Shallow quantum circuits are robust hunters for quantum many-body scars

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General motivation: NISQ devices

Quantum computers are prone to **noise** and **decoherence:**

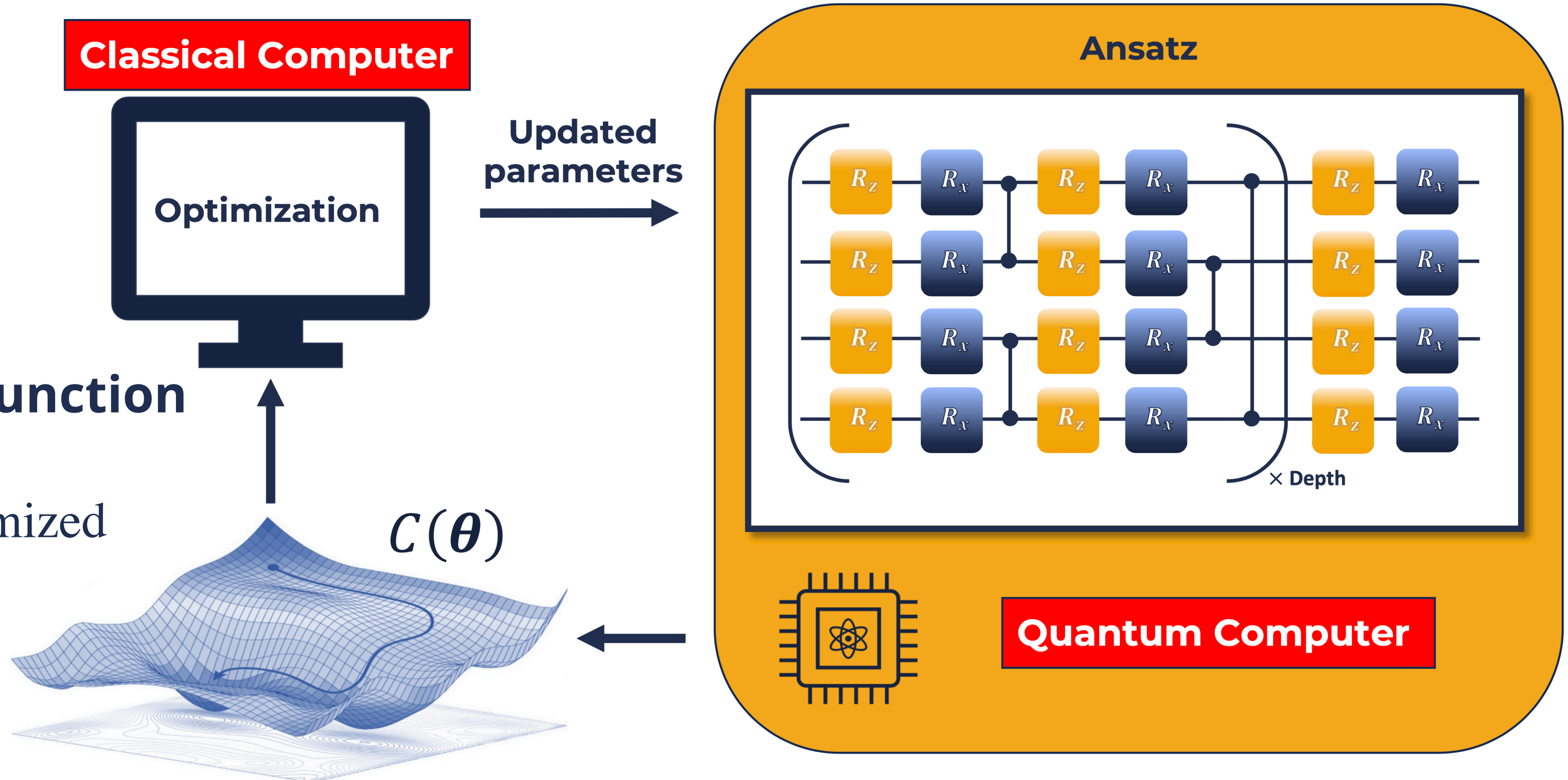
- ❖ **Coherent errors**
  - ➢ **Gate imperfections, drift in control parameters**
  - ➢ **Crosstalk**
- ❖ **Incoherent errors**
  Due to the interaction with the enviroment
  - ➢ **Bit-flip, phase-flip**
  - ➢ **Amplitude damping**
  - ➢ **Phase damping**
  - ➢ **Depolarizig noise**
  - ➢ **Readout errors**

→

- ➢ **Error mitigation**
- ➢ **Hybrid (Variational) Quantum Algorithms**

Variational Quantum Algorithms (VQAs)

A **variational quantum algorithm** combines **quantum circuits** with **classical optimization techniques:** designed to solve optimization problems and are particularly well-suited for **NISQ devices**

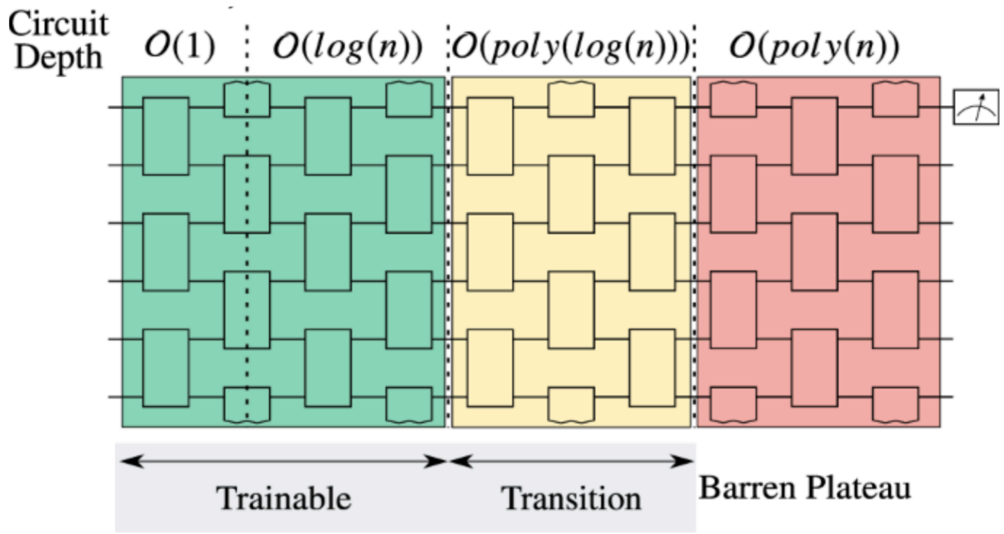


- **Cost function** $C(\theta)$ to be optimized

- **Ansatz:** parametric quantum circuit

Barren plateau problem

Noisy and **deep** ansatze induce **Barren plateau**, i.e., vanishing gradient:



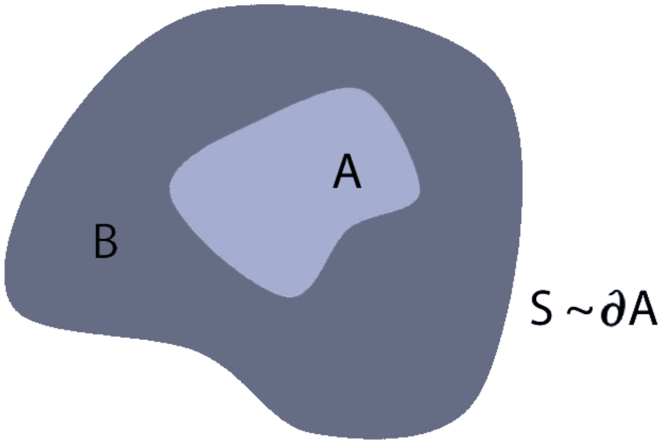
M. Cerezo, et al. Cost function dependent barren plateaus in shallow parametrized quantum circuits. *Nature communications* **12**, 1791 (2021).

➔ Ansatz depth must be kept **shallow**

Why many-body quantum scars?

Eigenstates of **non-integrable** many-body systems (in a lattice) which present peculiar characteristics such as **area-law entanglement** entropy and breaking of **eigenstate thermalization hypothesis (ETH)**

Area-law →



ETH (volume law) states:

Ergodic dynamics of the observables, thermal behaviour
➔loss of the information about the initial state
➔no dynamical revivals

Scar (area law) states:

States with **overlap** with scars exhibit long-lived oscillations (revivals) and thus **non-thermal behaviour** [see T. Iadecola and M. Schecter, PRB **101**, 024306 (2020)].

Why are scars interesting?

(Lack) of thermalization in isolated systems, a problem of broad interest in view of the rapid development of quantum simulators that evolve in near-perfect isolation from their environment

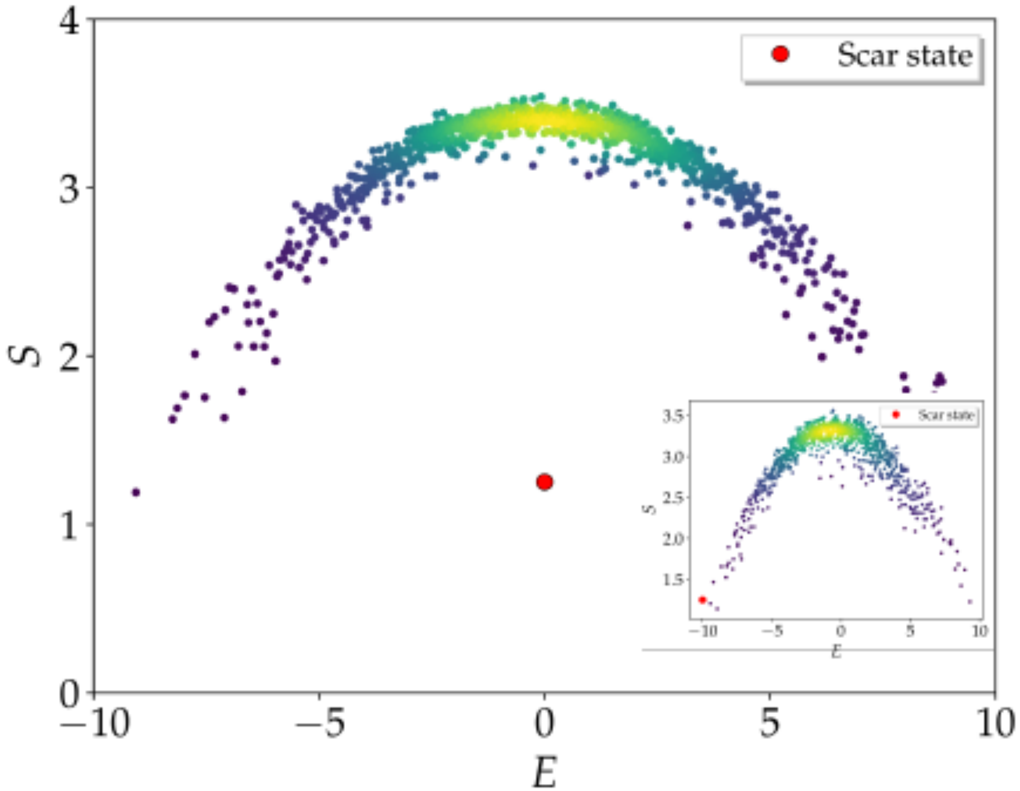
Scars are "rare" and immersed in a sea of thermalizing (ETH) states: can we nevertheless detect them?

Considered models

Model1: 1D Hamiltonian of *N* **hardcore bosons** placed in a circular lattice:

$$H_1 = \sum_{i\neq j} G^A_{ij} d^\dagger_i d_j + \sum_{i\neq j} G^B_{ij} n_i n_j$$

$$+ \sum_{i\neq j\neq l} G^C_{ijl} d^\dagger_i d_l n_j + \sum_i G^D_i n_i + G^E$$



A single scar state whose position can be moved in the spectrum adjusting parameters

Constant of motion: total number of bosons

$$\hat{N}_b = \sum_i n_i, \quad n_j = d^\dagger_j d_j$$

Considered models

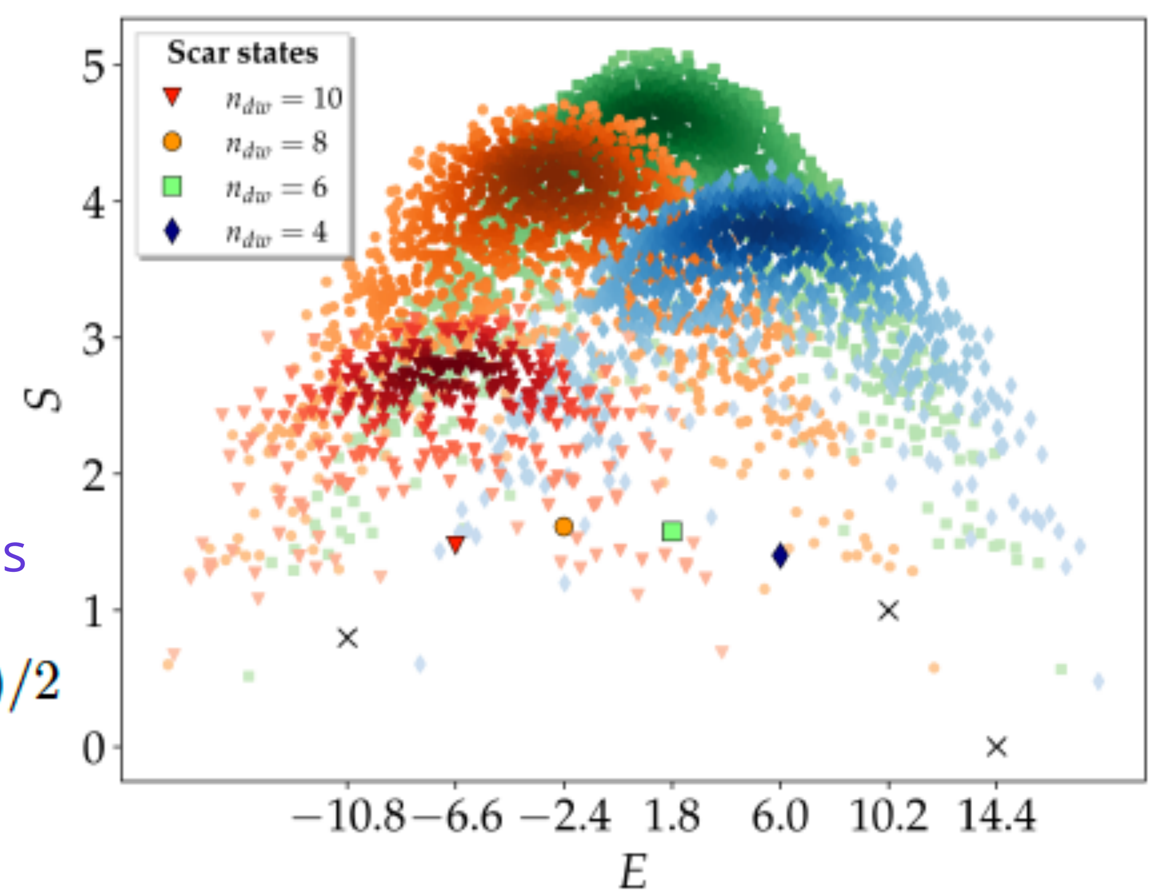
Model 2: 1D Hamiltonian of **spin-1/2 model** on a *N*-length chain

$$H_2 = \lambda \sum_{i=2}^{N-1} (\sigma_i^x - \sigma_{i-1}^z \sigma_i^x \sigma_{i+1}^z) + \Delta \sum_{i=1}^{N} \sigma_i^z + J \sum_{i=1}^{N-1} \sigma_i^z \sigma_{i+1}^z$$

Tower of scars

Constant of motion:
number of domain walls

$$\hat{N}_{dw} = \sum_{i=1}^{N-1} (1 - \sigma_i^z \sigma_{i+1}^z)/2$$



Cost function

$$C(\theta) = a\left\langle (H-E)^2 \right\rangle + b\left(\langle H^2 \rangle - \langle H \rangle^2 \right) + c f_{symm}$$

Target energy and symmetry sector, reduce variance

Multi-objective (Pareto) optimization problem

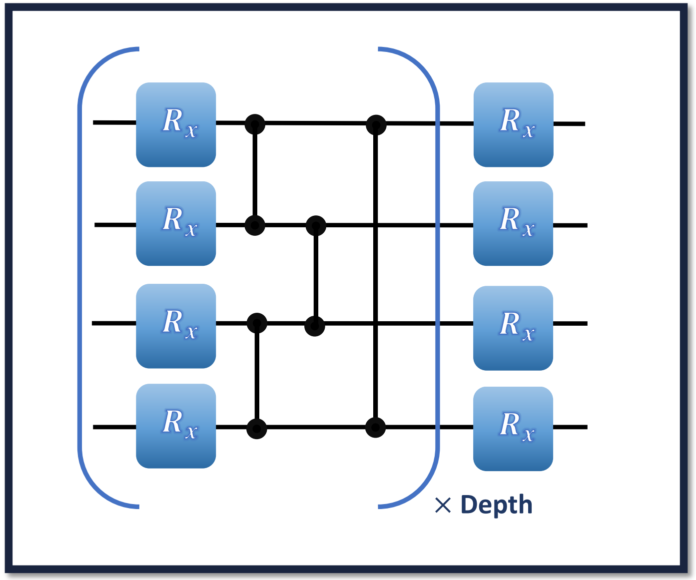
$$a,\ b, c \in [0,1]\ :\ a+b+c=1$$

$$\theta_{opt} = \mathrm{argmin}_{\theta} C(\theta)$$

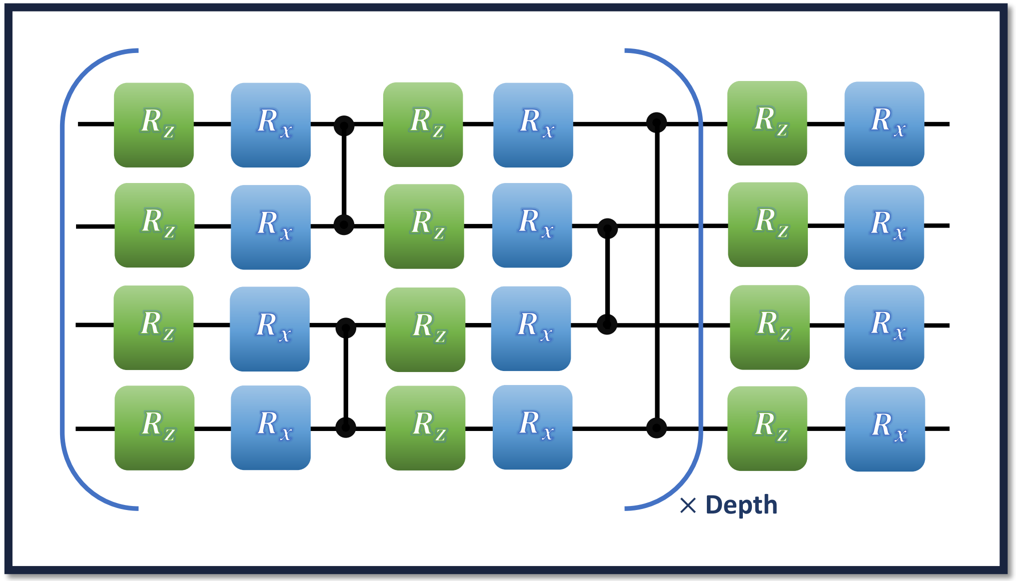
$$\left|\psi_{vqe}\right\rangle = U\left(\theta_{opt}\right)|0\rangle$$

Explored ansatze

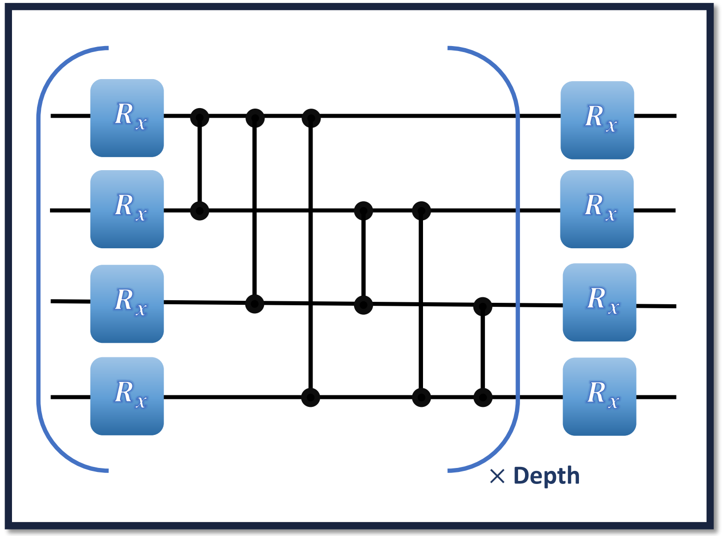
Nearest-neighbour (NN) ansatz:



Hardware-efficient (HE) ansatz:

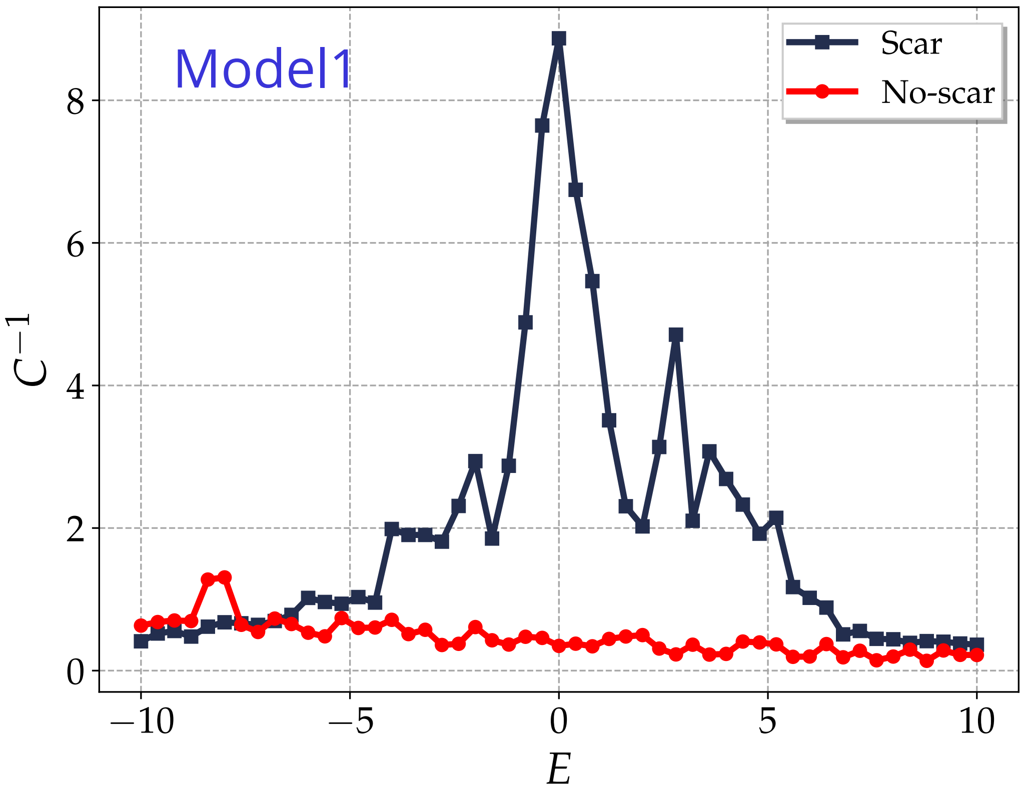


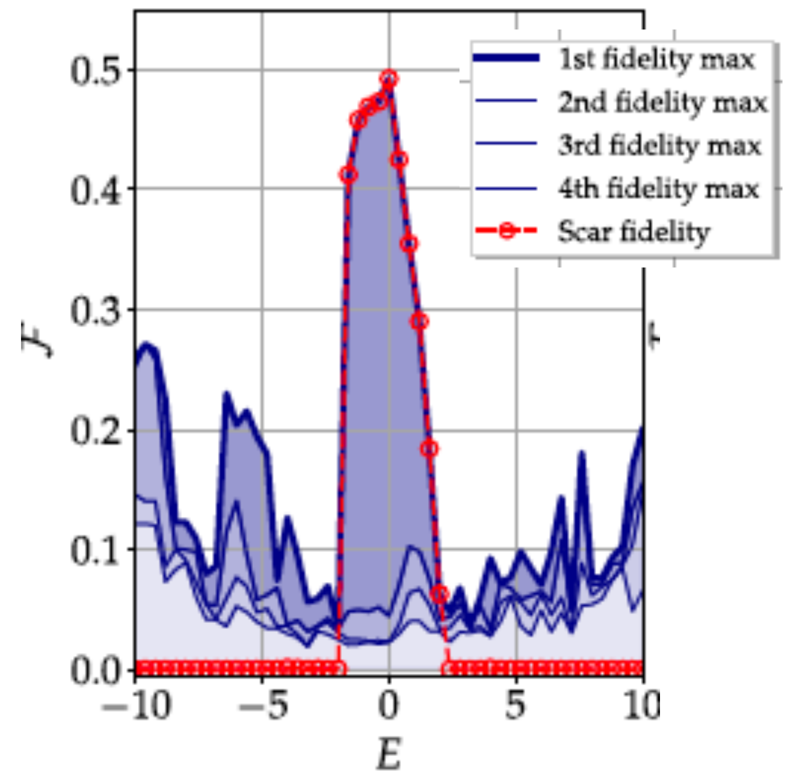
All-to-all (AA) ansatz:



Detecting scars

Agnostic ansatz: assume no previous knowledge besides system symmetries



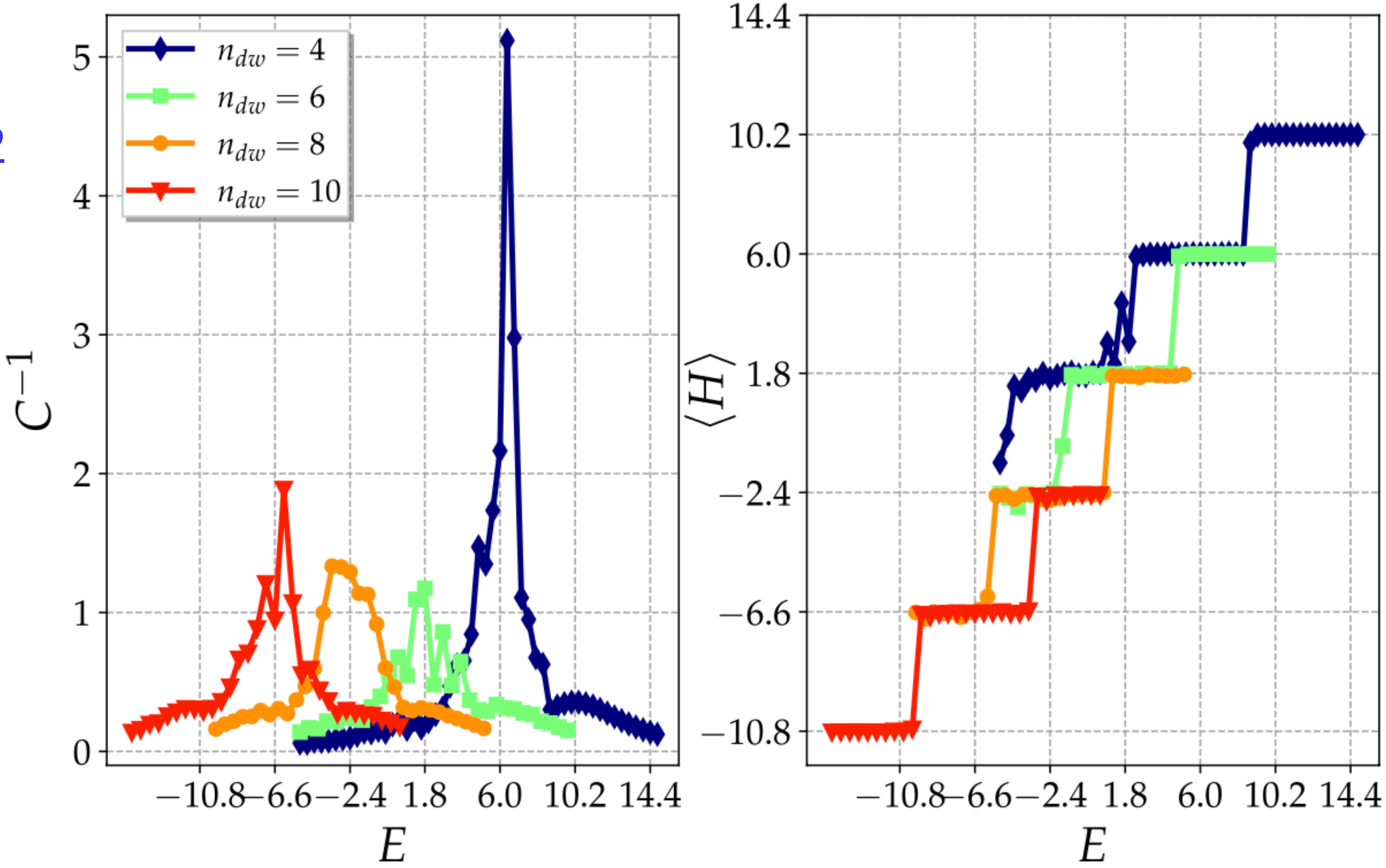


$$\mathcal{F} = \left| \left\langle \psi_{vqe} \middle| \psi_{SCAR} \right\rangle \right|^2$$

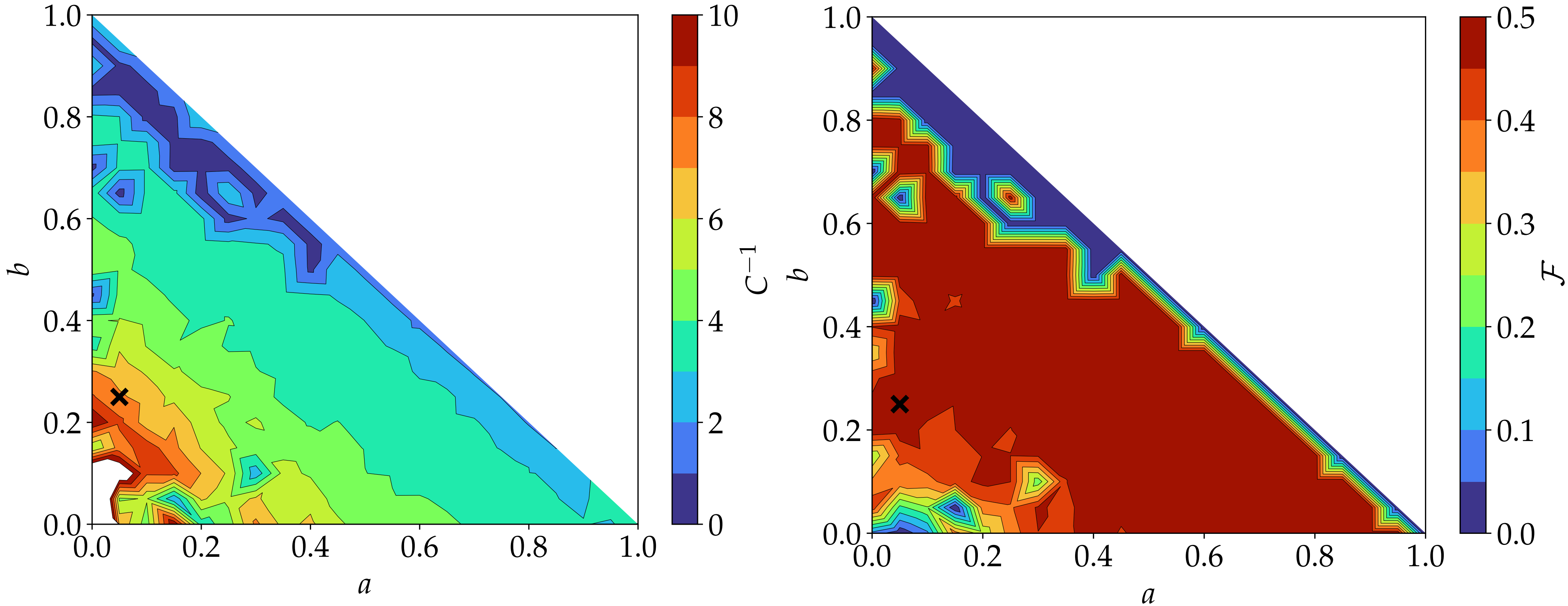
Detecting scars

Energy sweep: detecting scars also via mean energy

Model2



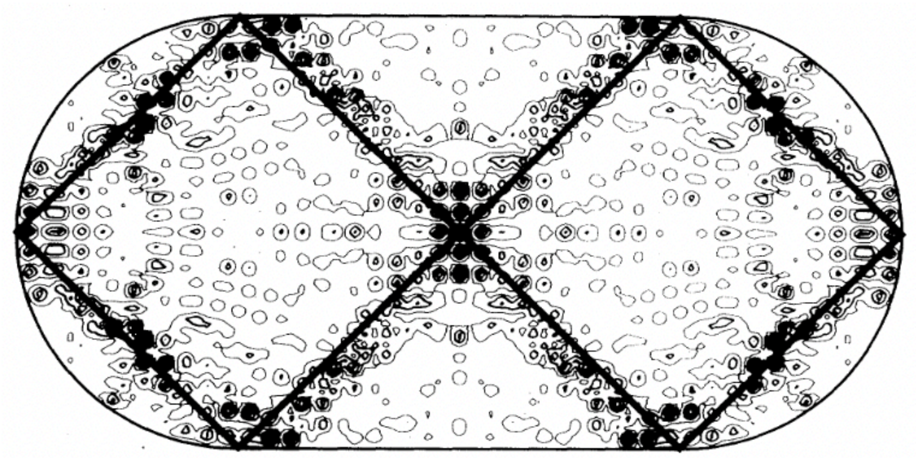
Algorithm robustness



Conclusions

Variational quantum algorithm converge despite the fact scar states are immersed in a sea of thermalizing states

VQE very versatile tool to search for scar states even in 2D and 3D and at the semiclassical limit (not easy for classical MPS methods)



[see E. J. Heller, PRL **53**, 1515 (1984)]

Real quantum hardware implementations seem possible