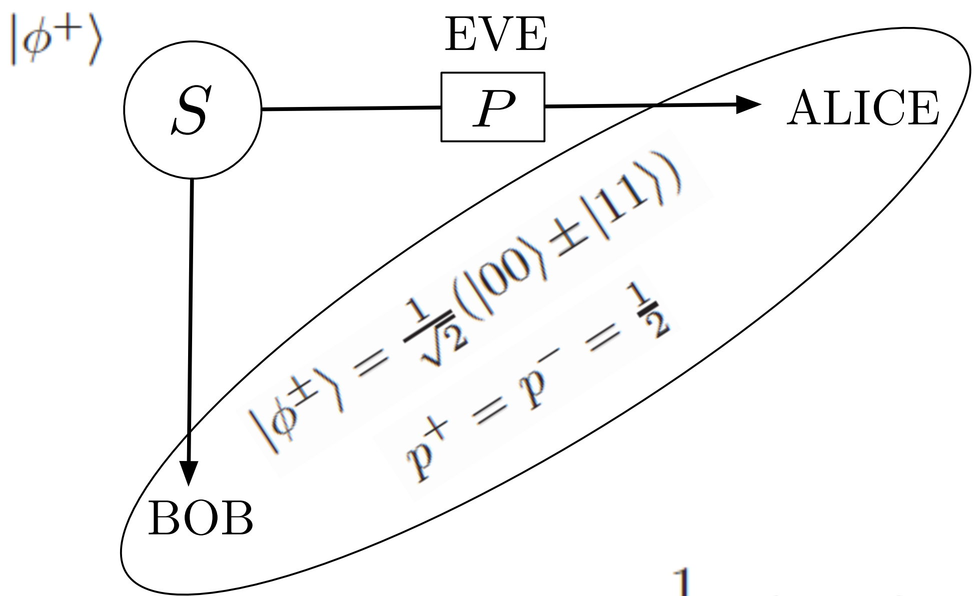
$$E_{av} = 0$$

$$E_h = 0$$



No recovery!

Quantum information scheme



$$\rho_{ABE} = \frac{1}{2}|\phi^+\rangle\langle\phi^+| \otimes |\kappa^+\rangle\langle\kappa^+| + \frac{1}{2}|\phi^-\rangle\langle\phi^-| \otimes |\kappa^-\rangle\langle\kappa^-|$$

$\{|\kappa^+\rangle, |\kappa^-\rangle\}$ orthonormal states for Eve

$$\rho_{AB} = \tfrac{1}{2}(|00\rangle\langle 00| + |11\rangle\langle 11|)$$

Quantum mutual information

$$I(Q_1 : Q_2) = S(\rho_{Q_1}) + S(\rho_{Q_2}) - S(\rho_{Q_1 Q_2})$$

$$I(A : B) = 1$$

$$I(AB : E) = 1$$

$$I(A : E) = I(B : E) = 0$$

Quantum key distribution possible due to hidden entanglement

$$\mathcal{A} = \{(1/2, |\phi^+\rangle), (1/2, |\phi^-\rangle)\}$$

$$E_{av}(\mathcal{A}) = E_h(\mathcal{A}) = 1$$

Summary

Defined **"Hidden" Entanglement (HE)** on the basis of the ensemble description of the system dynamics

HE>0: a **recovery** of **entanglement** by **local operation** and **classical communication** is possible

Practical relevance in **solid state quantum computing**: **HE** indicates the amount of entanglment recoverable by local pulses (**dynamical decoupling**: echo, bang-bang....)

Non-Markovian dynamics:
classical vs **quantum** environments