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N Two-level Atoms in a Two-mode Electromagnetic Cavity. J. E. BAYFIELD AND L.C . PEROTTI, University of Pittsburgh. --- The system of a large number N of two-level atoms interacting with a self-consistent nearly-resonant cavity mode classically can exhibit homoclinic chaos when a second, nonresonant cavity mode is externally driven [1]. Recent progress on the fully quantized problem indicates that near-classical evolution ceases after an experimentally accessible quantum breaktime proportional to ln N [2]. We report numerical results for a classical model that includes the dissipation and parameter averaging present in a real experiment. The single-orbit Lyapunov exponent remains positive and the early near-classical orbit ensemble evolution should be observable.

[1] D. D. Holm and G. Kovacic, Physica D 56, 270 (1992)
[2] G. P. Berman, E. N. Bulgakov and D. D. Holm, Los Alamos Report LA-UR-93-2187 (1993)

Prefer Poster Session

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N Two-level Atoms in a Two-mode Electromagnetic Cavity

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An ensemble of N two-level Rydberg atoms is placed in a microwave cavity resonant at frequency ω with the atoms' level separation. In addition, an externally applied microwave field is present in resonance with a second cavity mode. The variables for the classical spin model for the collective evolution of the atoms in
their common electromagnetic fields are the normalized mean value of the ensemble angular momentum $\vec{\sigma}(t) = \vec{J}(t) / J$ and the complex self-consistent electric field at frequency ω:
$E(t) + i E'(t)$ [1].
The parameters in the problem are

1) the collective frequency $\omega_c = [2\pi n d^2 \omega / h]^{1/2}$ where -dE is the atomic dipole-field interaction at frequency ω and n is the atom density;
2) the frequency detuning Δ of the externally applied field away from ω ; and
3) the frequency ω_R of atom Rabi-flopping induced by the applied field.

If there are cavity energy losses, then a fourth parameter is the cavity field decay time T_D .
If we take the unit of time to be the collective period $2\pi/\omega_c$, then the five coupled differential equations for the system's evolution in rotating wave approximation are

$$\sigma_z = -2[E\sigma_x + E'\sigma_y + (\omega_R/2\omega_c)\cos(\Delta t/\omega_c + \xi)\,\sigma_x + (\omega_R/2\omega_c)\sin(\Delta t/\omega_c + \xi)\,\sigma_y]$$

$$\sigma_x = 2[E + (\omega_R/2\omega_c)\cos(\Delta t/\omega_c + \xi)]\sigma_z$$

$$\sigma_x = 2[E' + (\omega_R/2\omega_c)\sin(\Delta t/\omega_c + \xi)]\sigma_z$$

$$E = \sigma_x/2 - E/(2\omega_c T_D)$$

$$E' = \sigma_y/2 - E'/(2\omega_c T_D)$$

where ξ is the external field phase at $t = 0$.
We are interested in the situation where $E(0) = E'(0) = 0$. The other initial conditions can be parametrized as

$$\sigma_z(0) = \cos\theta_0$$
$$\sigma_x(0) = \sin\theta_0 \cos\varphi_0$$
$$\sigma_y(0) = \sin\theta_0 \sin\varphi_0$$

We numerically integrate the differential equations using a forth order Runge-Kutta routine, and compute the atom mean upper state population $P_z(t) = [1 + \sigma_z(t)]/2$. The results will follow below.

Berman, Bulgakov and Holm (BBH) have considered this problem without the dissipation, both classically and quantum mechanically [2]. After changes in variables and the appropriate choice of surface of section, classically one can see the separatrix in the integrable Dicke-model limit $\omega_R = 0$, the weak homoclinic chaos when $\omega_R \approx \omega_c < \Delta$, and the strong chaos when $\omega_R \approx \omega_c \approx \Delta$:

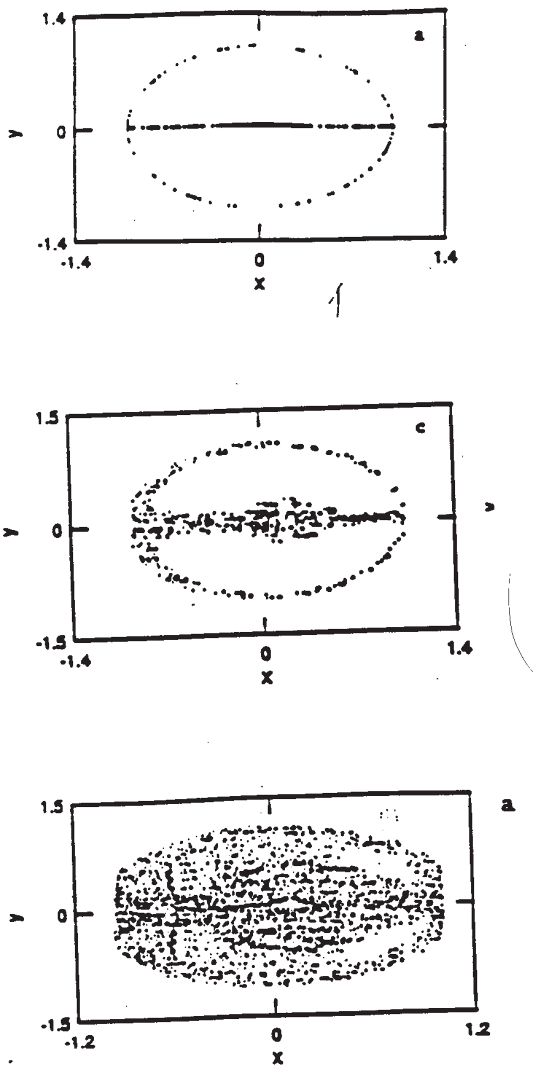

Figure 1. Surfaces of section for classical trajectories for the regular regime, $\omega_R = 0$ (top), weak chaos regime (middle) and strong chaos regime (bottom).

We are interested in the weak chaos, where θ_0 must be taken close to unity. There BBH find a difference between the quantal and classical evolutions that grows exponentially with time. They also find that the time τ for a 1% difference roughly varies as ln(N):

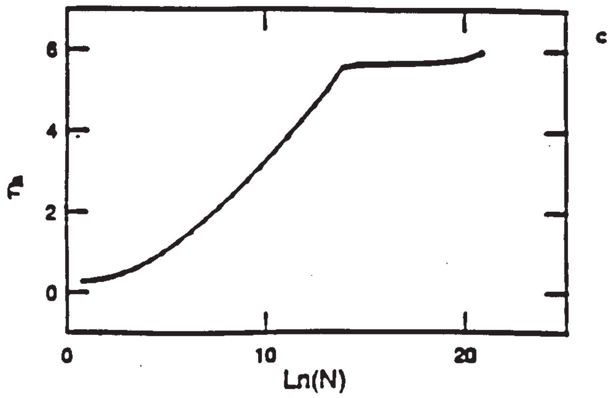

Figure 2. The predicted dependence of the quantum breaktime τ on ln(N).

Our intention is to experimentally observe the near-classical quantum evolution for $t < \tau$, the quantum evolution for $t > \tau$, and to compare these with classical numerical calculations. We hope to verify the ln(N) dependence.
An ensemble of Rydberg atoms would be used in a picosecond pulsed laser pump-probe experiment. As the experiments will not be perfect, the measured $P_z(t)$ will be averaged over distributions of the parameters, such as θ_0, φ_0, ξ, and ω_R. In the following we take estimates of achievable distributions and show that the averaging does not significantly blur the early near-classical orbit ensemble evolution that should be observed for $t < \tau$. We also show that while the dissipation reduces the single-orbit Lyapunov exponent, the latter does remain positive and the chaos persists.

In our averaging over θ_0 we took the distribution to have the form

$$P(\theta_0) = (2\,\theta_0 / \bar{\theta}_0^{2}) \exp(- q_0^{2} / \bar{\theta}_0^{2})$$

The averaging over φ_0 was uniform in [0, 2π), the averaging over ξ was uniform over [0, 2π) and the averaging over ω_R was uniform over a 6% range. The damping time constant T_D was taken to be 300 collective periods. The total number of orbits included was 7500. Figure 3 shows the orbit-ensemble evolution in the regular regime, with $\bar{\theta}_0$= 10(-6), ω_R/ω_c= 2x10(-4) and $\Delta/\omega_c = 4.5$:

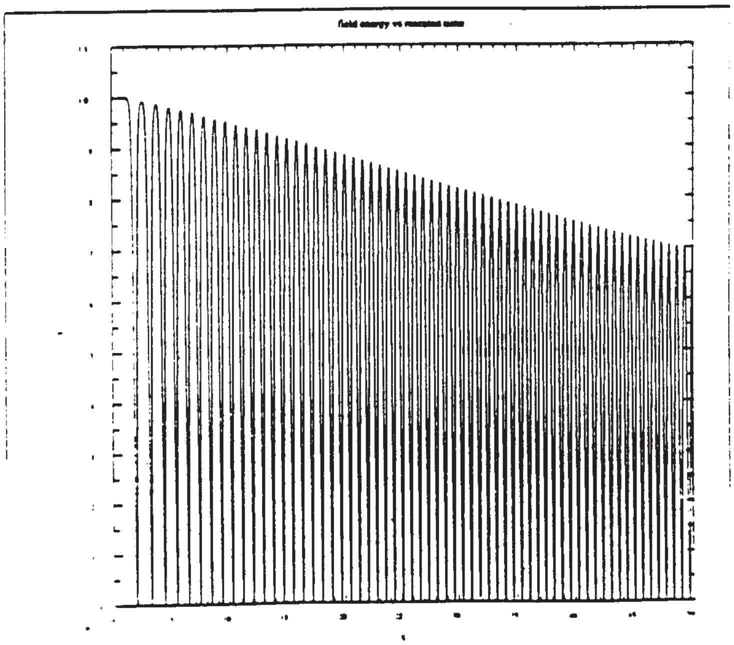

Figure 3. Classical evolution in the damped Dicke-model regime.

In Figure 4 we show the evolutions for $\bar{\theta}_0 = 2.6$x10(-3), $\omega_R / \omega_c = 0.6$, and Δ / ω_c having the three values 0.3, 1.0 and 4.5. The chaotic evolution is evident.

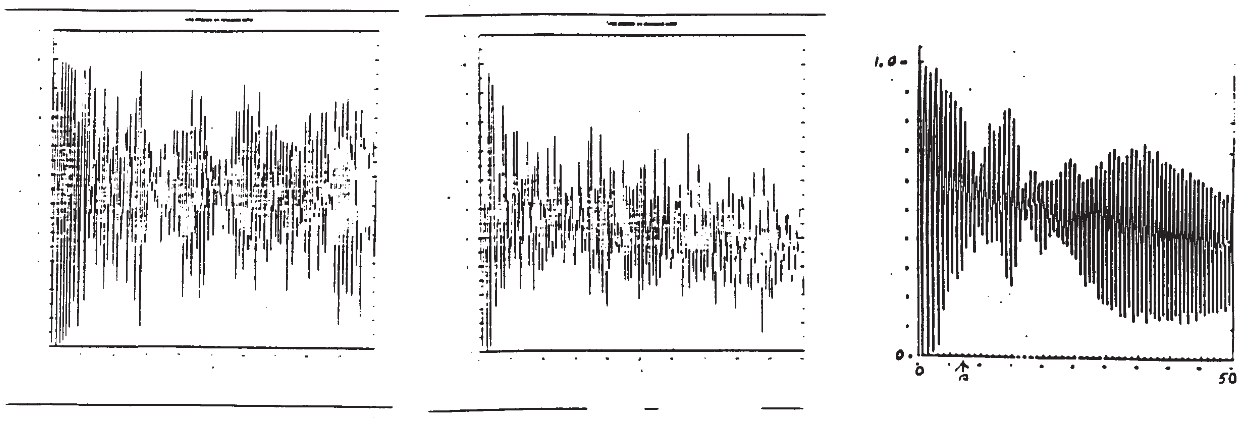

Figure 4. Evolution in the strong chaos (left) to weak chaos (right) regime. Collapses and revivals induced by field inhomogeneity dephasing and rephasing (the spread in ω_R) are seen in these evolutions.

The Lyapunov exponents characterize the stability of an individual trajectory in the system's phase space, at least one of them being positive when the trajectory is chaotic. They are defined as the long time limit of the Lyapunov functions. We have computed these using the approach of Wolfe et al [3]. An initial infinitesimal five-dimensional spherical volume in phase space becomes an evolving ellipsoid. One computes the long term evolution of its principal axes by simultaneous integration of the exact differential equations above and their linearized equations in the tangent space. This permits probing the local "stretch" while avoiding the effects of the global "fold" that together with stretch generates chaos in bound systems. To reach long times without numerical overflows, one carries out repeated Gram-Schmidt reorthonormalizations, which are axis orientation preserving.

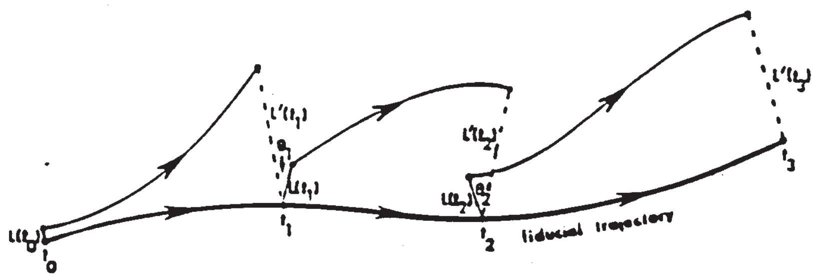

Figure 5. The procedure for computing Lyapunov exponents, sketched in one dimension.

Some results are shown in Figure 6 in three different regimes: $\Delta / \omega_c = 1.5$, $T_D = \infty$; $\Delta / \omega_c = 1.5$, $T_D = 300$ and $\Delta / \omega_c = 4.5$, $T_D = 300$; in all three cases $\theta_0 = 2.6x10(-3)$ and

$\omega_R / \omega_c = 0.6$

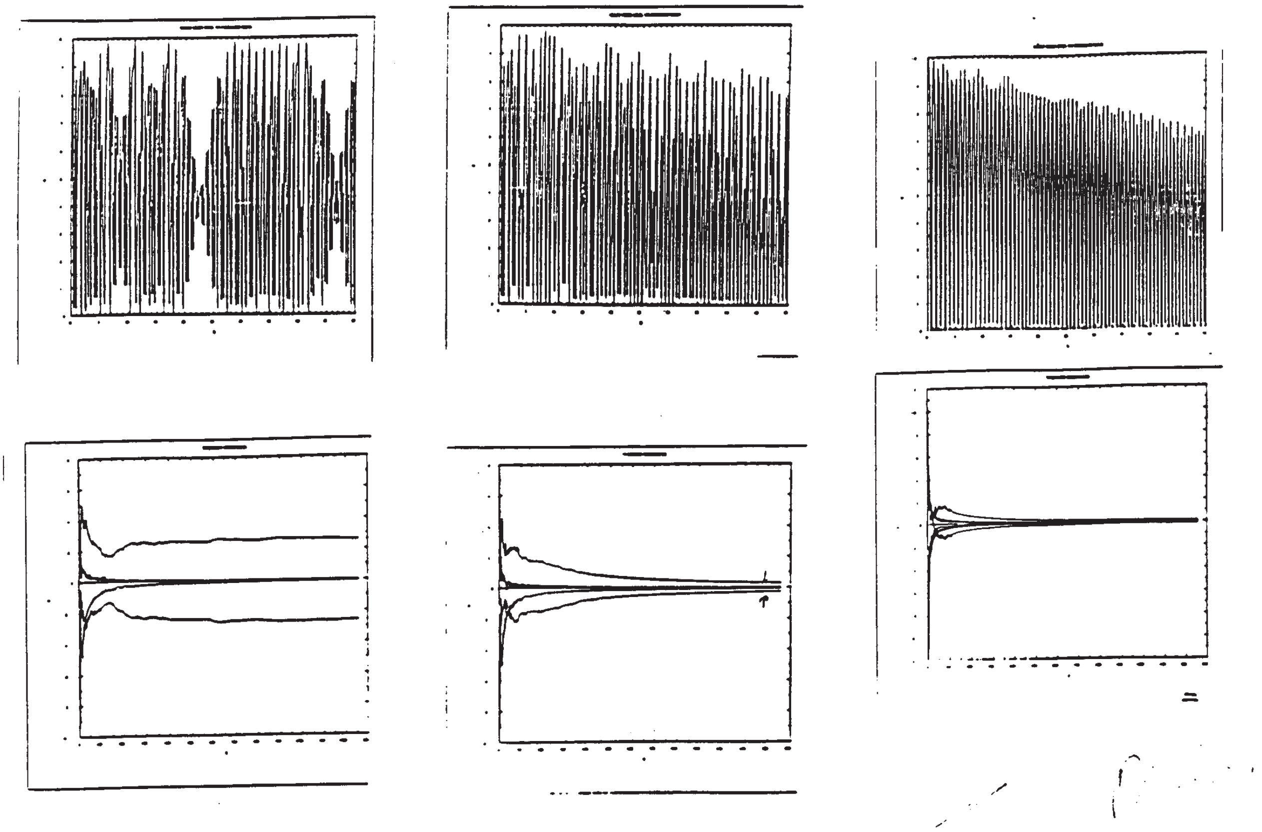

Figure 6. Single trajectory evolutions (top) and evolution of the corresponding Lyapunov characteristic functions (bottom) for strong chaos without damping (left), strong chaos with damping (middle) , and weak chaos with damping (right).

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[3] A.Wolfe, J. B. Swift, H. L. Swinney and J. A. Vastano, Physica D 16, 285 (1985)