Exotic states in the dynamical Casimir effect

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OUTLINE

The dynamical Casimir effect: a fundamental limitation to the maximum speed of quantum protocols

Interaction between a qubit and a single mode of the field in the ultrastrong coupling regime: the rotating wave approximation cannot be applied

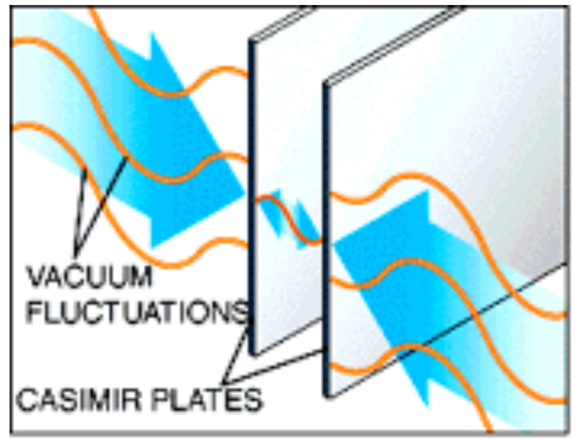
Non-adiabatic regime (finite-time quantum electrodynamics): emission of photons (manifestation of the dynamical Casimir effect)

Quantum communication protocol: coherent information degradation vs. dynamical Casimir effect

(Static) Casimir effect

Casimir (1948): Two uncharged conducting parallel plates experience attractive forces

Effect explained in terms of quantum mechanical vacuum fluctuations of the electromagnetic field: The two plates impose boundary conditions on the field, so that the density of electromagnetic modes between the plates depend on their distance d.

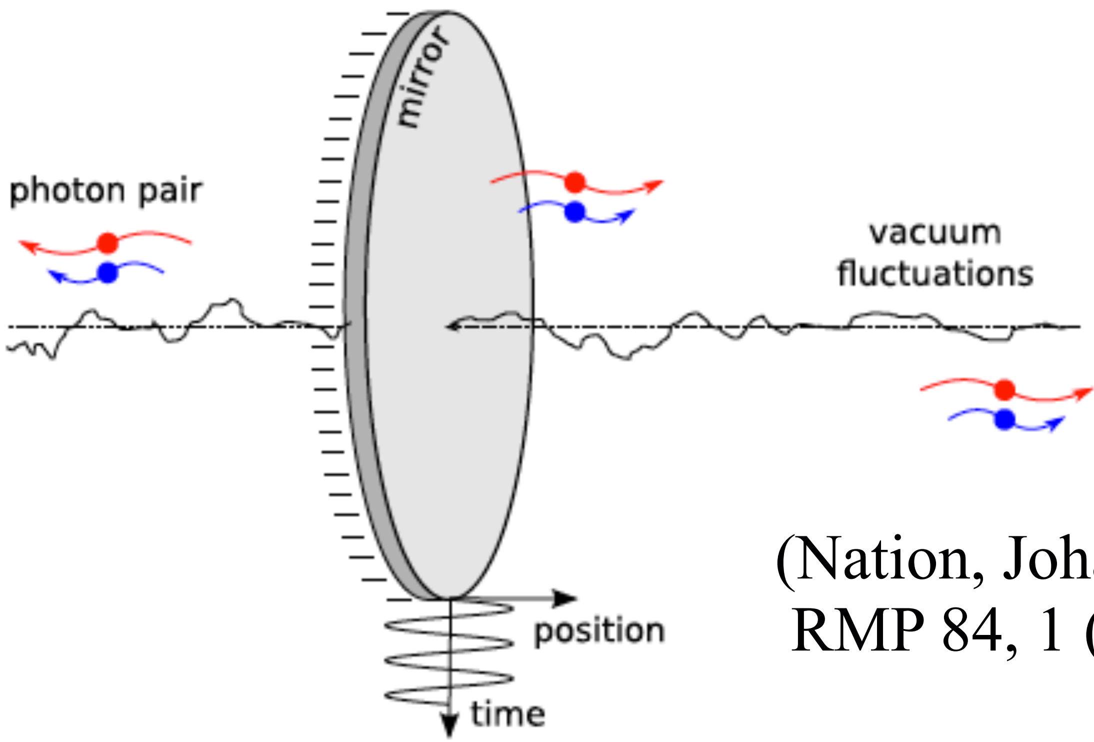


$$F \propto d^{-4}$$

Casimir effect important both for investigations in fundamental physics and for basic limitations of nanomechanical technologies

The dynamical Casimir effect

The dynamical Casimir effect concerns the generation of real photons from the vacuum due to time-dependent boundary conditions



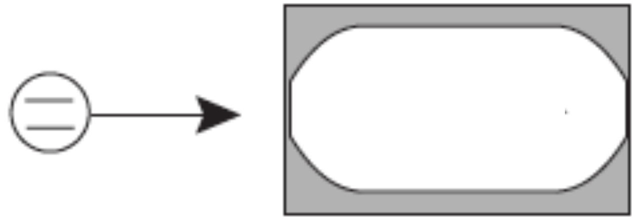
(Nation, Johansson, Blencowe, Nori, RMP 84, 1 (2012))

Related to other vacuum amplification effects, the Hawking radiation (radiation released by black holes) and Unruh effect (for an accelerated observer)

Experimental observation in superconducting circuits (SQUIDs embedded in a cavity, with the effective length of the cavity modulated -at speeds of the few per cent of the speed of light- by time-varying the external flux)

(Wilson et al., Nature 479, 376 (2011); Lähteenmäki et al., PNAS 110, 4234 (2013))

Qubit-oscillator system



Rabi Hamiltonian with a switchable coupling:

$$H(t) = H_0 + H_I(t),$$

$$\sigma_+|g\rangle = |e\rangle,\ \sigma_+|e\rangle = 0,$$

$$\sigma_-|g\rangle = 0,\ \sigma_-|e\rangle = |g\rangle$$

$$H_0 = -\frac{1}{2}\omega_a\sigma_z + \omega\left(a^\dagger a + \frac{1}{2}\right),$$

$$a^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle,$$

$$a|n\rangle = \sqrt{n}|n-1\rangle$$

$$H_I(t) = f(t)\left[g\,\sigma_+\left(a^\dagger + a\right) + g^\star\sigma_-\left(a^\dagger + a\right)\right]$$

Focus on the resonant case $(\omega = \omega_a)$

Rotating-wave approximation

RWA: neglect the term which simultaneously excites the two-level system and creates a photon or de-excites the two-level system and annihilates a photon

Jaynes-Cummings model:

$$H_I(t) = f(t)[g\sigma_+ a + g^\star \sigma_- a^\dagger]$$

For two-level atoms in a cavity the frequency of Rabi oscillations between states |g>|n+1> and |e>|n-1> is much smaller than the cavity frequency:

$$\Omega_n = g\sqrt{n} \ll \omega, \quad (\Omega \equiv \Omega_1 \sim 10^{-6}\omega)$$

Ultra-strong coupling regime

In circuit quantum electrodynamics the ultrastrong coupling regime can be achieved, with $\Omega > 0.1\,\omega$

The RWA is no longer valid

This regime beyond the RWA is unavoidable in the search for fast quantum gates, to operate fault-tolerantly, with many gates within the decoherence time scale

Differences with standard QED

A single mode of the field rather than an infinite number of modes is considered

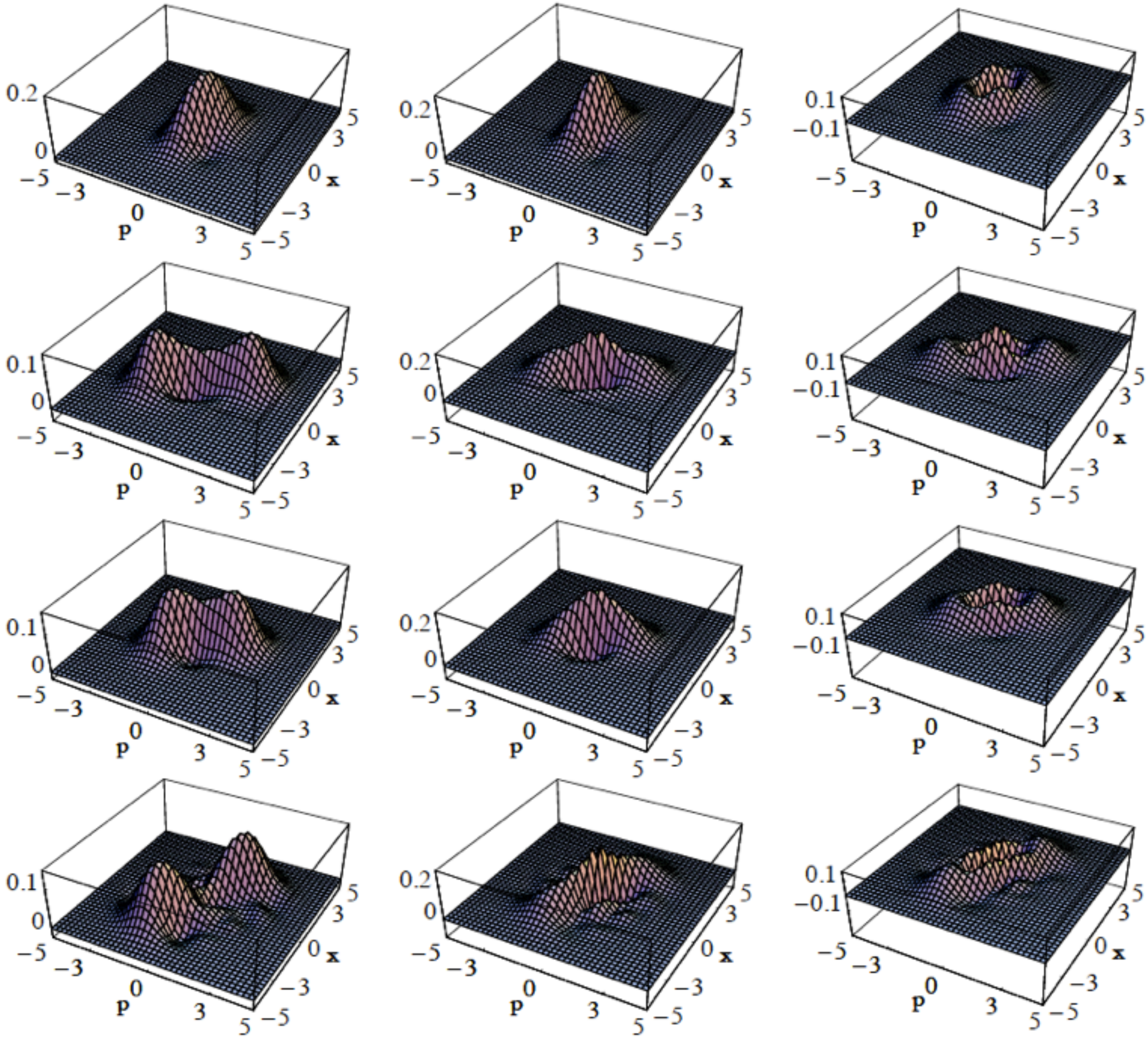
The quantization volume (of the cavity) is fixed and the limit of infinite volume is not taken at the end of the computation

Non-adiabaticity: the interaction is switched on abruptly and we focus on transient phenomena (finite-time QED)

Ultra-strong-coupling: Quantization volume much smaller than λ^3

Evolution starting from $|\Psi_0\rangle = |g,0\rangle$

unconditional conditional (g) conditional (e)



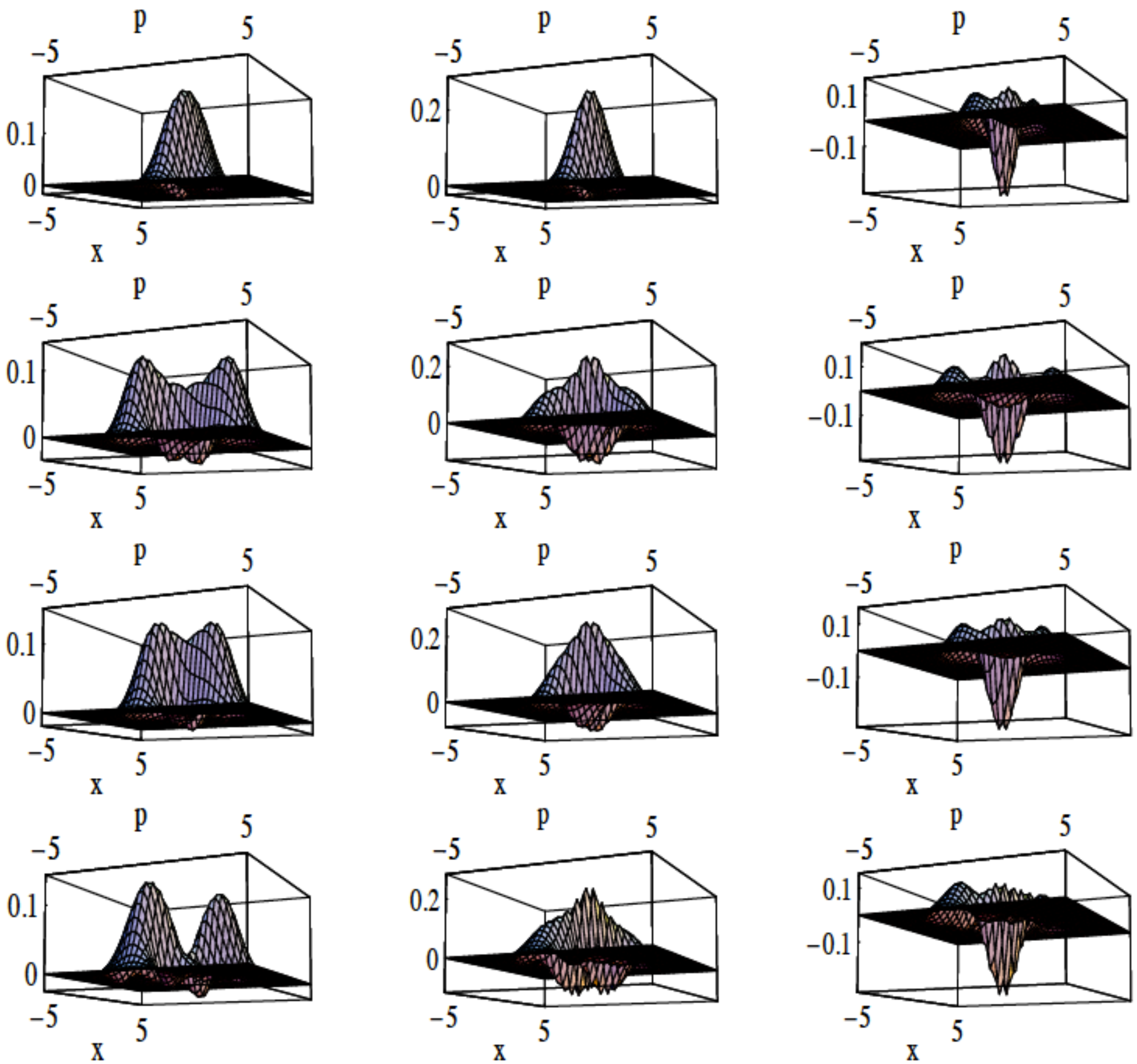
$g = 0.5, \ \tau = \pi/2g$

$g = 1.5, \ \tau = \pi/2g$

$g = 1, \ \tau = 0.75\pi/2g$

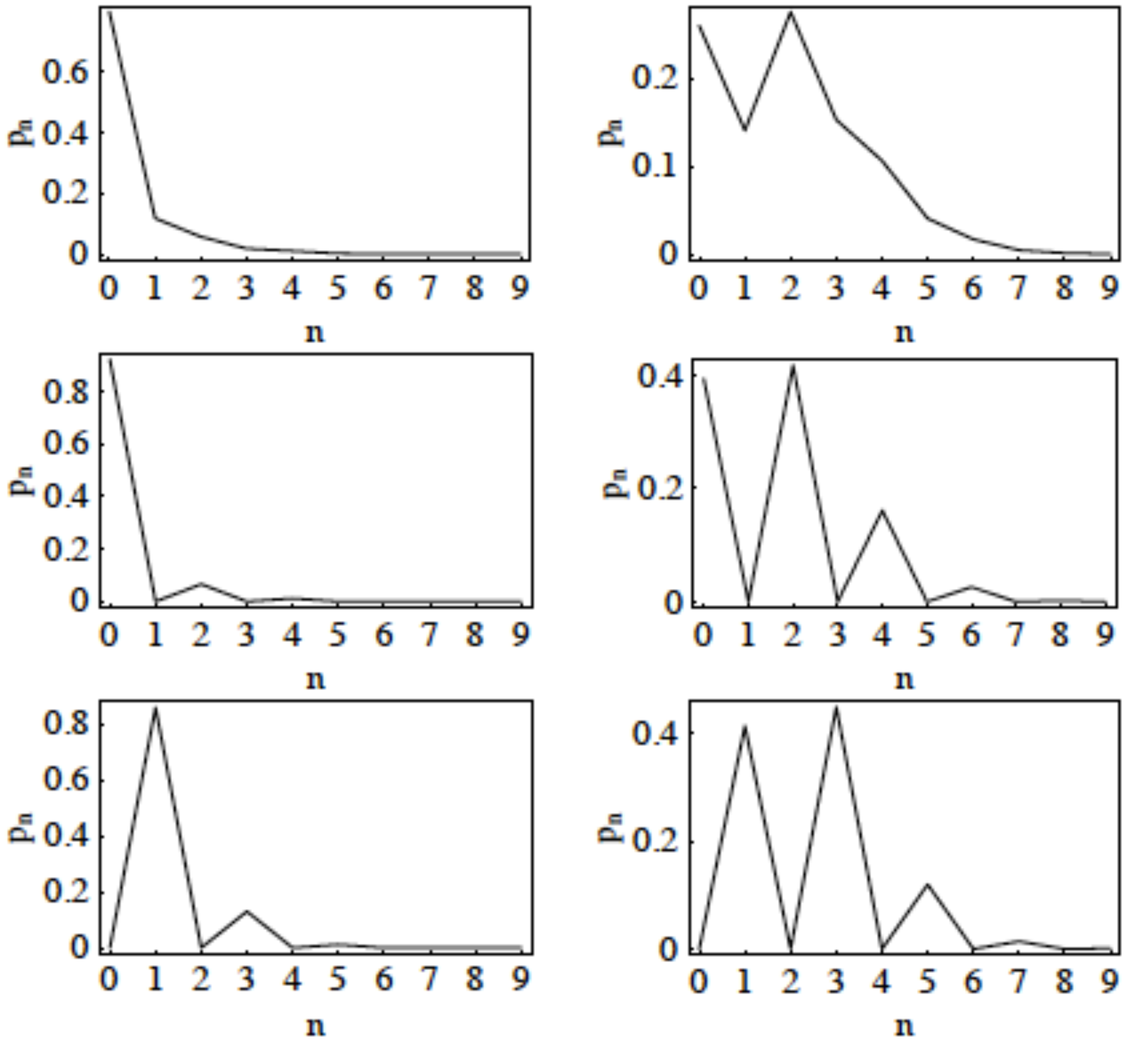
$g = 1, \ \tau = 1.5\pi/2g$

The final field states are exotic in that different from the squeezed states usually associated with the DCE (consequence of the intrinsic nonlinearity of the model)



Negativity of the Wigner function

Populations of the final field states



unconditional

conditional (g)

conditional (e)

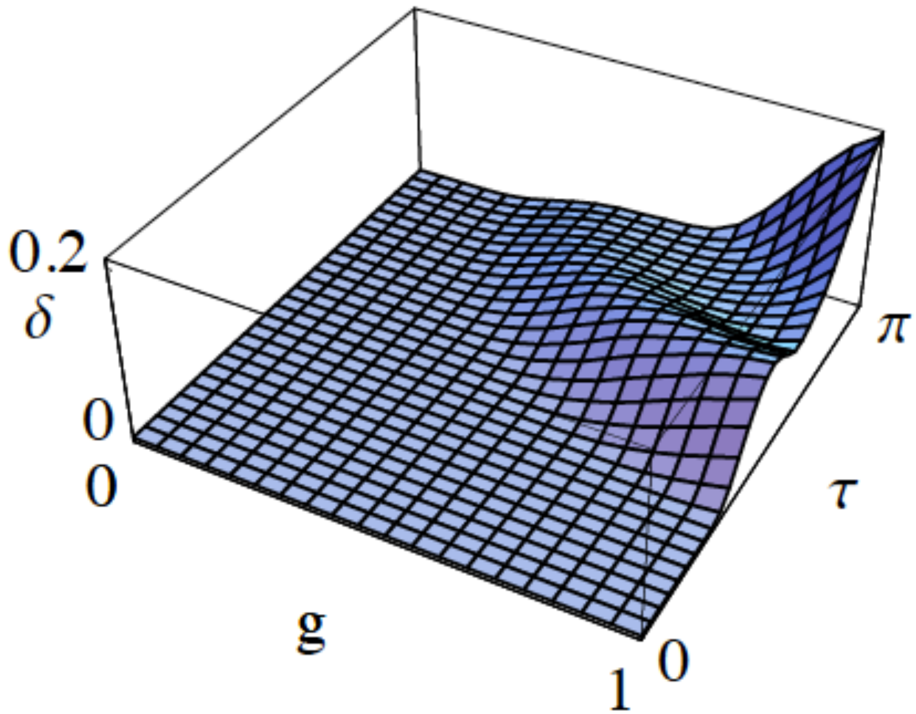
$\tau = \pi/2g,\ g = 0.5$

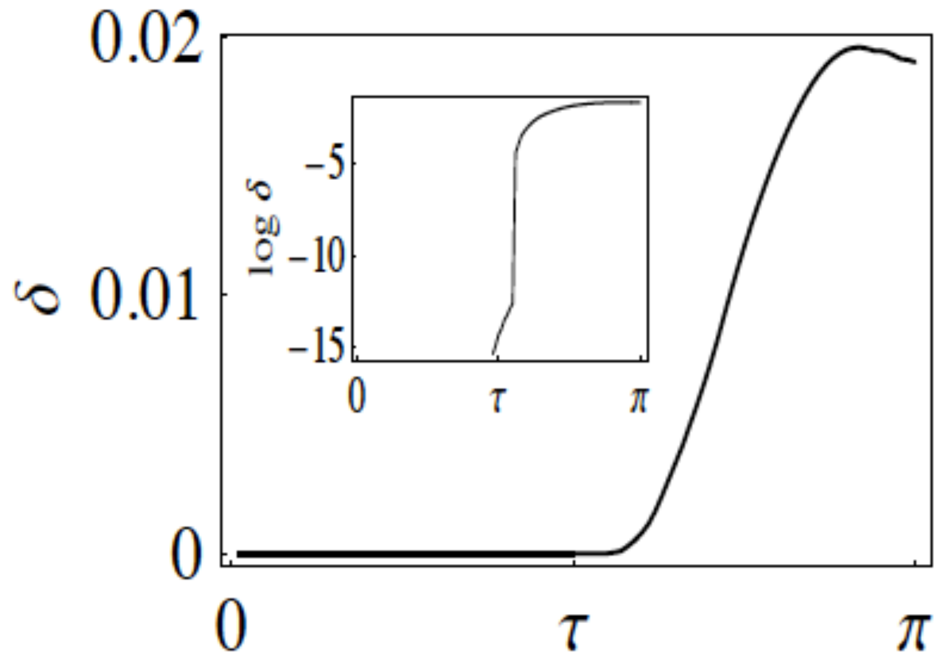
$g = 1.5$

Emission of real photons

Sharp transition to non-classical states

Negativity parameter $\delta = \int\int [\,|W(x,p)| - W(x,p)\,]\,dx\,dp$





Time-dependent perturbation theory (Picard series)

$$|\Psi(t)\rangle = \sum_{l=g,e} \sum_{n=0}^{\infty} C_{l,n} |l, n\rangle$$

$$\begin{cases} i\,\dot{C}_{g,n}(t) = \Omega_n\, C_{e,n-1}(t) + \Omega_{n+1} e^{-2i\omega t}\, C_{e,n+1}(t), \\ i\,\dot{C}_{e,n-1}(t) = \Omega_n^\star\, C_{g,n}(t) + \Omega_{n-1}^\star e^{2i\omega t}\, C_{e,n-2}(t), \qquad \Omega_n \;=\; g\sqrt{n} \end{cases}$$

$$\psi(\tau) = \psi(0) - i \int_0^\tau H_I(t)\, \psi(t)\, dt$$

$$\psi(\tau) = \psi(0) - i \int_0^\tau H_I(t) \left[\psi(0) - i \int_0^t H_I(t')\, \psi(t')\, dt' \right] dt$$

Picard series (for finite-time QED)

$$\psi^{(0)}(t) = \psi(0)$$

zeroth-order approximation

$t = \tau$

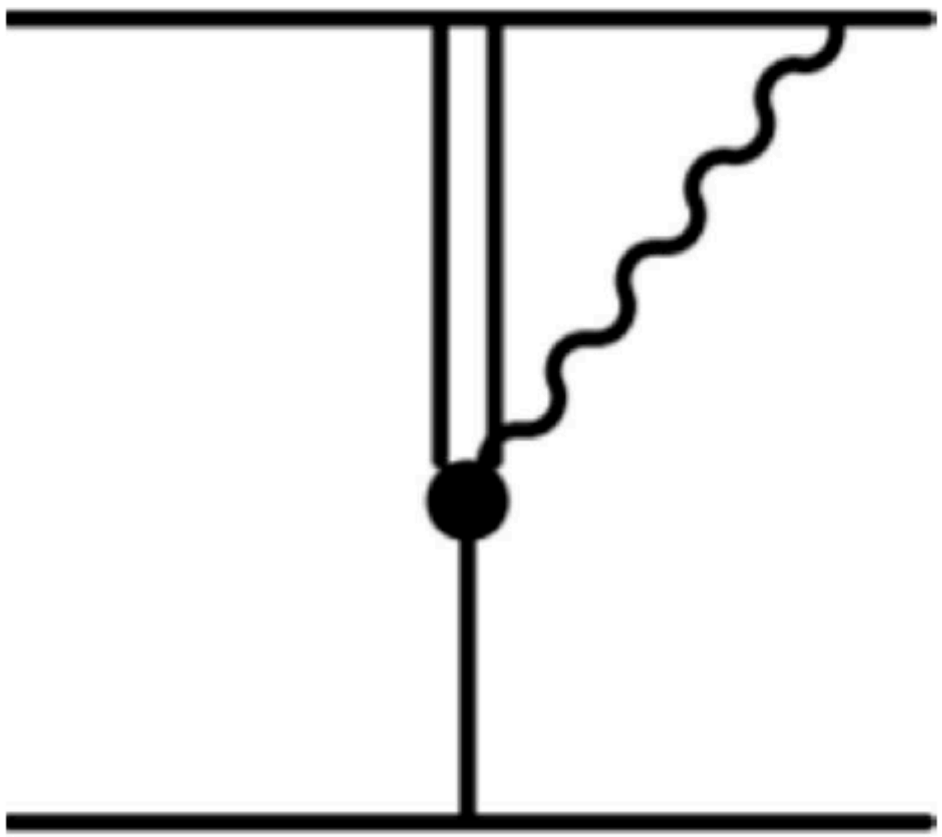
Start from ground state of the atom and the field (zero photons)

$t = 0$

$$\psi^{(1)}(t) = -i \int_0^\tau H_I(t)\,\psi(0)\,dt$$

first-order correction

$$|\psi^{(1)}(\tau)\rangle = \frac{g^\star}{2\omega}\left(1 - e^{2i\omega\tau}\right)|e, 1\rangle$$

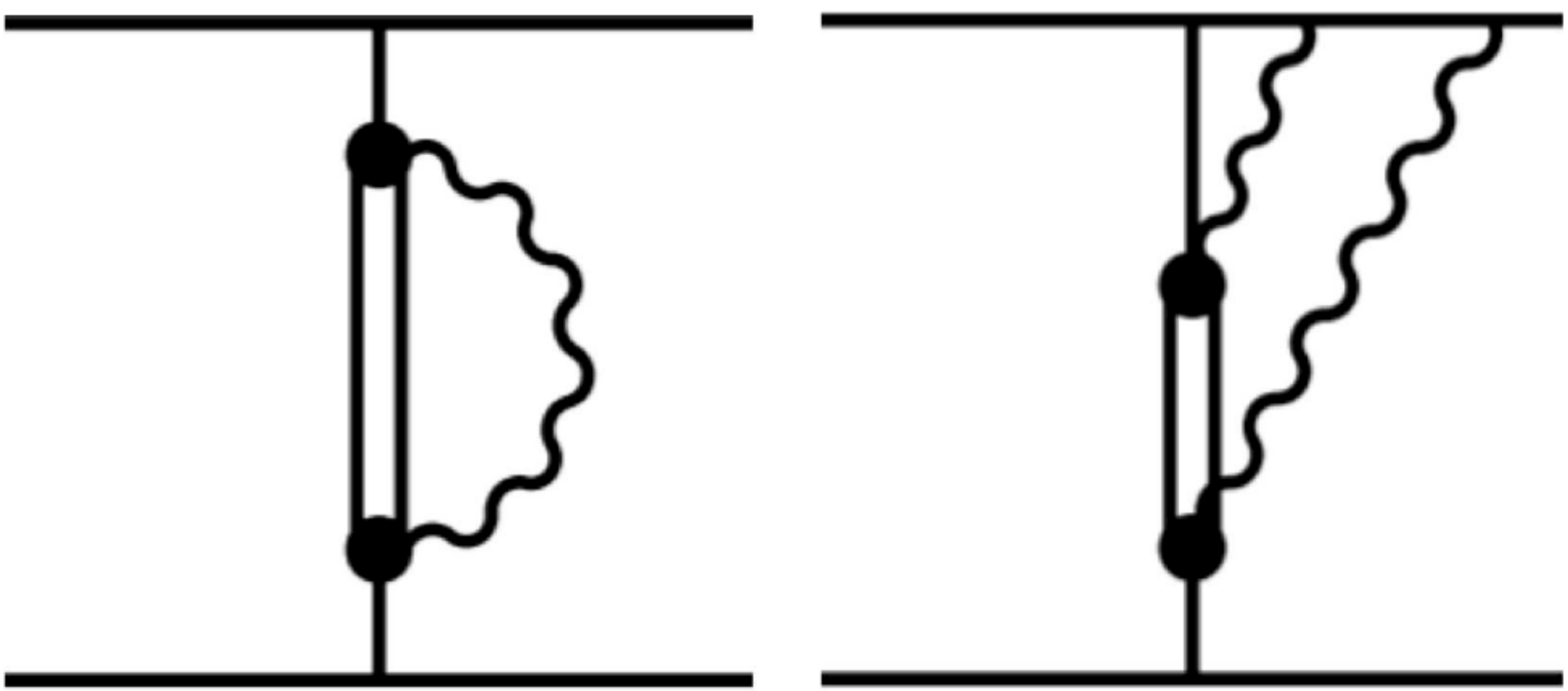


$$\psi^{(2)}(t) = -i \int_0^\tau H_I(t)\, \psi^{(1)}(t)\, dt$$

$$= - \int_0^\tau H_I(t) \int_0^t H_I(t')\, \psi(0)\, dt\, dt'$$

second-order correction

$$|\psi^{(2)}(\tau)\rangle = i\, \frac{|g|^2}{2\omega} \left[\tau + \frac{i}{2\omega}\left(1 - e^{-2i\omega\tau}\right)\right] |g, 0\rangle$$
$$+ i\, \frac{\sqrt{2}|g|^2}{2\omega} \left[-\tau + \frac{i}{2\omega}\left(1 - e^{2i\omega\tau}\right)\right] |g, 2\rangle.$$



Fidelity of quantum gates

IOP PUBLISHING PHYSICA SCRIPTA
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Validity of resonant two-qubit gates in the ultrastrong coupling regime of circuit quantum electrodynamics

Y M Wang[1], D Ballester[2], G Romero[2], V Scarani[1,3] and E Solano[2,4]

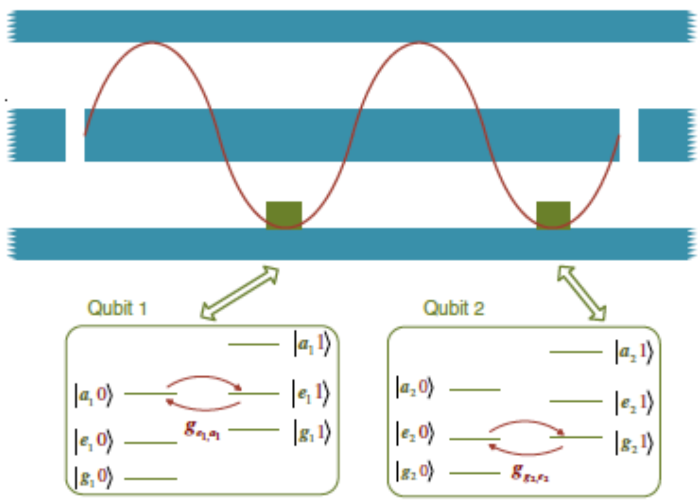
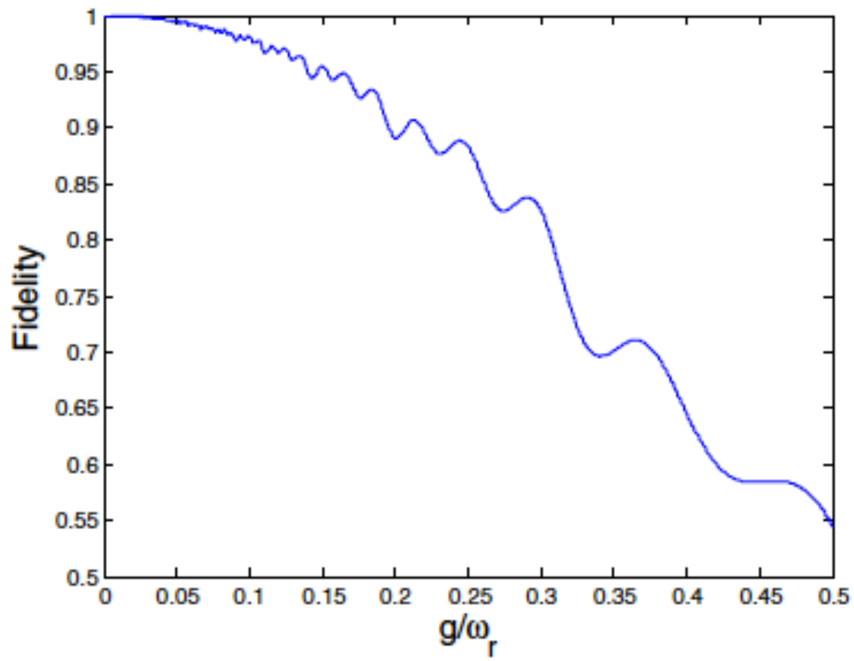
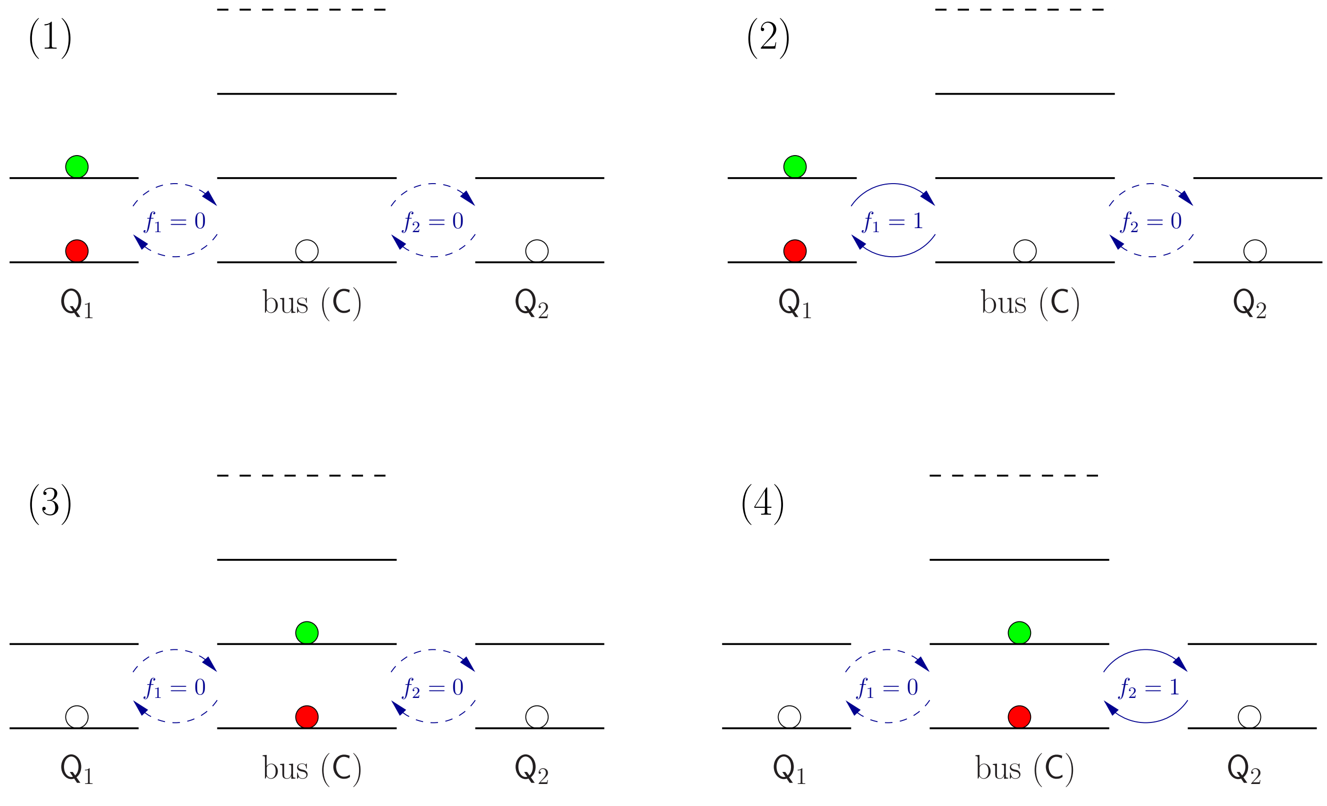


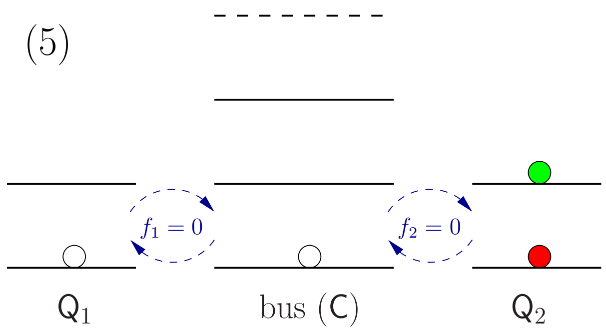
Figure 1. Schematic of protocol I for a resonant CPHASE gate.



Relation between fidelity decay and DCE?

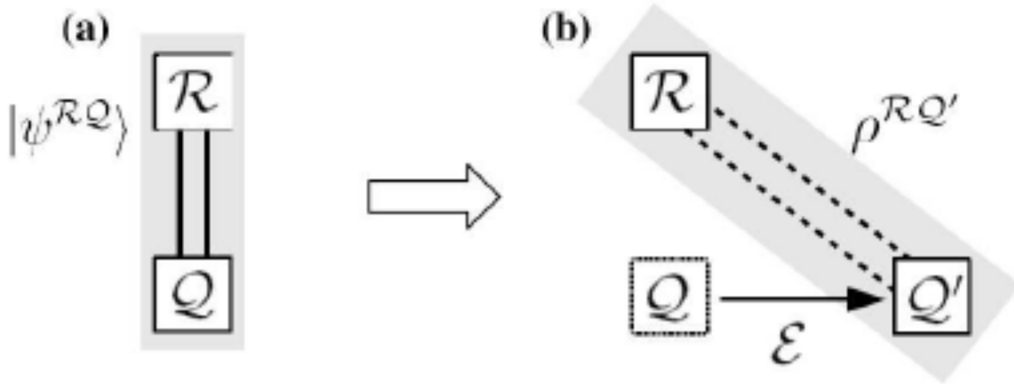
Quantum communication channel

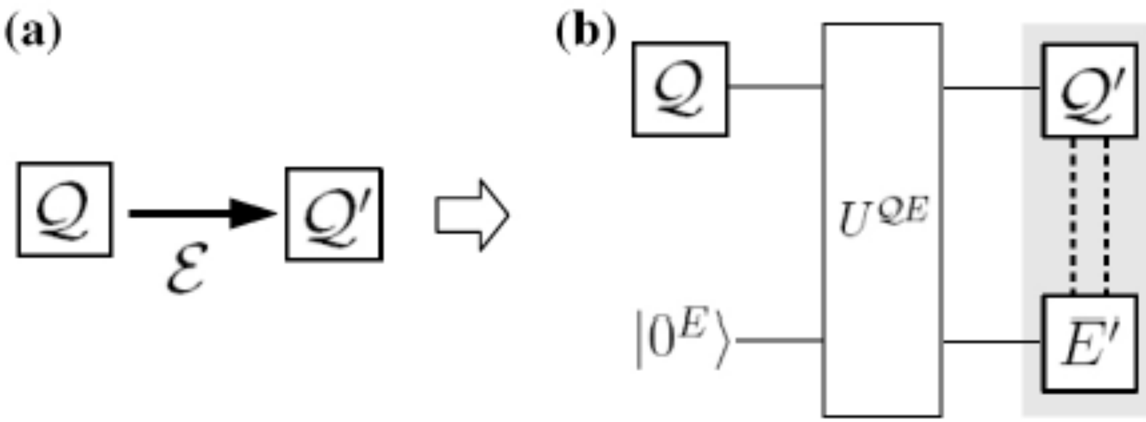




(see Sillanpää et al., Nature 449, 438 (2007))

Coherent information





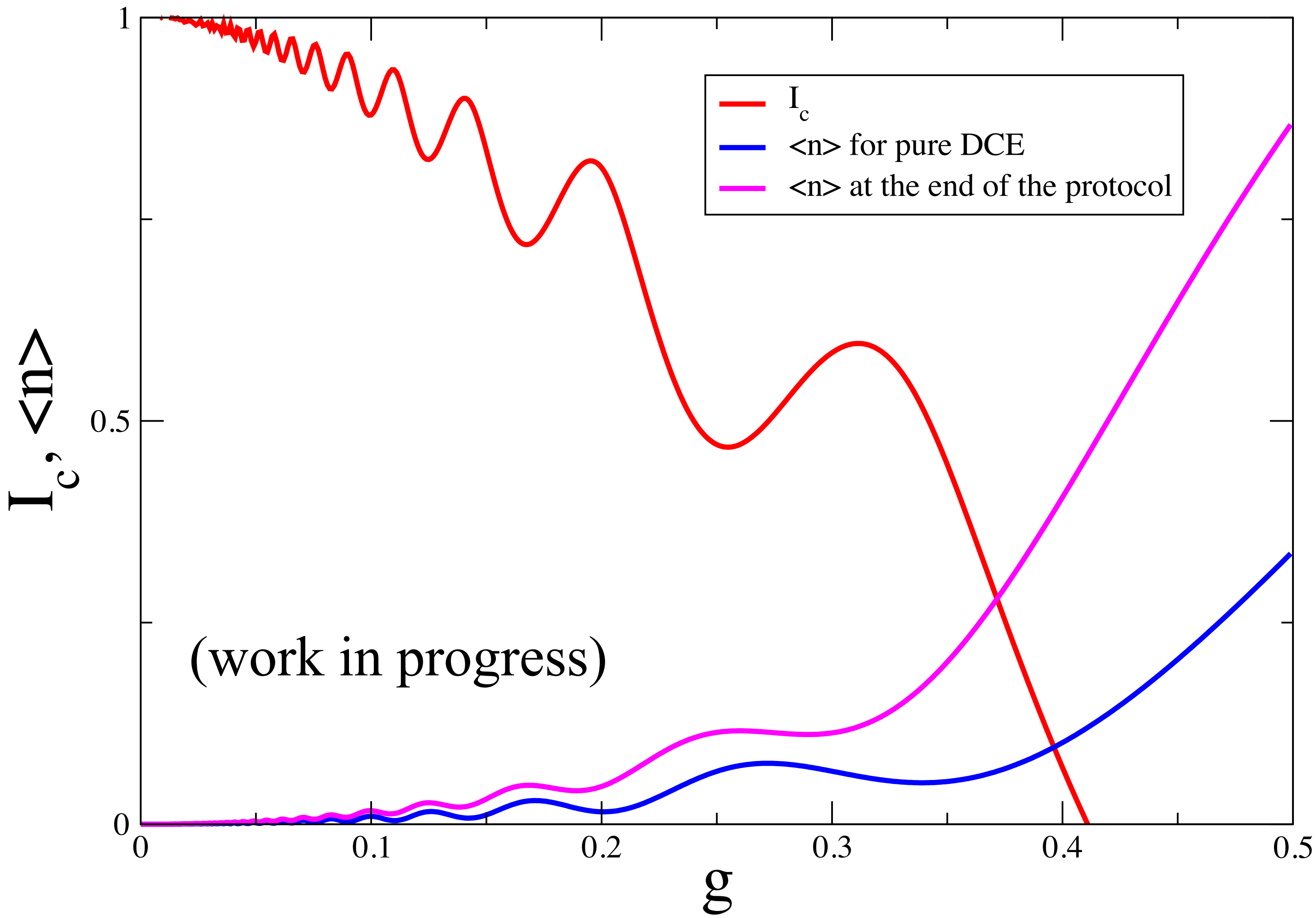
$$S_e = S_e(\rho, \mathcal{E}) = S(\rho^{E'})$$

entropy exchange

$$I_c(\rho, \mathcal{E}) = S[\mathcal{E}(\rho)] - S_e(\rho, \mathcal{E})$$

coherent information

Coherent information degradation vs photon generation



$$\text{Maxima of } I_c \text{ for } \frac{1}{g} = 2N+1, \quad N = 1, 2, 3, \ldots$$

Summary

Generation of exotic states of the field in the ultra-strong coupling regime, beyond the RWA

Transition to states (different from squeezed states) with negative Wigner distribution

Dynamical Casimir effect fundamental limitation to the fidelity of high speed quantum gates

How to counteract DCE? Adiabatic switching of the interaction,...