Berry phase in the presence of noise

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The Berry phase

Hamiltonian H depending on a set of control parameters $\vec{R}(t)$

$$H(\vec{R}(t))\,|\psi(t)\rangle = E(\vec{R}(t))\,|\psi(t)\rangle \qquad \vec{R}(T) = \vec{R}(0)$$

The control parameters are changed adiabatically and cyclically

Adiabatic Theorem: A non degenerate energy eigenstate at the end of the cyclic evolution differs from the initial one by a phase factor

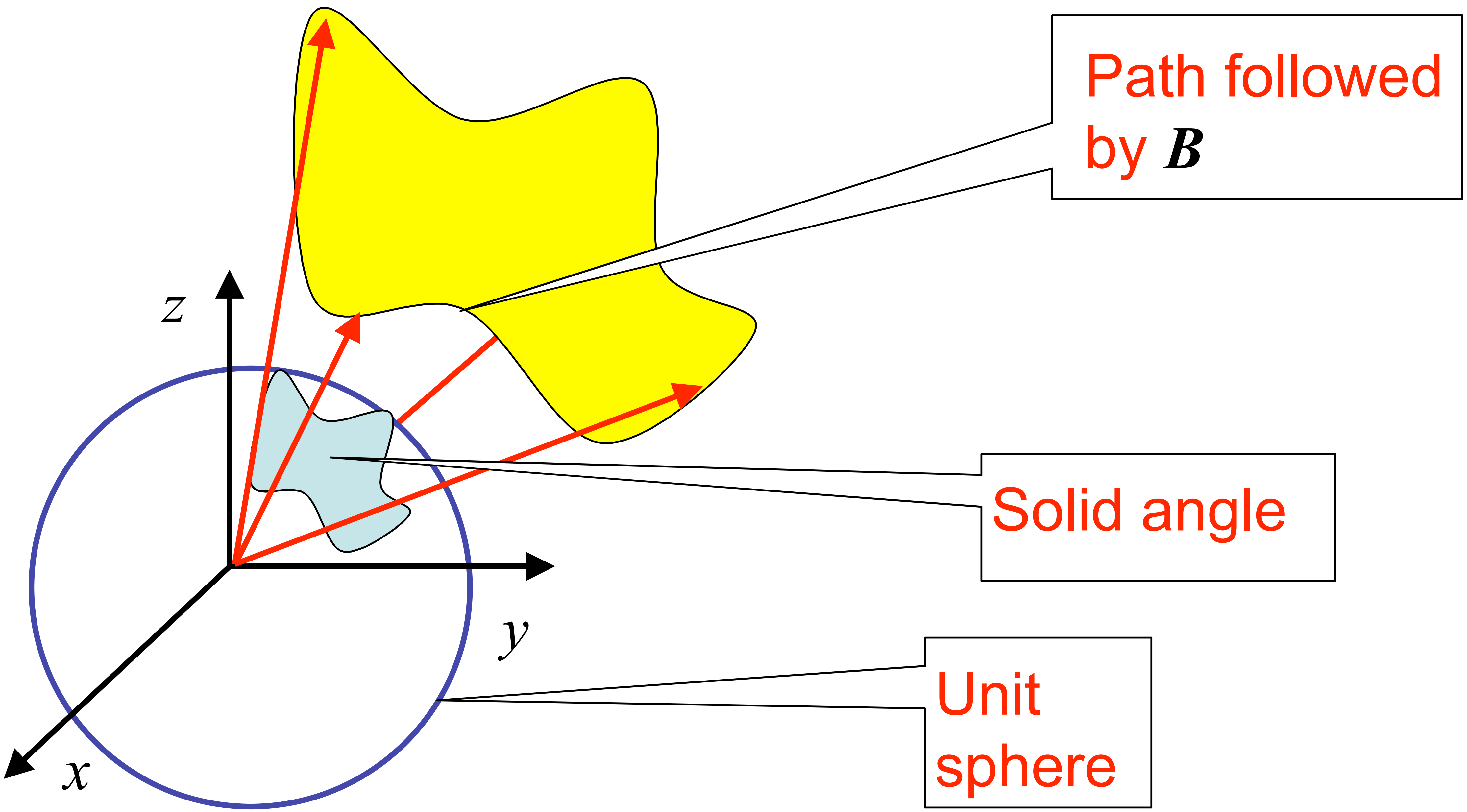
$$|\psi(T)\rangle = e^{i\delta}e^{i\gamma}\,|\psi(0)\rangle$$

$$\delta = -\int_0^T E(t)dt$$

$$\gamma = \oint_C \vec{A}\cdot d\vec{R}$$

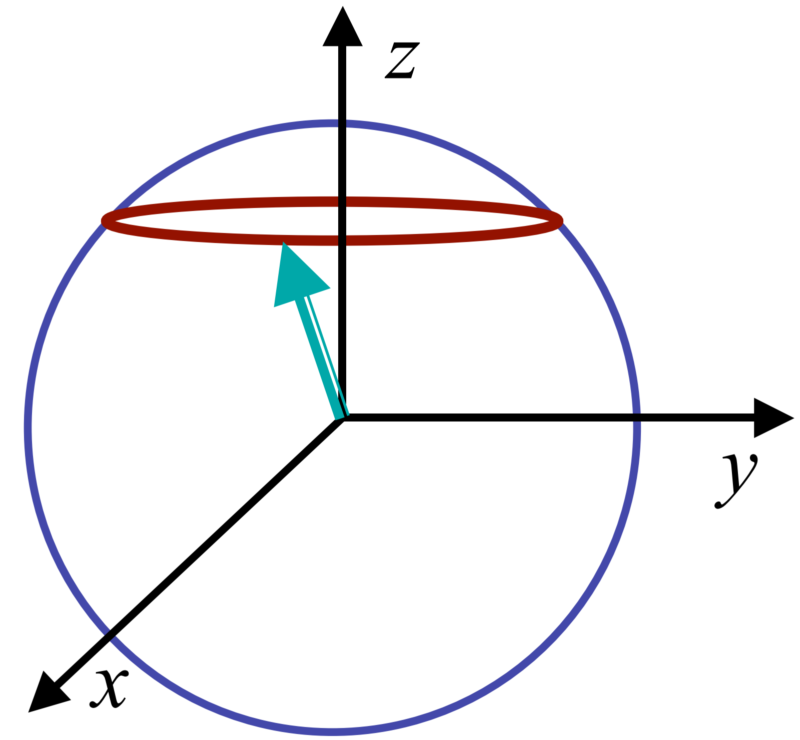
An example a spin ½ in a magnetic field

Berry phase is proportional to the solid angle subtended by B at the degeneracy point.



An example: precession around a parallel

B precesses around z axis at an angle ϑ with angular velocity Ω

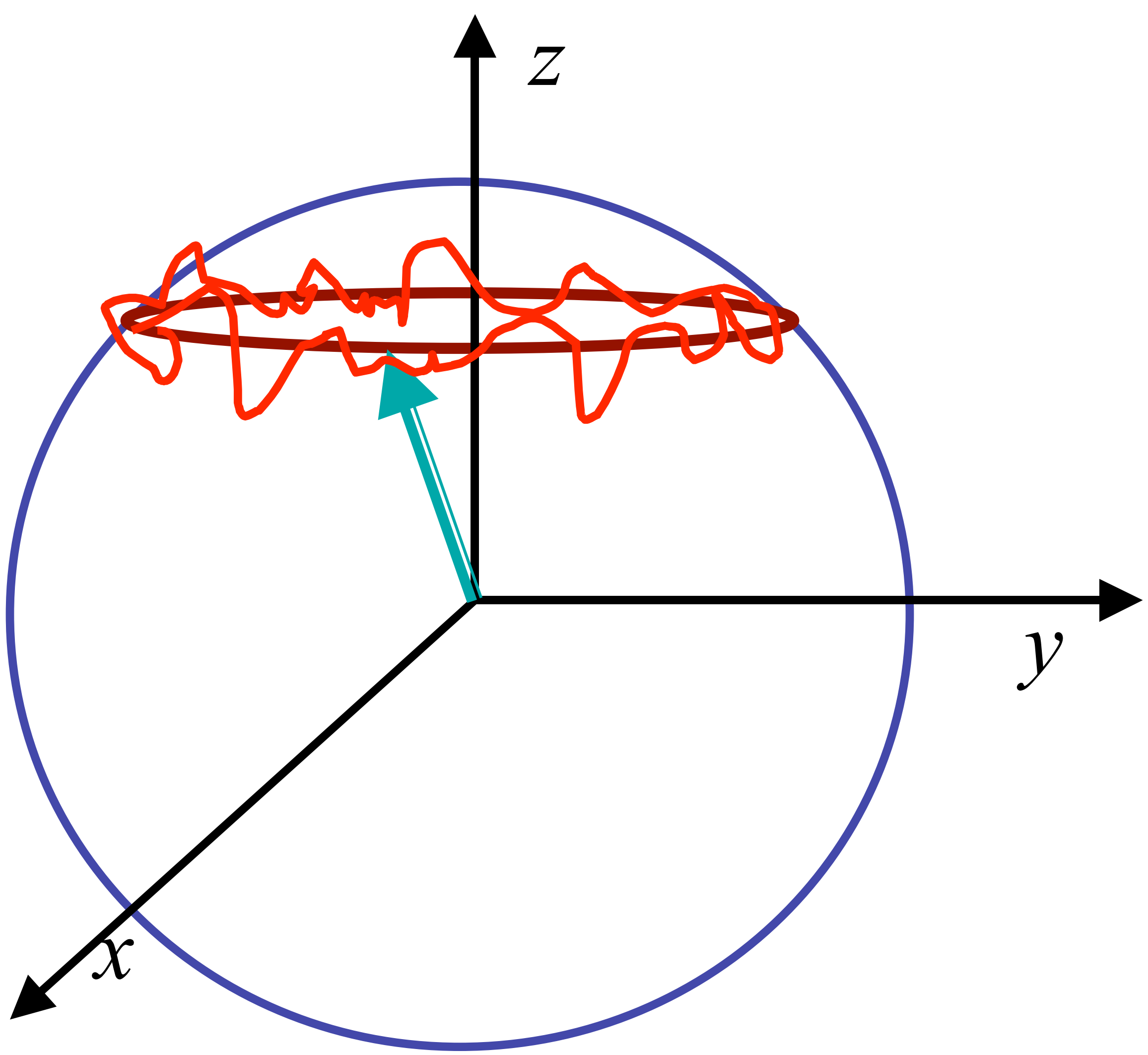


$$\gamma_\uparrow = -\gamma_\downarrow = \int_0^{2\pi} A_\varphi^\uparrow d\varphi = \pi \cos\vartheta.$$

Berry phase is independent from Ω!

Fault tolerant quantum computation

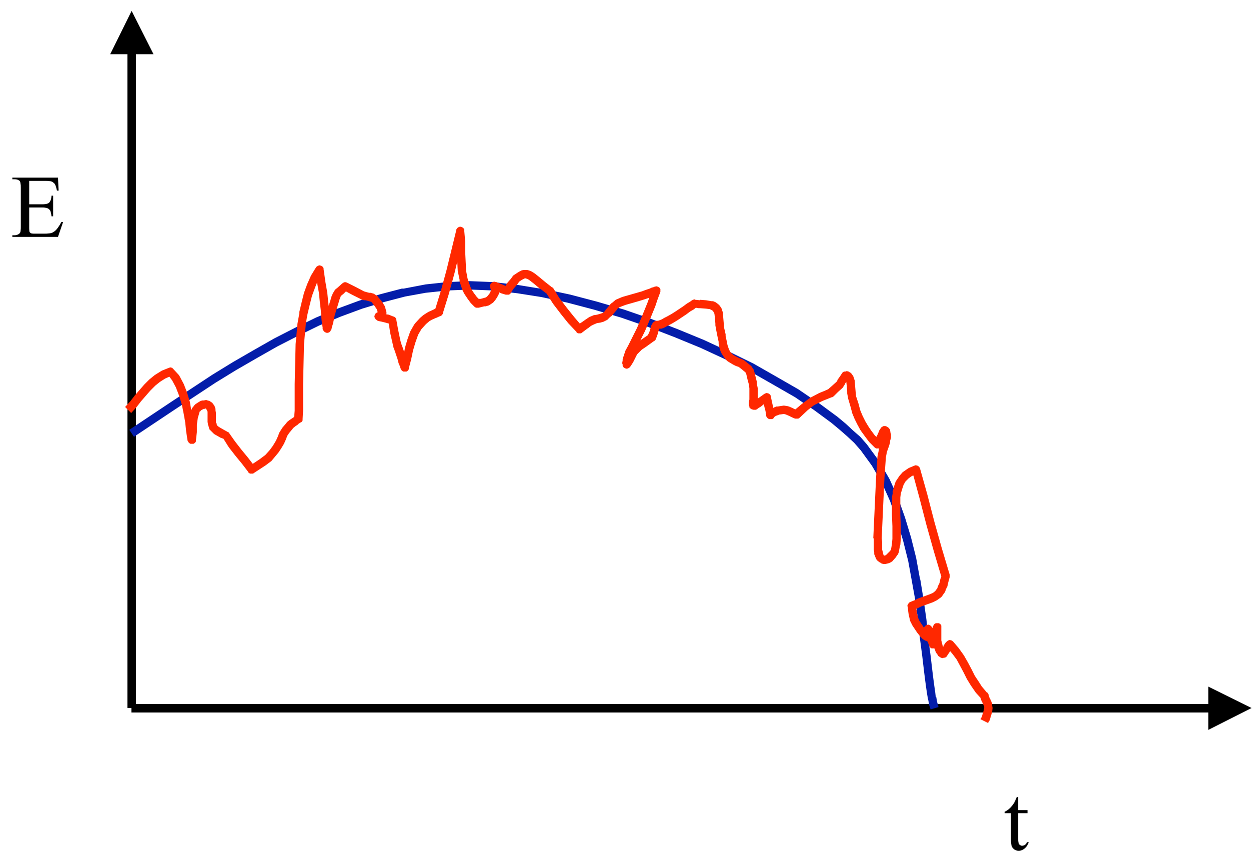
Geometric quantum computation is believed to be intrinsically more robust against random errors



As the geometric phase is proportional to the overall area traced on the unit sphere i.e. to a global property of the path in parameter space, errors with zero time average should not introduce errors

Objection

Dynamic phase $\delta = \int_0^T E(t)dt$



- The dynamical phase is proportional to the area of $E(t)$ vs. t
- Dynamic phase fluctuations are known to introduce decoherence

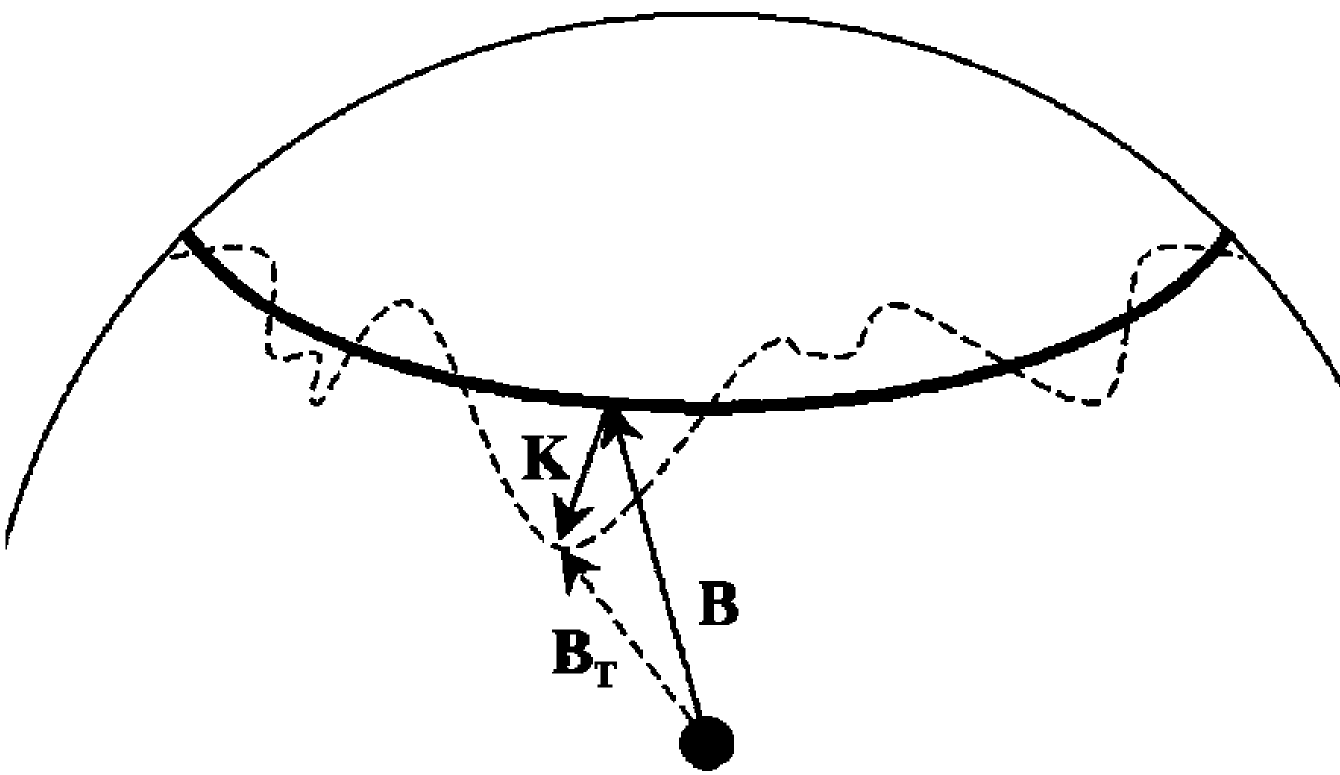
Do fluctuations play a different role in geometric and in dynamic phases?

Noise model

$$H(t) = \frac{1}{2}\mathbf{B}_T \cdot \vec{\sigma} = \frac{1}{2}[\mathbf{B}(t) + \mathbf{K}(t)],$$

Control field

Fluctuating field



A spin interacting with a classical magnetic field with a small fluctuating component

DeChiara&Palma (PRL 2003)

Noise model

We assume small and slow fluctuations:

$$\vec{K} \ll \vec{B}$$

K is assumed to be a Ornstein –Uhlenbeck process with zero average and variance σ^2. It is therefore:

- Gaussian
- Markovian
- Stationary

Lorentian Spectrum

$$\langle K_i(t) K_j(t+\tau) \rangle = \delta_{ij} e^{-\Gamma\tau} \sigma^2$$

DeChiara&Palma (PRL 2003)

Decoherence

Initial state: $$|\psi\rangle = a|\uparrow\rangle + b|\downarrow\rangle.$$

Final state: $$|\psi'\rangle = ae^{i\alpha}|\uparrow\rangle + be^{-i\alpha}|\downarrow\rangle,$$

Total phase: $$\alpha = \gamma_B + \delta$$

Averaging: $$\rho = \int |\psi'\rangle\langle\psi'|P(\alpha)d\alpha.$$

Decoherence

Density matrix:
$$\rho = \int |\psi'\rangle\langle\psi'| P(\alpha) d\alpha.$$

Using:
$$\langle e^{2i\alpha} \rangle = e^{2i\alpha_0} e^{-2\sigma_\alpha^2}$$

$$\rho = \begin{pmatrix} |a|^2 & ab^* e^{2i\alpha_0} e^{-2\sigma_\alpha^2} \\ a^* b e^{-2i\alpha_0} e^{-2\sigma_\alpha^2} & |b|^2 \end{pmatrix}$$

Geometric and Dynamic Decoherence

The main contribution to σ_α^2 comes from dynamic phase and is proportional to B while geometric contribution is proportional to Ω.

Dynamic phase variance grows linearly in T

Geometric phase variance goes to zero as 1/T

Decoherence is mainly DYNAMIC!

Intuitive picture

Suppose to double T:

Here the domain doubles

Here fluctuations accumulates