Cross-Section for the Hard-core Scattering from a sharp-edged Body with Cylindrical Symmetry: a High-school Introduction

Marco Giliberti and Luca Perotti[a]

Sezione Didattica della Fisica, Dipartimento di Fisica dellUniversitá di Milano, via Celoria 16, 20133 Milano Italy

[a]*Center for Nonlinear and Complex Systems, Universitá degli studi dell'Insubria, Via Valleggio 11, Como 22100, Italy*

(July 19, 2005)

Abstract: The differential cross-section, for the elastic scattering of material points off a rigid bodies, obtained by the rotation of a generic derivable convex function, is calculated. The calculation is developed using elementary notions of calculus and is therefore suitable for European High-School students. Three particular cases are presented as examples of the general procedure and of the physical considerations about the found cross-sections on which a guided discussion can ensue in class.

01.40.Gm, 01.40.Ej

I. INTRODUCTION

The present paper focuses on the concept of cross-section, which is fundamental to describe and interpret the results of scattering experiments, especially in Nuclear and Particle Physics, where collisions between "elementary" particles are -if not the only- certainly the principal tool to investigate interactions and probe structures.

As the "effective surface" the target presents to the probing particles, it contains all the information about the nature of the interactions between probes and target we can extract from the experiment. In a few words, the cross-section is a key concept for the comprehension of Modern Physics.

Unfortunately, the calculation of cross-sections for physically relevant cases is usually rather long and often too difficult for European High-School students. Examples, if given, are therefore generally very few. To overcome this difficulty a simple approach and a general formula will be here introduced for a class of interactions for which the concept of cross section is expecially easy to grasp: that of elastic scattering of point-like particles by rigid surfaces having cylindrical symmetry. Although based on simple geometrical considerations, this approach allows an explicit calculation of cross-sections in many different situations and with very little effort. The application of the formula is simple and can lead to many interesting classroom discussions about the concept of cross-section, thus providing a clear High-School level introduction to the concept, useful for further developments.

The paper is organized as follows: section II introduces through an example the concept of total cross-section; section III introduces the class of interactions we use as a model and through it illustrates the concept of differential cross-section; section IV outlines the proposed approach to the calculation of the differential cross-section for the chosen class of interactions; section V presents three particular significant cases, namely that of ellipsoids, which reduces in the case of equal semiaxes to the classical example of spherical targets, the case of paraboloids, which give the same angular dependence of the differential cross-section as the Rutherford experiment and finally the case of targets generated by the rotation of an inverse sine curve which presents a curious similarity to our second example; in section VI we discuss our approach and propose further developments; finally, section VII summarizes the advantages of the proposed approach.

The order of our presentation, the relevance given in it to the concept of total cross section, and the use of several examples have been prompted by the problems evidentiated by a preliminary test (whose results we shall discuss in section VII) on 18 year old last year High-school students of an Italian "Liceo Scientifico", which had been introduced to the algorithm we propose only at the end of a traditional presentation of the subject.

II. THE CONCEPT OF TOTAL CROSS-SECTION

In the experience of many a teacher, a whimsical story can be a good starting point for the introduction of a new concept to a High-School audience. To introduce that of cross-section we can start with the following one: during a party someone releases a large number of coloured balloons, so that they rise in the sky. Suppose now that a lunatic starts to shoot against them, as in figure 1.

What is the probability that a bullet strikes one of the balloons? It is evident that the answer depends:

a) on the number of balloons per unity volume, that is on the density n of the balloons,

b) on the height h of the layer of balloons,

c) and on the section σ_T that each one of them shows to the bullets.

This last quantity is what is called the total cross-section for the bullet-balloon interaction.

Lets suppose, for simplicity, that:

i) the lunatic is moving inside a circle of surface S while he shoots straight upwards at random times: figure 1;

ii) the balloons are sufficiently spaced from each other and h is small, so that $nh\sigma_T \ll 1$ and the probability that a bullet hits more than one balloon is negligible (thin target hypothesis).

In this case the total area shown to the bullets by the balloons contained in the cylinder of base S and height h is $nSh\sigma_T$; and the probability P of a bullet-balloon collision is given by its ratio to the total base surface S of the same cylinder:

$$P = \frac{nSh\sigma_T}{S} = nh\sigma_T \tag{1}$$

Relation (1) shows the connection between the probability of collision and the total cross-section. Even if obtained in this very simple and special case, it has a much wider range of applicability. In fact it may be regarded as a general definition of the total cross-section, provided the target is thin, in the sense of hypothesis ii).

III. THE CONCEPT OF DIFFERENTIAL CROSS SECTION

Let's now imagine another very simple situation, which will be helpful to introduce a deeper and more fundamental concept: that of differential cross-section. Suppose to have hard solids with cylindrical symmetry, held fixed at some points of the space to form a thin target, and imagine to shoot at them, along the direction of their symmetry axis, a well collimated beam of point-like particles which ricochet on hitting the individual targets: see figure 2.

The probability that one of the particles suffers a collision is evidently given by (1). However this second example differs from the previous one for the kind of bullet-target interaction. In this second case also more refined questions can be asked, questions that, in the first case, either have no meaning or a trivial answer. One of them is the following: what is the probability that an incoming particle is scattered more than a given angle? In the first case, the answer is trivially zero: the balloons do not scatter the bullets shot at them.

To instead answer this question in the second case, we can start considering one particle, moving toward one of the fixed bodies, on a straight line, at a distance b from the axis of the body (b is called impact parameter). We limit our analysis to elastic scattering, so that the particle will be deflected according to the reflection laws: the trajectory of incidence, the perpendicular to the reflecting surface at the impact point and the trajectory of reflection all lie in the same plane and the incidence angle is equal to the reflection angle. Lets call ϕ the scattering angle, that is the angle of deviation from the initial trajectory caused by the collision; see figure 3.

One can see that if the target is convex (the second derivative of the generating curve is stricly positive), then the smaller the impact parameter b the larger is the scattering angle ϕ. That means that the particles striking a disc of area πb^2, perpendicular to their velocity, will suffer an angle of scattering greater than ϕ; see again figure 3. We can now answer our question by saying that the probability that an incoming particle is scattered through an angle greater than ϕ is:

$$P(\phi) = nh\pi b^2 \tag{2}$$

In other words and keeping in mind equation (1), it can be said that the cross-section for scattering through an angle greater than ϕ is:

$$\sigma(\phi) = \pi b^2 \tag{3}$$

We observe that equation (3) is not "fundamental" in that it relates the cross-section to the impact parameter b which, in a scattering experiment, where the positions of the individual targets are not known, is not a measurable quantity. Nonetheless this equation will be of great importance for the next considerations.

We now further refine the question to: what is the probability that a particle is scattered through an angle between ϕ and $\phi + d\phi$?

The particles scattered, through an angle between ϕ and $\phi + d\phi$, are given by those ones deflected through an angle greater than ϕ minus those particles deflected more than $\phi + d\phi$. These are the particles that hit the fixed body on a cross surface of area $|d\sigma| = \sigma(\phi) - \sigma(\phi + d\phi)$; the required probability is then:

$$P_\phi = nh|d\sigma| \tag{4}$$

After the scattering, these particles are contained into a solid angle of amplitude $d\Omega$ given by the ratio between the area of the spherical zone Z, of figure 4, and r^2, that is:

$$d\Omega = \frac{2\pi(r\sin\phi)rd\phi}{r^2} = 2\pi\sin\phi d\phi \tag{5}$$

and therefore the probability that a single particle is scattered around the angle ϕ, per unit solid angle, is:

$$P_\phi = nh\frac{|d\sigma|}{d\Omega} \tag{6}$$

Equation (6) is of general interest and its valid for all scattering experiments in the thin target hypothesis.

The fundamental quantity $|d\sigma|/d\Omega$ is called the differential cross-section. To better understand its physical meaning we can extend our description of the experiment to include the detection process. Consider a particle beam shot against a fixed target and an ideal particle detector of effective section A, located at an angle ϕ at a distance R from the target and perpendicular to the scattered particles. In this way it detects all the particles in the solid angle of amplitude $\Omega \sim A/R^2$. If Ω is sufficiently small, P_ϕ can be considered to be constant over the surface A, and the ratio between the number of detected particles and the number of incident particles is given by P_ϕ multiplied by Ω which is:

$$P_{\phi,\Omega} = nh\frac{|d\sigma|}{d\Omega}\frac{A}{R^2} \tag{7}$$

Equation (7) shows a clear way of calculating the differential cross-section from given experimental measures and a comparison between (6) and (7) helps students enlighten the conceptual meaning of this useful quantity.

However it should be noted that, even if we have focused our attention to (7), $|d\sigma|/d\Omega$ is not the fundamental quantity for every scattering process, for instance in the case of inelastic scattering the relevant quantity is

$$\frac{d^2\sigma}{d\Omega dE} \tag{8}$$

(where E is the energy of the scattered particle), that is the cross-section per unit solid angle and per unit energy.

IV. DETAILED CALCULATION

We state in advance that, at a first reading, the following analysis might seem a little bit formal and that, during classroom lessons, High-School students should be warned not to get discouraged. In fact, at the end of the following calculations, students will soon be able to deal with many different scattering problems with just a "touch" of guide by their teacher. In this way their comprehension of cross-section will grow rapidly deeper.

Let's consider the rigid solid produced by the complete rotation around the y axis of the increasing convex function $y = f(x)$ with x between 0 and a; figure 3. Our aim is to calculate the total and differential cross-sections for the elastic scattering of point-like particles, that are shot, in the direction of the y axis, against this fixed solid. The total cross-section is calculated straightforwardly as:

$$\sigma_T = \pi a^2 \tag{9}$$

For what concerns the differential cross-section we make reference to figure 3.

Since the incident line, the perpendicular to the surface and the line of reflection lie all on the same plane, the problem can be considered in the $x-y$ plane. Let b be the impact parameter, t the tangent to the curve in the collision point, p the perpendicular, j the line of incidence and d the line of reflection. Now c is orthogonal to the x axis and p is orthogonal to t, therefore the angle α, between t and the x axis, is equal to the angle $\hat{\imath}$ between j and r, that is the incidence angle.

It follows that:

$$g(b) \equiv \left.\frac{df}{dx}\right|_{x=b} = \tan\alpha = \tan\hat{\imath} = \tan\left(\frac{\pi-\phi}{2}\right) = \cot\frac{\phi}{2} \tag{10}$$

We are now able to obtain the differential cross-section. The principal steps are the following:

I.Solve equation (10) with respect to b (with the geometric limitation $0 \le b \le a$).

In formulae, defining the inverse function g^{-1} through the equation $g^{-1}(g(x)) = x$:

$$\begin{cases} b(\phi) = g^{-1}\left(\cot\frac{\phi}{2}\right) \\ \pi - 2\arctan(g(a)) \le \phi \le \pi - 2\arctan(g(0)) \end{cases} \tag{11}$$

II.Calculate $\sigma(\phi) = \pi b^2$.

III.Differentiate $\sigma(\phi)$ to obtain $|d\sigma(\phi)|$.

IV.Divide the found expression by $d\Omega$ given by eq.(5).

V. THREE EXAMPLES

1) Let's consider the ellipsoid obtained by the rotation, around the y axis, of the function

$$f(x) = -c\sqrt{1-\left(\frac{x}{a}\right)^2}; \qquad 0 \le x \le a, \tag{12}$$

with a and c the two semiaxes of the generating ellipse[1].

Its derivative is:

$$g(b) \equiv \left.\frac{df}{dx}\right|_{x=b} = \frac{cx}{a^2}\frac{1}{\sqrt{1-\left(\frac{x}{a}\right)^2}}. \tag{13}$$

Therefore from step I, equation (11), we get

$$\begin{cases} b^2 = a^2\frac{\cot^2\left(\frac{\phi}{2}\right)}{\frac{c^2}{a^2}+\cot^2\left(\frac{\phi}{2}\right)} \\ 0 \le \phi \le \pi \end{cases} \tag{14}$$

and then (steps II., III., and IV.):

$$\frac{|d\sigma(\phi)|}{d\Omega} = \frac{a^2}{4}\left(\frac{ac}{c^2\sin^2\left(\frac{\phi}{2}\right)+a^2\cos^2\left(\frac{\phi}{2}\right)}\right)^2. \tag{15}$$

A simple change of signs in eq. (12) gives us the equation of a hyperboloid:

$$f(x) = c\sqrt{1+\left(\frac{x}{\tilde{a}}\right)^2}; \qquad 0 \le x \le a, \tag{16}$$

which results in the cross-section

$$\begin{cases} \frac{|d\sigma(\phi)|}{d\Omega} = \frac{\tilde{a}^2}{4}\left(\frac{\tilde{a}c}{c^2\sin^2\left(\frac{\phi}{2}\right)-\tilde{a}^2\cos^2\left(\frac{\phi}{2}\right)}\right)^2. \\ \pi - 2\arctan\left(\frac{ca}{\tilde{a}}\frac{1}{\sqrt{c^2+a^2}}\right) \le \phi \le \pi \end{cases} \tag{17}$$

where, as the curve (16) is not bound in x, we had to distinguish between the target size parameter a (introduced as to avoid an infinite total cross-section) and the curve patrameter $\tilde{a}$.

For $c = a$ eq. (15) reduces the well known expression of the differential cross-section of a rigid sphere[2], which depends only on the sphere radius a, which is a constant, but is independent of ϕ. This means that after the collision with a sphere the particles are isotropically scattered, that is they are deflected to every angle with equal probability. This result depends on the particular interaction here considered. If in a scattering experiment, in which we dont know the nature and composition of the target, we get results (fraction of particles revealed at certain angles) that are independent both from the scattering angle and from other physical quantities such as, for example, the kinetic energy of the incident particles or their electric charge, from which they could a priori depend, we have strong indications that the interaction between particles and target is a hard-core one (elastic collision with the surface of the individual target) and that the target is "made of" rigid spheres.

Its worth noting that from the obvious relations:

$$\int_{all\ \ directions} d\Omega = 4\pi; \qquad \int \frac{d\sigma}{d\Omega} d\Omega = \sigma_T, \quad (18)$$

and from equation (15), one gets immediately the trivial result (equation (9)) $\sigma_T = \pi a^2$. This can be a useful check for most students.

2)Let us consider the solid obtained by the rotation of the curve[3]

$$f(x) = \frac{x^2}{c}; \qquad 0 \le x \le a. \quad (19)$$

Then, reminding equation (9), we have that the total cross section again is:

$$\sigma_T = \pi a^2. \quad (20)$$

For the differential cross-section we again start with the derivative of $f(x)$ in b:

$$g(b) \equiv \left.\frac{df}{dx}\right|_{x=b} = 2\frac{b}{c}. \quad (21)$$

From step I. (equation (11)) we then get:

$$\begin{cases} b = \frac{c}{2}\cot\left(\frac{\phi}{2}\right) \\ \pi - 2\arctan\left(2\frac{a}{c}\right) \le \phi \le \pi \end{cases} \quad (22)$$

and from step II.:

$$\sigma(\phi) = \pi b^2 = \frac{\pi c^2}{4}\cot^2\left(\frac{\phi}{2}\right). \quad (23)$$

then (step III.):

$$|d\sigma| = \left|\frac{\pi c^2}{4} 2\cot\left(\frac{\phi}{2}\right) \frac{-1}{\sin^2\left(\frac{\phi}{2}\right)} \frac{1}{2} d\phi\right| = \frac{\pi c^2}{4} \frac{\cos\left(\frac{\phi}{2}\right)}{\sin^3\left(\frac{\phi}{2}\right)} d\phi. \quad (24)$$

and finally, dividing (24) by (5), (step IV.):

$$\frac{|d\sigma|}{d\Omega} = \frac{\pi c^2}{4} \frac{\cos\left(\frac{\phi}{2}\right) d\phi}{\sin^3\left(\frac{\phi}{2}\right)} \frac{1}{2\pi \sin\phi d\phi} = \frac{c^2}{16} cosec^4\left(\frac{\phi}{2}\right). \quad (25)$$

This result shows some similarities with the Rutherford cross-section[4,5]:

$$\frac{|d\sigma_R|}{d\Omega} = \frac{K^2}{16} cosec^4\left(\frac{\phi}{2}\right); \qquad K = \frac{zZe^2}{4\pi\epsilon_0 K_0} \quad (26)$$

where Ze is the target charge, ze and K_0 are the charge and kinetic energy of the incident particle and ϵ_0 is the vacuum permeability. Clearly, the same angular dependence appears; moreover, both of the cross sections are not valid for small angles, but for very different reasons: in the present case because of the discontinuity of $f(x)$ in $x = a$, and in the Rutherford case because when the impact parameter is large so is the distance of closest approach to the nucleus and the shielding effect of the atoms' elecronic cloud is no more negligible. How can we then distinguish the two cases? First of all, the differential cross-section given by eq. (25) shows no dependence from the kinetic energy of the incident particles (the variable in the Rutherford experiment which can be most easily changed); moreover eq. (25) is only valid when the incidence direction is that of the symmetry axis of the individual targets; a rotation of the target with respect to the incident beam would result in a different result.

3)Let's finally consider the surface obtained by the rotation, around the y axis, of the function

$$f(x) = \tilde{c}\arcsin\frac{x}{a}; \qquad 0 \le x \le a. \quad (27)$$

Its derivative reads

$$g(b) \equiv \left.\frac{df}{dx}\right|_{x=b} = \frac{\tilde{c}}{a}\frac{1}{\sqrt{1-\left(\frac{x}{a}\right)^2}}. \quad (28)$$

Therefore from step I, equation (11), we get

$$\begin{cases} b^2 = a^2 - \tilde{c}^2\tan^2\left(\frac{\phi}{2}\right) \\ 0 \le \phi \le \pi - 2\arctan\left(\frac{\tilde{c}}{a}\right) \end{cases} \quad (29)$$

then (step II. and III.):

$$|d\sigma| = \left|\pi\tilde{c}^2 2\tan\left(\frac{\phi}{2}\right)\frac{1}{\cos^2\left(\frac{\phi}{2}\right)}\frac{1}{2}d\phi\right| = \frac{\pi\tilde{c}^2}{2}\frac{\sin\phi}{\cos^4\left(\frac{\phi}{2}\right)}d\phi. \quad (30)$$

and finally, dividing (30) by (5), (step IV.):

$$\frac{|d\sigma|}{d\Omega} = \frac{\tilde{c}^2}{4}\frac{1}{\cos^4\left(\frac{\phi}{2}\right)} = \frac{\tilde{c}^2}{4} sec^4\left(\frac{\phi}{2}\right), \quad (31)$$

which is a mirror image of our previous example eq. (25), obtained by the substitutions $\phi \to \pi - \phi$ and $c \to 2\tilde{c}$. The transformation between the two ranges in ϕ follows the same simple rule, as for $\alpha \in (0, \pi/2)$,

$$\frac{\pi}{2} - \arctan \alpha = \arctan \left(\frac{1}{\alpha} \right) . \qquad (32)$$

VI. DISCUSSION

The concept of cross-section in the case of hard-core scattering has been introduced on the basis only of simple statistic and geometric considerations. This choice has been made because of didactic reasons: with this approach, the intuition is helped both by the "material" existence of the cross-sections both total and differential (which are real parts of the rigid surface of the body) and by the existence of the trajectories of the particles.

In physically more significant cases, as e. g. the Rutherford scattering itself, the interaction is between the incoming particle and the force field of the target. It is true that in these cases the total cross-section can be geometrically conceived as the "effective surface" presented by the target force field to the incoming particles and the differential cross section as the "part" of that "surface" scattering the particles in a given direction, but this only in an abstract sense, which usually is not easy for the students to grasp: for example, even in the simple geometrical case of particles randomly scattered by a rough surface, the differential cross section, even if mathematically defined, cannot be associated with any geometrically definite part of the scattering surface.

Besides this difficulty, the general approach, in which the scattering is introduced from the beginning in its abstract meaning, often allows students only to discuss existing experimental data, thus missing important relations and theoretical considerations, an understanding of which can be gained by the direct calculation of several simple cross-sections.

The approach we propose can moreover be easily adapted with only minor changes to the case of light beams of suitable wavelength being reflected according to the rules of geometrical optics by the targets' surfaces, thus smoothing the transition to the Quantum Mechanical treament where the concept of trajectory of a point-like particle must be abandoned. In this case the differential cross section $|d\sigma|/d\Omega$ is defined by the equation

$$\frac{dP}{d\Omega} = \frac{dP}{dS} \frac{|d\sigma|}{d\Omega} \qquad (33)$$

where the functions dP/dS and $dP/d\Omega$ are the incoming energy flux per unit surface (which is convenient to assume constant) and the scattered energy flux per unit sold angle respectively.

VII. CONCLUSIONS

In brief, this paper gives a simple formula to calculate with little effort many different cross-sections, three of which are explicitely given (in the case of a rigid ellipsoid, in that of a rigid paraboloid, and in that of the solid generated by the rotation of an inverse sine curve). The formula itself is of limited utility because of the very simple interaction taken into account but, at the introductory level, it can help students understand the link between experiments and theoretical explanations.

An extended case study, about the students' understanding of the cross-section concept at High-School level, following the ideas outlined in the present paper, is under way and will be presented in a forthcoming paper. Only preliminary results are currently available, relating to a previous study on the same subject, when the algorithm outlined above was first tried on (18 year old) last year High-school students of an Italian "Liceo Scientifico", whose curriculum includes simple calculus.

Before presenting those results we should point out that on that occasion the students were first presented with the Rutherford scattering, where it has no sense to introduce the concept of total cross section (as it is infinite if the electronic cloud is neglected), and only afterwards both the algorithm and the concept of total cross section were introduced, but only in the case of elastic scattering off rigid spheres, which is isotropic.

The drawbacks of that approach were evident from the test presented to the students, and prompted the revised approch otlined here.

Among other questions, the students were asked to calculate the total and partial cross sections and the minimum deflection angle for a target composed of solids obtained by the rotation, around the y axis, of the function[6]

$$f(x) = x^3; \qquad 0 \le x \le 1. \qquad (34)$$

Out of 12 students, only one correctly answered the question about the minimum deflection angle, suggesting the need for elastic scattering examples more general than the rigid spheres one: there, the minimum deflection angle is zero and in the only other example given to the students, namely the Rutherford scattering, it arises only from limitations of the model used.

Only three students correctly answered the calculationally simple first question about the total cross section, and four thought it to be a function of the deflection angle. This suggests a confusion between $\sigma(\phi)$ and σ_T which could be avoided by some more examples of finite total cross sections.

Most students on the other hand directly tackled the calculation of the partial cross section, 8 correctly reaching step II. and 6 completing the calculation, a rather encouraging result.

VIII. ACKNOWLEDGEMENTS

We are very grateful to Profs Ludovico Lanz, Lucio Rossi and Guido Vegni for very helpful discussions and their comments on the manuscript. The resent work was conducted within the frame of the Italian inter-university project "F21 - Percorsi di formazione in fisica per il 21^o secolo".

---

[1] this example is discussed in detail in: F. Corni , M. Michelini , L. Santi , F. Soramel and A. Stefanel "The Concept of the Cross Section", in Proc. Girep Conf. *Teaching the Science of Condensed Matter and New Materials* (Forum Editrice, Udine, 1996) pp. 192-198.

[2] see e.g.: L.D. Landau and E.M. Lifshits, *Mechanics*, English trans. by J.B. Sykes and J.S. Bell (Pergamon, Oxford, 1976), 3rd ed., pp. 48-57.

[3] this particular model has already been presented with similar finalities in: J. Evans, "Elastic scattering by a paraboloid of revolution", Am. J. Phys. **56**, 423-425 (1988).

[4] E. Rutherford, "The Scattering of α and β particles by Matter and the Structure of the Atom", Phil. Mag. ser. 6,xxi 669-688 (1911).

[5] W.S.C. Williams *Nuclear and Particle Physics* (Oxford University Press, Oxford, 1992).

[6] For the general case $f(x) = x^n/c; \quad n \geq 2; \quad 0 \leq x \leq a$, we have

$$\begin{cases} \sigma_T = \pi a^2 \\ \frac{|d\sigma|}{d\Omega} = \frac{\pi}{4(n-1)} \left(\frac{2^n c}{n} \frac{\cos^2 \frac{\phi}{2}}{\sin^n \phi} \right)^{\frac{2}{n-1}} \\ \pi - 2\arctan\left(\frac{n a^{n-1}}{c} \right) \leq \phi \leq \pi; \end{cases} \tag{35}$$

at this point, substituting the values $n = 3$, $a = 1$, and $c = 1$, the solution in the particular case proposed to the students is:

$$\begin{cases} \sigma_T = \pi \\ \frac{|d\sigma|}{d\Omega} = \frac{\pi}{24} \frac{1}{\sin^3 \frac{\phi}{2} \cos \frac{\phi}{2}} \\ \pi - 2\arctan(3) \leq \phi \leq \pi. \end{cases} \tag{36}$$

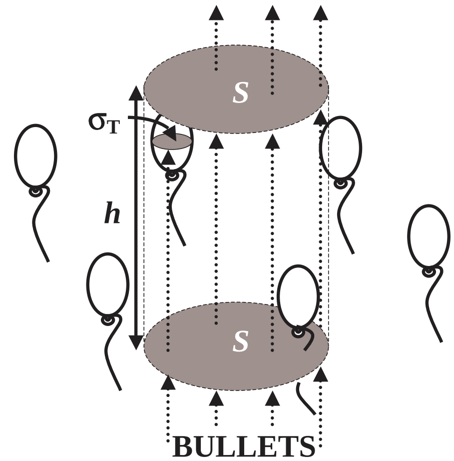

FIG. 1. A lunatic shoots some bullets against some toy balloons contained into a cylinder of base surface S and height h. If σ_T is the section shown by each toy balloon to the bullets, the probability that a bullet strikes a toy balloon is $P = nh\sigma_T$, where n is the density of the layer of toy balloons.

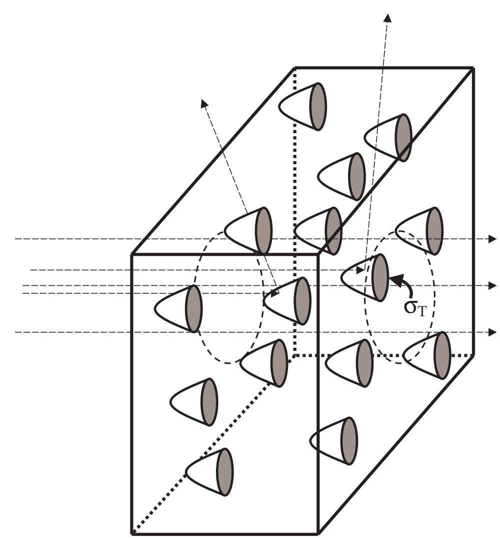

FIG. 2. A collimated beam of point-like particles is shot against a thin target of hard solids, each one of total cross-section σ_T, fixed at some points of the space. Most of the particles will be undeflected but some of them will suffer a collision.

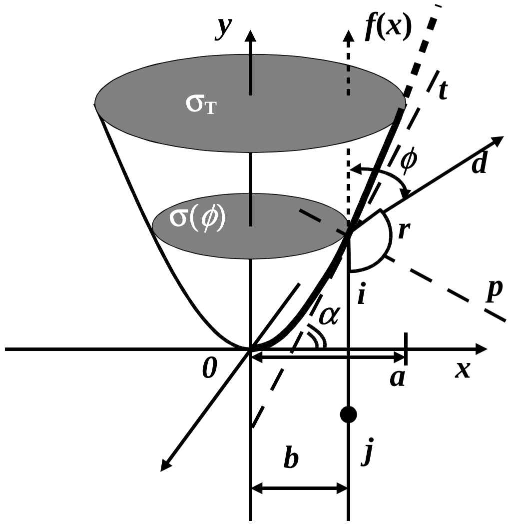

FIG. 3. A sharp-edged solid is given by the complete rotation around the y axis of the increasing convex function $y = f(x)$ with x between 0 and a. A point-like particle, moving in the y-direction, is scattered by the solid through the angle ϕ: b is the impact parameter, t the tangent and p the perpendicular to the solid at the collision point, c and d are respectively the line of incidence and the line of reflection.

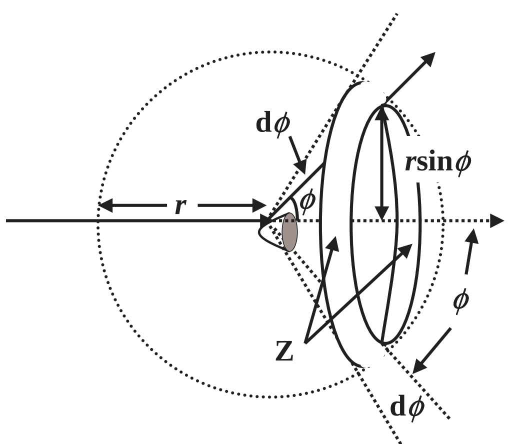

FIG. 4. The particles scattered between ϕ and $\phi + d\phi$ are contained into a solid angle of amplitude given by the ratio between the area $2\pi r^2 \sin\phi d\phi$, of the spherical zone Z, and r^2.