A new technique for fluid velocimetry based on near field scattering

M.A.C. Potenza\textsuperscript{a,}\textsuperscript{*}, M.D. Alaimo\textsuperscript{a}, D. Pescini\textsuperscript{b}, D. Magatti\textsuperscript{b}, F. Ferri\textsuperscript{b}, M. Giglio\textsuperscript{a}

\textsuperscript{a}University of Milan, via Celoria 16, Milan, Italy
\textsuperscript{b}University of Insubria, via Valleggio 11, Como, Italy

Available online 22 August 2005

Abstract

We show that the time evolution of near-field scattering speckles, originated by a fluid suspension of particles, provides information about the velocity field in the fluid. This information can be extracted from a statistical analysis of speckle fields taken at different times, either by measuring their cross-correlation function or by recovering the power spectrum corresponding to the difference between the two speckle fields. Experimental data are in accordance to the expected behaviors. The results are independent of the scatterer’s size, allowing one to exploit the technique also with sub-wavelength tracking particles.

\textsuperscript{c} 2005 Elsevier Ltd. All rights reserved.

Keywords: Scattering; Speckles; Optical velocimetry; Fluids

1. Introduction

The measurement and mapping of fluid velocity are a main issue in studying fluids under many different conditions, and many techniques have been developed and widely used to this aim. Among the most interesting methods, optical analysis provides a great advantage because it is a non-invasive measure, one of the most important requirements in studying fluids. Imaging, Doppler effect and speckle

\textsuperscript{c}Corresponding author.

\textit{E-mail address:} marco.potenza@unimi.it (M.A.C. Potenza).
analysis [1–3] are widely used to map the fluid velocity in many different application fields, from the study of wings aerodynamics to the simulations of blood flow inside heart valves. In particular, speckle analysis has been deeply used in many different configurations, from the old speckle photography method to the more up-to-date digital techniques, often supported by pulsed lasers and in some cases by nonlinear optics (see for example [4–11]).

Recent studies [12–14] devoted to the characterization of the so-called near-field scattering (NFS) technique, that have been extensively performed on colloidal suspensions of micron- and sub-micron-sized particles in water, have provided strong evidence that the technique is influenced by particle motions. Thus, the principles of NFS can be exploited for application to fluid velocimetry: we called this novel technique near-field scattering velocimetry (NFSV) [15]. As we will describe later on, NFSV introduces some important advantages with respect to the far-field speckle methods, which have been known since long ago [3].

NFSV works by illuminating the sample containing the scattering tracer particles with a large laser beam and recording the intensity distribution with a CCD camera placed at a close distance from the sample. On this plane, with many interference patterns among scattered and transmitted wavefronts, originates a low contrast, non-uniform speckle field whose time-averaged characteristics depend on the way the light is scattered by tracers. At the same time, if tracers move in a direction transversal to the incident light, the speckle field displaces accordingly. Thus, from a statistical analysis of speckle fields taken at different times, a measure of tracer velocity can be easily recovered. This can be done either by measuring the cross-correlation function between the two fields or by recovering the power spectrum associated to the two speckle field difference.

In this work we apply NFSV to the quantitative study of convective motions of polystyrene particles inside a thin cell and discuss the data analysis procedures that can be adopted for recovering the fluid velocity. An example of the technique for mapping the two-dimensional velocity flow around an obstacle inserted along the fluid flow is presented. In the following sections, we first introduce the experimental apparatus and a brief description of the expected effects depending on the fluid velocity, and then we present the experimental results describing both the spectral analysis and the cross-correlation analysis.

2. Experimental setup and near-field speckles

The experimental apparatus has been extensively described and discussed elsewhere [12,13]. A sketch of the setup used for the experiment is shown in Fig. 1. A spatially filtered, collimated and expanded laser beam is sent through a cell filled with the fluid to be studied (pure water), in which latex colloids of known size are suspended as tracking particles. The speckle intensity distribution at a given distance \(z\) from the sample is recorded by a CCD sensor through a magnifying collection optics.

The intensity distribution at the observation plane is the result of interference between transmitted and scattered light. If the scattered field \(e_s\) is negligible with
respect to the transmitted field $e_0$, the intensity distribution can then be written as

$$f(r, t) = i_0 + \delta f(r, t),$$

where $i_0 = |e_0|^2$ is the transmitted intensity and $\delta f(r, t) = e_0 e_0^*(r, t) + e_0^* e_S(r, t)$ is called the heterodyne signal and depends on the scattered field $e_S(r, t)$ at position $r$ and time $t$. Notice that while $i_0$ is a static term, $\delta f(r, t)$ may vary with time because of particle motion. In that case, $\delta f(r, t)$ is a stochastic zero-average fluctuating term that can be recovered as

$$\delta f(r, t) = f(r, t) - \langle f(r, t) \rangle_t$$

in which the time-average $\langle f(r, t) \rangle_t$ has to be carried out over a period of time corresponding to a large number of independent sample configurations.

For any given time $t$, the heterodyne signal $\delta f$ depends stochastically on the position $r$ over the sensor plane. This speckle-like appearance is due to the sum of many independent interference patterns originating from interference between the transmitted beam and the field scattered by each particle, whose position inside the scattering cell is random.

Close enough to the sample, in the so-called near-field condition, the sensor receives light from a region $D^* \sim 2z_{\text{max}}$ smaller than the illuminated region $D$ (see Fig. 1). This can be due either to the limited scattering angle of the scatterer or to the limited angular aperture of the collection optics. As a consequence, if all scatterers are rigidly displaced transversally to the optical axis, provided that their mutual positions remain unchanged, the speckle field is completely preserved but displaced accordingly. This is true as long as the set of scatterers responsible for a given speckle are subjected to a constant illumination, a condition that is met only when the region $D^*$ is well inside the illuminated region $D$.

Notice that the one-to-one mapping between particle motions and speckles displacement is a feature of the NFSV technique. Indeed, in the far-field zone the speckles do not move upon particle motions, but simply fluctuate stochastically in time. This is due to the fact that, in the far-field, each speckle is the result of contributions arriving from all the illuminated sample, and therefore no correspondence between speckles and particles is anymore possible.
3. Time evolution of near-field speckles

In this section, we describe how we can recover information on particle motions by analyzing the spatial–temporal correlations of the heterodyne signals taken at different times. We will consider a water suspension of particles that are only subjected to an ordered motion caused, for example, by convection or sedimentation, while the disordered, diffusive motions will be neglected. Under this condition the particle velocity can be easily determined by means of two different analyses, in the space and the spectral domain.

3.1. Data analysis in the space domain

Let us indicate the heterodyne signal at time \( t_1 \) and \( t_2 \) with \( \delta f_1(r) = \delta f(r, t_1) \) and \( \delta f_2(r) = \delta f(r, t_2) \). The first way in which data can provide evidence for the particle motion from the speckle fields is the most direct and intuitive one, namely the spatial cross-correlation of two different signals \( \delta f_1(r) \) and \( \delta f_2(r) \) (see for example [3]):

\[
G_{1,2}(\mathbf{x}) = \langle \delta f_1(\mathbf{r}) \delta f_2(\mathbf{r} + \mathbf{x}) \rangle. \tag{3}
\]

When all the particles undergo a uniform motion characterized by a constant velocity \( \mathbf{V} \), the heterodyne signal \( \delta f(r,t) \) at the time \( t = t_2 \) is simply a “shifted version” of the signal at the time \( t = t_1 \), the shifting being represented by the displacement \( \Delta \mathbf{r} = \mathbf{V}(t_2-t_1) \). Thus, we can write

\[
\delta f_2(r) = \delta f_1(r - \Delta \mathbf{r}), \tag{4}
\]

and consequently

\[
G_{1,2}(\mathbf{x}) = G_{1,1}(\mathbf{x} - \Delta \mathbf{r}), \tag{5}
\]

which shows that the cross-correlation function \( G_{1,2} \) is simply a shifted version of the field auto-correlation function \( G_{1,1}(\mathbf{x}) \). Thus, by measuring \( \Delta \mathbf{r} \) and knowing \( \Delta t = t_2-t_1 \), it is straightforward to recover the velocity \( \mathbf{V} = \Delta \mathbf{r}/\Delta t \).

An evident drawback of this method deals with the average procedure (see Eq. (2)) needed for recovering the heterodyne signals. This introduces a very strong requirement on the stability of the optical system, which has to be maintained over the measuring time \( T = N\Delta t \), \( N \gg 1 \) being the number of independent frames. If this stability is not guaranteed, part of the static contribution changes with time and will be accounted for as a real signal. A thorough discussion of this issue as well as a solution for this problem has been developed in Ref. [13].

Following Ref. [13], one can adopt a different approach for the data analysis, and define an alternative heterodyne signal \( \delta f'_{1,2}(\mathbf{r}, t) \) as

\[
\delta f'_{1,2}(\mathbf{r}, t) = f(\mathbf{r}, t + \Delta t) - f(\mathbf{r}, t) = \delta f(\mathbf{r}, t + \Delta t) - \delta f(\mathbf{r}, t), \tag{6}
\]

which corresponds to the difference between the frames taken at a temporal distance \( \Delta t = t_2-t_1 \). Notice that \( \delta f'_{1,2}(\mathbf{r}, t) \) is also equal to the difference between the heterodyne signals frames taken at a distance \( \Delta t \). Thus, we have obtained a zero-fluctuating stochastic signal which is independent of \( i_0 \) without the necessity of any
averaging procedure as required for \( \delta f(\mathbf{r},t) \) [see Eq. (2)]. As a result, the requirement about system stability is less stringent than above, because it is needed only over the time interval \( \Delta t \ll T \). Notice also that while \( \delta f(\mathbf{r},t) \) is independent of the frame difference \( \Delta t \), \( \delta f'_{1,2}(\mathbf{r},t) \) does depend on it. If we compute the auto-correlation function of \( \delta f'_{1,2}(\mathbf{r},t) \), we obtain

\[
G'_{1,2}(\mathbf{x}) = 2G_{1,1}(\mathbf{x}) - G_{1,2}(\mathbf{x}) - G_{2,1}(\mathbf{x}),
\]

in which \( G_{i,j}(\mathbf{x}) (i, j = 1, 2) \) are the same as defined in Eq. (3). Notice that in Eq. (7) both the auto- and the cross-correlation functions of the heterodyne signals appear, but when the particles undergo a uniform motion corresponding to a constant displacement \( \Delta \mathbf{r} \), it turns out that

\[
G'_{1,2}(\mathbf{x}) = 2G_{1,1}(\mathbf{x}) - G_{1,1}(\mathbf{x} - \Delta \mathbf{r}) - G_{1,1}(\mathbf{x} + \Delta \mathbf{r}).
\]

The first term represents the signal auto-correlation function and brings no information about the motion, while the other two are shifted auto-correlation terms whose sum makes \( G'_{1,2}(\mathbf{x}) \) symmetrical with respect to the center. Since these last two terms are always present together, the versus of the velocity cannot be determined through Eq. (8). This procedure is much more robust than the one associated to Eq. (5) because it does not require the averaging procedure associated to Eq. (2). As a consequence, the mechanical stability of the system has to be guaranteed only over a very short time, i.e. the time interval between two acquisitions.

### 3.2. Data analysis in the spectral domain

A different approach deals with the spectral analysis of heterodyne signals, either \( \delta f(\mathbf{r},t) \) or \( \delta f'_{1,2}(\mathbf{r},t) \). This is usually carried out by computing the time-averaged two-dimensional power spectrum associated to such signals. However, there is a profound difference depending on whether we are processing \( \delta f(\mathbf{r},t) \) or \( \delta f'_{1,2}(\mathbf{r},t) \). In the first case, such a spectrum is given by

\[
S(\mathbf{q}) = \langle |F[\delta f(\mathbf{r},t)]|^2 \rangle \sim I(\mathbf{q}),
\]

where \( F \) denotes the two-dimensional Fourier transform operator and \( I(\mathbf{q}) \) represents the static intensity distribution scattered by the sample. Notice that \( S(\mathbf{q}) \) does not carry any information on particle motions, but is related only to \( I(\mathbf{q}) \), which in turn is ultimately related to particle size.

Conversely, if we use \( \delta f'_{1,2}(\mathbf{r},t) \), it can be easily worked out (making use of the Fourier shift theorem) that the corresponding spectrum is

\[
S'_{1,2}(\mathbf{q}) = \langle |F[\delta f'_{1,2}(\mathbf{r},t)]|^2 \rangle \sim I(\mathbf{q})[1 - \cos(\mathbf{q} \cdot \Delta \mathbf{r})],
\]

which therefore depends on particle motion through the ordered displacement \( \Delta \mathbf{r} = V \Delta t \). The particle motion appears in the spectrum as a set of straight fringes whose orientation is perpendicular to the velocity direction and characterized by a wavelength \( \lambda = 2\pi/|\mathbf{q} \cdot \Delta \mathbf{r}| \). Notice that for \( q \to 0 \), \( S'_{1,2}(\mathbf{q}) \to 0 \). This is because the heterodyne signals at time \( t \) and \( t + \Delta t \) are always correlated over length scales \( \xi \sim q^{-1} \gg \Delta r \), and therefore their difference tends to zero.
Eqs. (9) and (10) have been written in terms of the two-dimensional wavevector $q$, but usually intensity distributions are represented as a function of the scattering angle or a function of the modulus $q = |q|$. Thus, an azimuthal average over the polar angles $\varphi$ has to be performed in both Eqs. (9) and (10). Since the intensity distribution depends only on $q$, Eq. (9) remains unchanged. Conversely, taking into account that $\langle \cos[q \cdot \Delta r] \rangle_\varphi = J_0(q \Delta r)$, $J_0$ being the Bessel function of zero order, Eq. (10) becomes

$$S'_{1,2}(q) = 4I(q)[1 - J_0(q \Delta r)],$$

in which $\Delta r = |V| \Delta t$. We can conclude that the two-dimensional power spectrum of the difference provides the velocity vector $V$, while the monodimensional spectrum gives only the absolute value $V$. Of course, in this case also the measured velocity is the transverse component only.

4. Experimental results

Starting from what was discussed above, we will show evidence that ordered motions of the sample inside the cell are visible and, more important, quantitatively measurable. A central point is that the method is independent of the tracer particle size because the dimensions of the speckles are ultimately determined only by the angular acceptance of the collection optics. We provide evidence that the method can be used to map velocity fields in a fluid where the tracer particle size is of the order of 100 nm. With such small particles the disturbance to the fluid flow is minimum, the inertial behavior of tracers being really negligible. Also, such small particles are negligibly affected by gravity, so that a pure water suspension does not sedimentate over times comparable with days or more. No spurious effects from gravity are therefore present, and the speckle motion is only determined by the fluid behavior.

We first show experimental results obtained with polystyrene spheres, 3 $\mu$m in diameter, suspended in pure water. The sample was confined inside a square 2 mm thick cell in which we induced an ordered, convective flow much stronger than sedimentation settling (sedimentation velocity for this sample is $\sim 0.25$ m/s). A comparison between the two methods used for recovering the cross-correlation of two signals (Eq. (3)) and the auto-correlation of their difference (Eq. (7)) is shown in Fig. 2(a) and (b), respectively. Both figures were obtained by averaging the correlations over 60 independent couples of frames, whose temporal distance was $\Delta t = 0.5$ s. The distance of each point from the center of the frame (indicated by a cross) represents the displacement $\Delta r$ that the particles have traveled over that period of time, while the gray scale is a measure of the number of particles that have been subjected to that displacement. Fig. 2a shows that the displacements are distributed on the right of the frame center, varying from zero to a maximum value of about 25 $\mu$m. The latter corresponds to a speed of about 50 $\mu$m/s. This evident “comet effect” is due to the velocity distribution inside the sample. For a quantitative analysis of this distribution we forward the reader to Ref. [15].
The same information (with the exception of the versus of the velocity) can be retrieved by analyzing Fig. 2b. In this case, according to Eq. (7), the correlation function is symmetrical, with a strong positive maximum at \( x = 0 \) and two weaker negative minima at about \( \pm 25 \mu m \). The first one corresponds to the auto-correlation of the signal \( G_{1,1}(x) \) and is related to speckle size, while the other two derive from the cross terms \( G_{1,2}(x) \) and \( G_{2,1}(x) \) and give the displacement of the fluid in the sample.

Secondly, the flow around a tip inserted in the cell has been measured. In this case, the measurement has been performed by recording the speckle field generated by a suspension of polystyrene spheres 263 nm in diameter, flowing into a cell due to convection. To map the velocity field, the whole frame has been divided in a number of square regions 58 \( \mu m \) wide, and the cross-correlation method (Eq. (3)) has been used in each of them to determine the local velocity vector. The velocity field is shown in Fig. 3, in which the different velocities have been represented as arrows starting from the center of the frame to the maximum value of the cross-correlation function.

A key point here deals with the need to guarantee that the local displacement measured within a single frame in the speckle field image really corresponds to the local velocity in the fluid. This is not obvious since the intensity distribution in a point of the observation plane is determined by the light coming from an extended region as shown in Fig. 1. To overcome this problem we used small aperture collection optics and set the observation plane in the center of the cell.
Finally, the influence of particle motion on the power spectra is analyzed. The sample was the same as that of Fig. 3, namely 3 μm particles in water, and the analysis was carried out over the same set of data. Fig. 4a shows the static 2D spectrum corresponding to Eq. (9) obtained by averaging 60 independent sample configurations. The corresponding 1D spectrum is reported in Fig. 4b. Albeit these two spectra are dependent on particle size, no information on particle velocity is present. The situation is different when the static 2D and 1D spectra associated to the signal $\delta f_{1,2}(r,t)$ are considered. In this case, the analysis was carried out by averaging 60 couple of frames taken at a temporal distance $\Delta t = 0.1$ s. The 2D spectrum, reported in Fig. 4c, clearly shows the presence of straight fringes whose amplitude scales as $I(q)$, and whose direction and wavelength give information about particle velocity or particle displacement. The corresponding 1D spectrum is illustrated in Fig. 4d. The depression for $q \to 0$ corresponds to the correlation between two heterodyne signals over increasingly larger length scales.

Quantitatively, the 2D spectrum of Fig. 4c can be analyzed by using Eq. (10). Thus, since the distance between two adjacent fringes approximately corresponds to a spatial frequency $q^* \sim 1.2 \mu m^{-1}$, the corresponding displacement is $\Delta r = 2\pi / q^* \sim 5.2$ μm, which corresponds to a velocity $|V| \sim 52$ μm/s, in good accordance to the result obtained in the space domain.

Summarizing, while the spectrum associated to the signal $\delta f(r,t)$ depends on particle size, the one associated to the signal $\delta f'_{1,2}(r,t)$ contains information on both particle size and velocity.
5. Discussion and conclusions

In this paper, we have shown that the near-field speckles are sensitive to transverse motions of the sample, and that we can profitably study their temporal evolution for recovering and mapping velocity fields in the fluid. A key point that we have heavily used in the experiments is that the speckle motion is independent of scatterer dimensions, thus allowing one to work with very small tracers inside the fluid.

Different procedures for data analysis have been developed, based on either the cross-correlation of two speckle fields taken at different times, or on the auto-correlation of the difference between the two speckle fields. Similar information could be also retrieved by recovering the power spectrum associated to the two speckle field difference.

The technique was successfully applied for the quantitative characterization of ordered convective motions of latex particles suspended in pure water and for mapping of the two-dimensional velocity flow around a tip inserted inside the cell.

Fig. 4. Results of data analysis in the spectral domain. In (a) and (b) we show the 2D and 1D power spectra associated to the heterodyne signal $\delta f$ (Eq. (2)), while in (c) and (d) the 2D and 1D power spectra associated to the heterodyne signal $\delta f_0$ (Eq. (6)). The results are in accordance to what is expected from Eqs. (9) and (10). Data are the same used to obtain Fig. 2.
References