BIFURCATION-ASSISTED MICROWAVE IONIZATION OF HIGHLY EXCITED HYDROGEN ATOMS

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Experimental results for the highly excited hydrogen atom exposed to a partially-ionizing short pulse of microwave electric field often are near-classical, that is they agree reasonably well with the predictions of classical models. When the microwave frequency is within 20% of the initial Kepler electron orbiting frequency, the presence of a primary classical nonlinear resonance dominates the character of a number of experimentally relevant quantum quasienergy eigenstates at the peak of the pulse [1], leading to an observed near-classical resonant increase in the threshold field for ionization [2]. Bounding this resonance region in phase space is a separatrix that is crossed by the system during the rise and during the fall of the pulse [3], producing an explanation for observed near-classical double-peaked final bound state distributions [4].

Here we report the experimental observation of a third near-classical phenomenon in this frequency region, the presence of sequences of steps in the ionization probability with increasing microwave field strength. Classical numerical calculations that include the pulse envelope reveal a separatrix crossing early in the rise of the pulse, which places a large classical electron probability close to the center of the primary resonance region. At higher fields fixed-field calculations find the creation of a secondary nonlinear resonance at a field strength at the base of each step. Each secondary resonance appears to constitute a branch of a subharmonic bifurcation tree having the primary resonance as its trunk [5]. The diameter in phase space of a given secondary resonance grows rapidly with increasing field to reach the diameter of the primary resonance region itself. As the field further increases, the secondary resonance region locally goes chaotic, a golden-mean KAM surface is broken into cantori, and classical transport into the globally chaotic region of phase space [6] produces an ionization step.

We expect that quantum calculations of quasienergy states would reveal adequate support for the secondary resonances when the latter are large. There is an interesting unaddressed question of how quantum mechanics reflects the time evolution of an enlarging classical feature in phase space that had initial area much smaller than Planck's constant.

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[1]. R. V. Jensen et al, Phys. Rev. Lett. 63, 2771 (1989).

[2]. E. J. Galvez et al, Phys. Rev. Lett. 61, 2011 (1988).

[3] K. Dietz, J. Hankel and M. Holthaus, Phys. Rev. A 45, 4960 (1992).

[4] J. E. Bayfield and D. Sokol, Phys. Rev. Lett. 61, 2007 (1988).

[5] J. M. Greene, R. S. MacKay, F. Vivaldi and M. J. Feigenbaum, Physica 3D, 468 (1981).

[6] R. S. MacKay, J. D. Meiss and I. C. Percival, Physica 13D, 55 (1984).

The experiments utilized fast stretched highly excited hydrogen atoms exposed to collinear pulsed microwave and steady static electric fields, to achieve a nearly one dimensional system. Figure 1 shows a schematic of the heart of the apparatus.

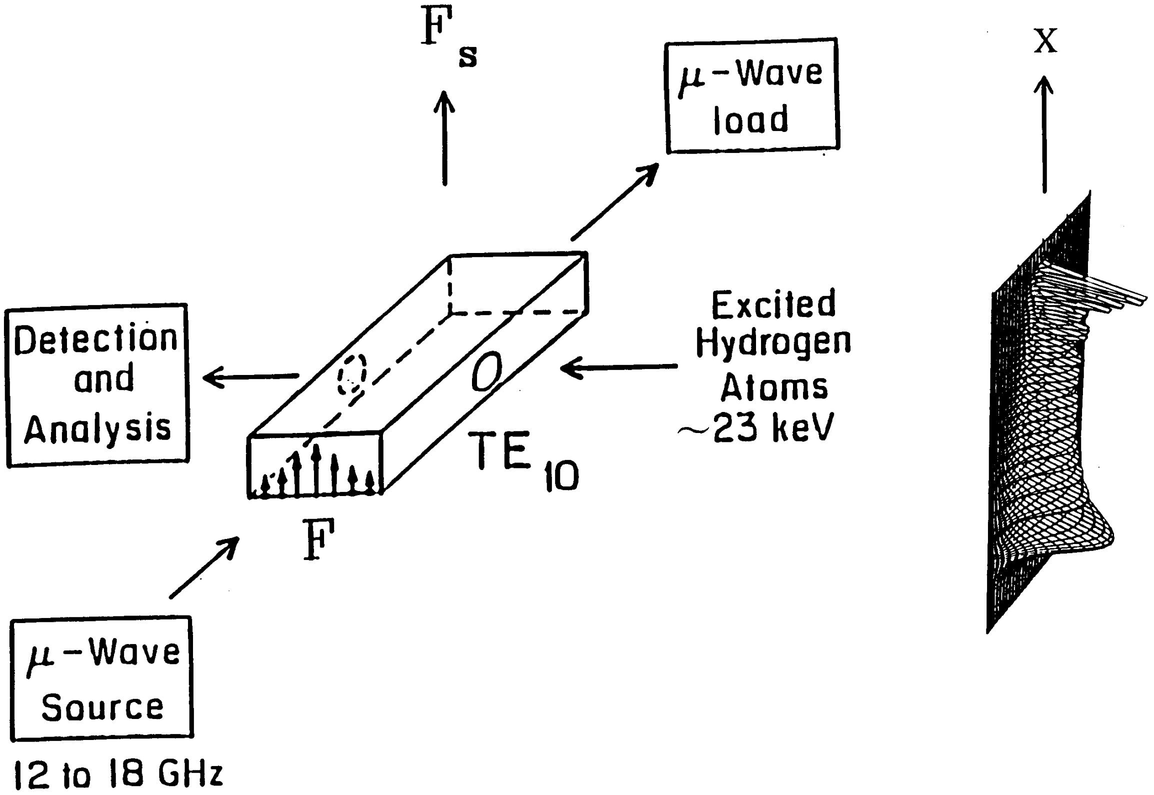


Figure 1. Schematic of a "one dimensional" experiment on hydrogen in microwaves. Fast hydrogen atoms are laser-excited in a static field F_S to a nearly one dimensional stretched atom quantum energy eigenstate (see right, not to scale!). The atoms are passed through holes in a rectangular microwave guide. In their rest frames, the atoms are exposed to a short half sinewave pulse of microwave electric field F. The static field, microwave field and atom stretch directions are all collinear. The principal quantum number of the atoms is selectable in the range 60 to 75. The microwave pulse induces transitions to bound stretched atom states with various principal quantum numbers, as well as transitions to the atomic continuum.

The stretched atoms are in initial quantum energy eigenstates having parabolic quantum numbers n,n_1,m = 69,0,0 prepared by an optical double resonance excitation technique. Detection and analysis using state-selective static electric field ionization was set up for measurement of microwave "ionization probability", defined as probability for transition outside the band $50<n<90$. The static field was 8.0 V/cm.

Let the static field be F_s and the microwave field be $F(t)=f(t)\,F\sin(\omega t+\phi)$, where for the half sinewave pulsed field $f(t)=\sin(\pi t/T_p)$ with T_p the temporal pulse length. We have carried out pulsed-field numerical calculations using a classical model with the one dimensional Hamiltonian

$$H = p^2/2 - 1/x + x[F(t)-F_s]$$

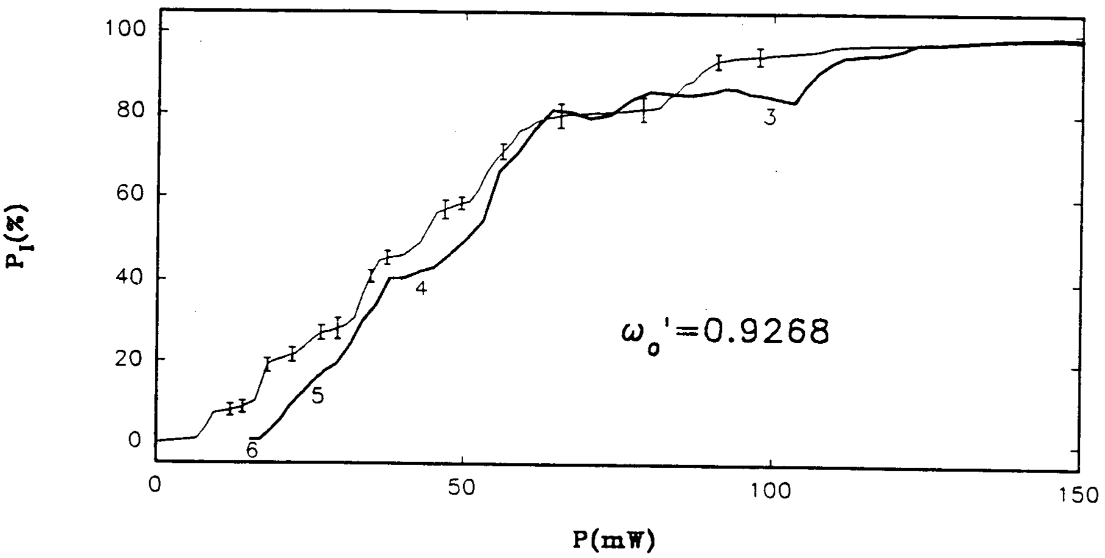
We also do fixed field calculations, with $f(t)=1$. We transformed to free-atom action-angle variables I,θ, introduced the eccentric anomaly ξ according to $\theta=\xi-\sin\xi$, and introduced the modified time $\eta=t/(1-\cos\xi)$. We then integrated Hamilton's set of ordinary differential equations of motion for an ensemble of electron trajectories.

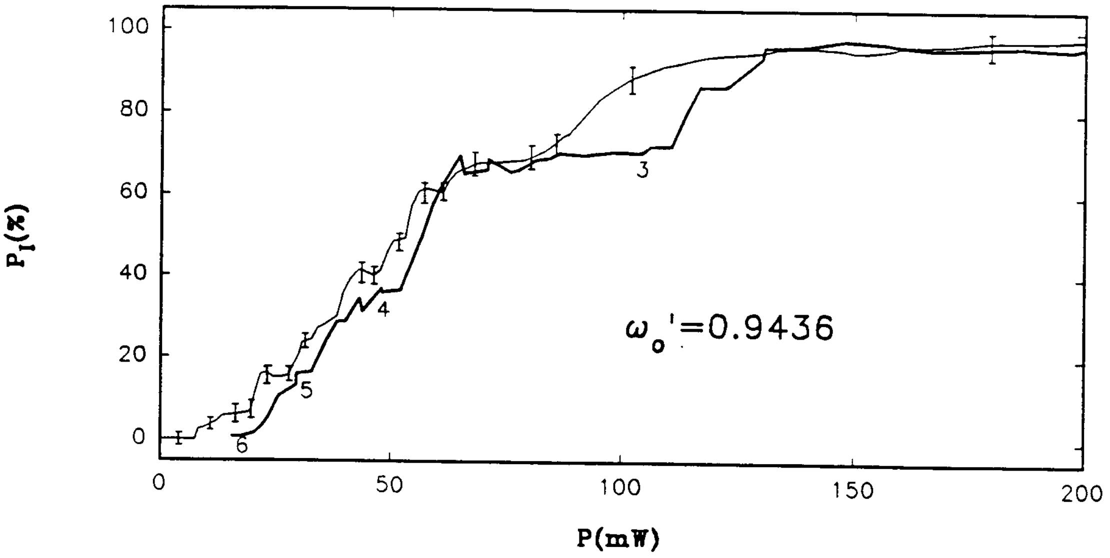
$$dI/d\eta=-I^2\sin\xi[f(t)F\sin(\omega t+\phi)-F_s]$$
$$d\xi/d\eta=1/I^3+2I(1-\cos\xi)[f(t)F\sin(\omega t+\phi)-F_s]$$
$$dt/d\eta=1-\cos\xi$$

The set of initial conditions I_0,θ_0 corresponded to the initially prepared quantum Stark energy eignenstate.

Up to fifty uniformly distributed values of θ_0 and thirty uniformly distributed values of ϕ were included.

Figure 2 shows some of our experimental data for the dependence of ionization probability on incident waveguide microwave power. The data is compared with pulsed-field numerical results.


P_I(%)
100
80
60
40
20
0
0
50
100
150
P(mW)
3
4
5
6
ω_o'=0.9268


P_I(%)
100
80
60
40
20
0
0
50
100
150
200
P(mW)
3
4
5
6
ω_o'=0.9436

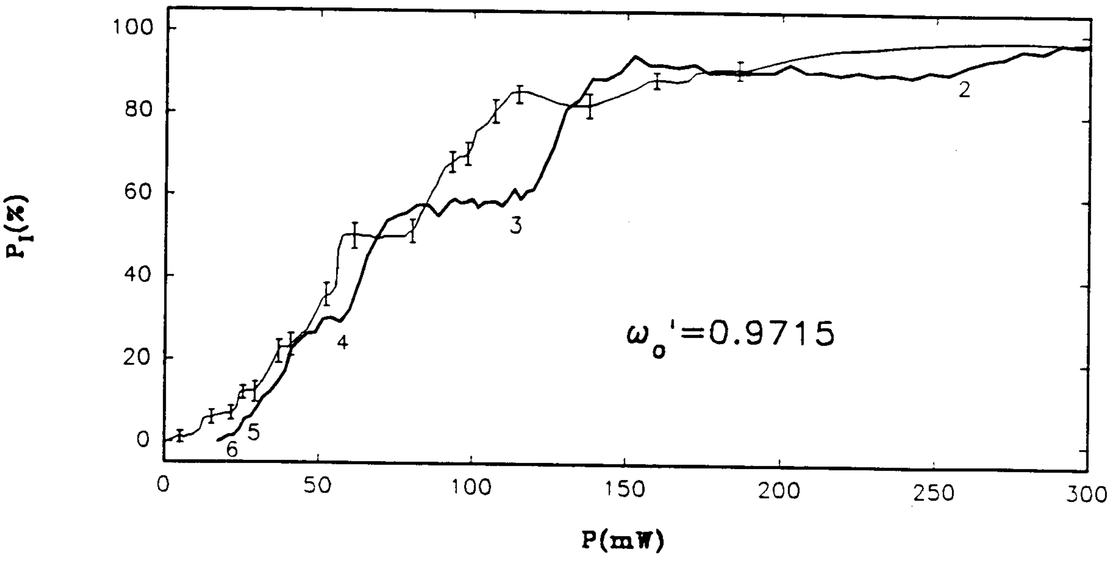

P_I(%)
100
80
60
40
20
0
0
50
100
150
200
250
300
P(mW)
2
3
4
5
6
ω_o'=0.9715

Figure 2. The probability of microwave ionization of n_0=69 stretched atoms, as a function of microwave traveling wave r.m.s. power in the waveguide. Experimental data (light lines) are compared with numerical predictions of a classical model (heavy lines). Results are shown for three microwave frequencies, all below the frequency for classical nonlinear resonance of the microwave oscillations with the Kepler orbiting.

Comparisons are shown for different Stark-corrected, classically scaled microwave frequencies $\omega_0' = n_0^3 \omega/(1-3n_0^4 F_s)$. Ionization steps are visible in both the data and numerical results. Each of the major steps is labeled by an integer q. The reasonable agreement seen in the comparisons indicates that the microwave ionization is largely near-classical. There are deviations, in that the experimental q=3 steps are at lower power than the classical ones. Also, the ionization probability is above classical at very low powers.

In order to uncover the origin of the ionization steps, it is useful to compare numerical calculations of pulsed-field instantaneous trajectory distributions in I,θ with fixed-field stroboscopic surfaces of section in I,θ. The latter reveal the structures in the phase space of this nonintegrable nonlinear system, for any fixed pair of values of ω_0' and $F_0=n_0^4F$. Figure 3 shows a few fixed-field and pulsed-field results.

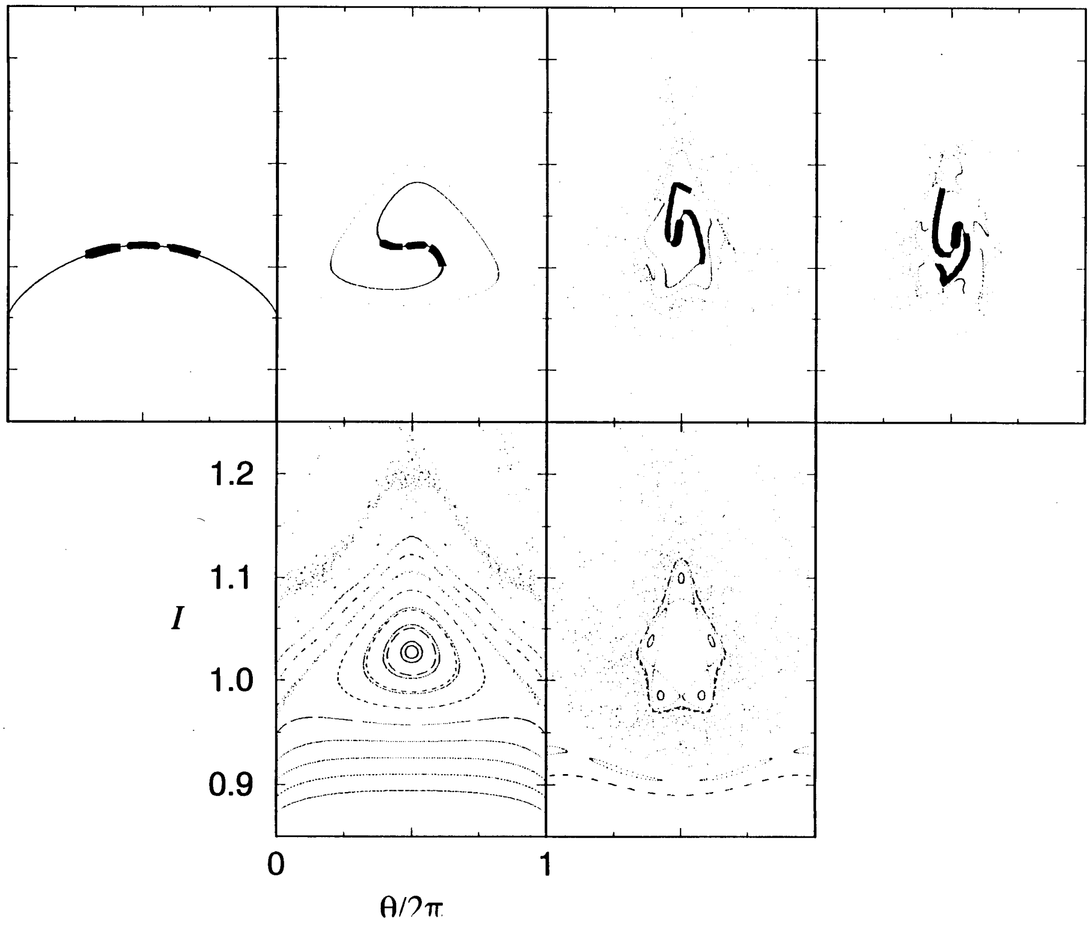


Figure 3. The top row of pictures contains snapshots of the instantaneous locations of the ensemble of classical electron trajectories in action-angle space. The parameters are $\omega_0'=0.9771$, $F_0=0.0290$ at the maximum of the pulse, and a pulse time of 136.5 microwave periods. This F_0 is just below the onset of a q=5 step. The points for three trajectory subsembles are enlarged. From left to right, the times are for the initial ensemble, for the ensemble at 31% of the maximum value of the field pulse, for the ensemble at the maximum value, and for the end of the pulse. The bottom pictures are stroboscopic surfaces of sections at fixed microwave field strength, with values equal to the instantaneous field values for the pictures directly above each of them.

When the instantaneous pulsed field is 31% of the maximum value, the trajectory ensemble is almost entirely within the primary nonlinear resonance island structure seen in the fixed-field surface of section calculated at the same field strength. When the instantaneous pulsed field is at maximum, a portion of the trajectory ensemble is entangled within a dominant secondary nonlinear resonance five-island (q=5) chain seen in the corresponding surface of section. At a slightly higher field strength, this entangled subemsemble would contribute to ionization.

The evolution of the structure in phase space can be determined by calculating a succession of fixed-field surfaces of section for increasing values of field strength F_0. One finds that over fairly narrow ranges of F_0, the structure within the primary nonlinear resonance region changes dramatically. Through calculating a large number of sufficiently detailed surfaces of section, each structural change was found to be initiated by a bifurcation of the primary periodic orbit ($\omega_0'=1/1$) of the system. A secondary nonlinear resonance island chain accompanies the new periodic orbit produced by each subharmonic bifurcation, such as the chain seen in Figure 3 at the pulse maximum. A secondary island chain grows with increasing F_0 to become quite large. When large enough, it is found to destabilize within a sharply defined range of F_0. Above the destabilization point the chain disappears, and a corresponding increase in the chaotic portion of the surface of section occurs. It is well known that chaotic electron trajectories are responsible for microwave ionization, in the classical limit.

Figure 4 shows a sequence of fixed-field results that show the evolution of phase space structure for a four-island secondary island chain, from bifurcation through destabilization to chaos.

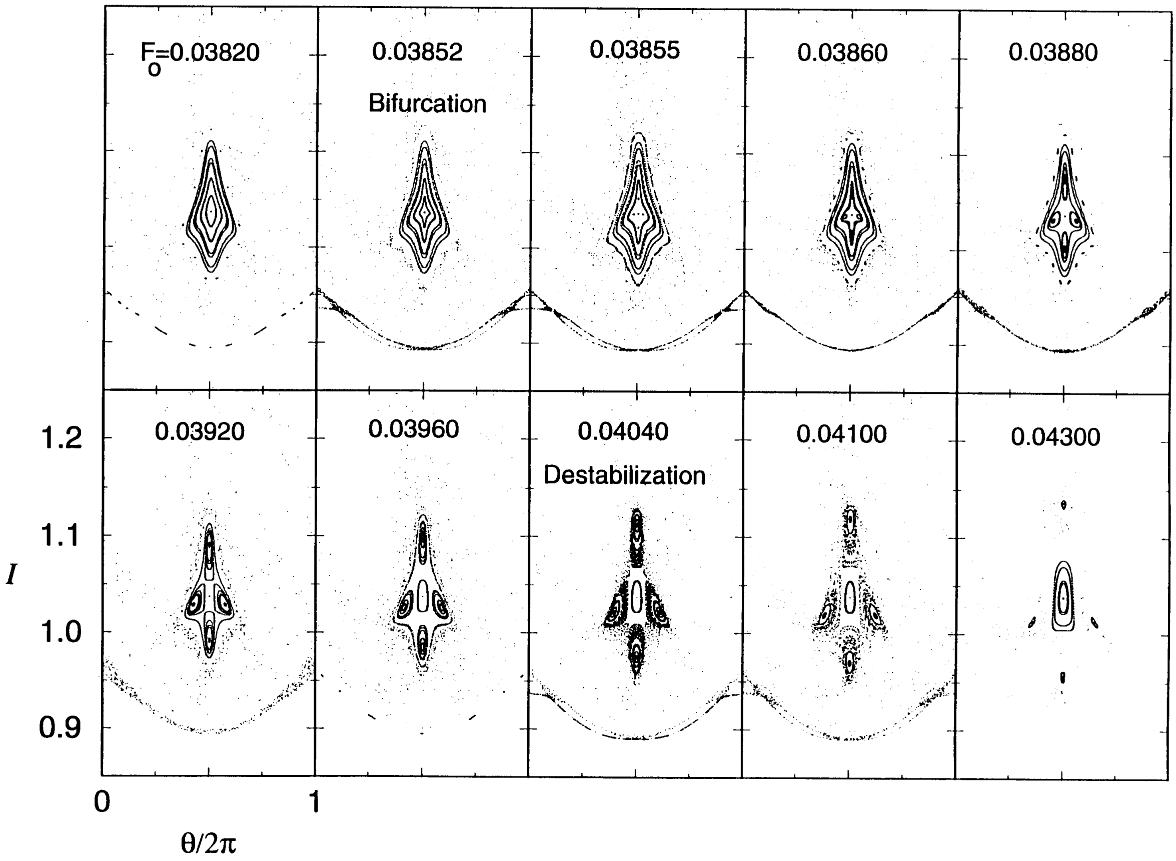


Figure 4. Fixed-field stroboscopic surfaces of section for $\omega_0'=0.9771$ and $\phi=\pi/2$. A series of pictures shows the nature of the primary nonlinear resonance region in action-angle space, as F_0 is increased from 0.0382 to 0.0430.

The bifurcation point is between $F_0=0.03852$ and 0.03855. The destabilizaton point is between $F_0=0.0396$ and 0.0410, and actually is very close to 0.0404. Let us define the height of a secondary island chain as the vertical distance (in action I or the corresponding principal quantum number n) between the centers of the uppermost and lowest islands in the chain. This distance Δn can be plotted as a function of F_0. The dominant secondary island chains are associated with periodic orbits of period q relative to the microwave oscillation period, where q=2,3,4,5,6,7..... . Plots of Δn for the case of $\omega_0'=0.9771$ are shown in the top portion of Figure 5.

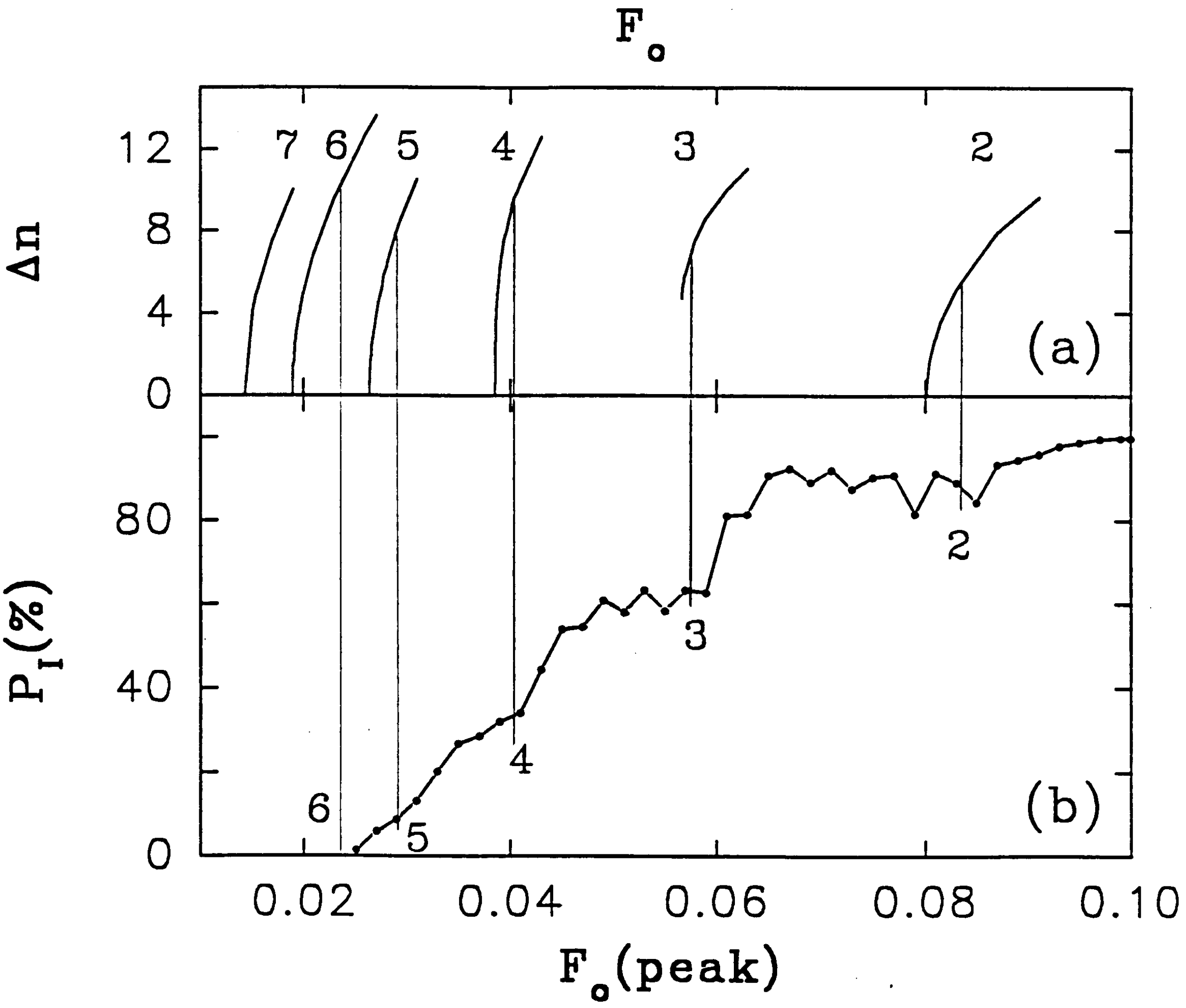

F$_o$
Δn
12
8
4
0
7
6
5
4
3
2
(a)
P$_I$(%)
80
60
40
0
6
5
4
3
2
(b)
0.02
0.04
0.06
0.08
0.10
F$_o$(peak)

Figure 5. Results of classical numerical calculations for $\omega_0'=0.9771$ and $\phi=\pi/2$. Top: a) The size $\Delta n=\Delta I/\hbar$ in reduced action of the q = 2 to 7 subharmonic resonance island chains, as a function of scaled fixed microwave field F_0. Bottom: b) The ionization probability as a function of the microwave field F_0 at the maximum of the pulse.

The bottom part of Figure 5 shows pulsed-field numerical results for the ionization probability at $\omega_0'=0.9771$. Again ionization steps are seen. The vertical lines indicate the destabilization field points for the secondary chains. The agreement of these points with the field onsets of the ionization steps makes a strong connection between the bifurcation-induced island chains and the steps.

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