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Summer school
May 21-25

Optimization techniques
for imaging
June 11-15

Numerical methods
for Astronomical Imaging
June 18-22

Reconstruction methods
for sparse-data tomography
June 25-29

Restoration of
Medical Imaging
July 9-13

Workshop
July 16-18

INDAM intensive period at University of Insubria DISAT

Via Valleggio 11 - Como, Italy

COMPUTATIONAL METHODS FOR INVERSE PROBLEMS IN IMAGING

MAY 21 - JULY 20, 2018

UNIVERSITÀ DEGLI STUDI DELL'INSUBRIA

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Program

	Monday 16th	Tuesday 17th	Wednesday 18th
9:15		Durou	Rodolà
10:00		Calatroni	Rebegoldi
10:30		Coffee Break	Coffee Break
11:00		Estatico	Falcone
11:45		Dell'Acqua	Tozza
12:15		Blanc-Feraud	Fornasier
13:00		Lunch	Closing Remarks
14:00	Opening Remarks		
14:15	Steidl	Rodriguez	
15:00	Bianchi	Franchini	
15:30	Coffee Break	Coffee Break	
16:00	Buccini	Huska	
16:30	Bonettini	Pragliola	
17:15	Concas		

Social Dinner
The social dinner will take place on Tuesday at 20.00 at the Restaurant "Funicolare" situated in Via Lungo Lario Trieste, 62. Please confirm your participation within 8th July by email to CMIPI2018@uninsubria.it.

Abstracts

Laplacian approximation on graphs for signal and image restoration

Marco Donatelli[a], Davide Bianchi[a]

[a]Università degli Studi dell'Insubria, Como, Italy

Abstract

In image restoration the noise is not negligible, usually producing discrete ill-posed problems that require regularization. Graphs, as generic data representation forms, are useful for describing the geometric structures of the underlying problem. In particular, in image processing there has been a recent spike in graph-based filtering methods that build non-local and semi-local graphs to connect the pixels of the image based not only on their physical proximity, but also on noisy versions of the image to be processed, see [7, 5, 6]. Such methods are often able to better recognize and account for image edges and textures. Getting into details, let $G = (V, E, W)$ be a weighted graph, where V is a finite set of vertices, E is a set of edges and W is a weighted adjacency matrix. A 2D image can be viewed like a graph, where the pixels are the vertex, the intensity of the colors a nonnegative function (signal) on the vertex and the edge weights play the role of diffusivity. In a noisy graph signal, where at the original signal acting on the vertex is added an uncorrelated additive Gaussian noise, in order to enforce some a priori informations on the signal, the norm of the penalty term in the Tikhonov regularization is usually weighted by L, where L is the combinatorial graph Laplacian associated to W and defined as $D - W$, with D the degree matrix of the graph.

Since L oversmooths the computed solution, wherever the true signal presents discontinuities, we are interested in investigating a new approach for preserving such discontinuities: first of all, we take into account the underlying geometry of the ambient space; secondly, we ask ourselves which weights and connectivity relationship between the points should be used. Indeed, we show that more the graph-Laplacian is spectrally close to the continuous Laplacian, and better the reconstructed solutions are.

A couple of meaningful numerical experiments are provided along with comparisons with other methods at the state of the art.

References

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Singular molecule localization by L_2–L_0 constrained optimization

Arne Bechensteen[a], Simone Rebegoldi[b], Laure Blanc-Féraud[a], Gilles Aubert[c]

[a]Université Côte d'Azur, CNRS, INRIA, Laboratoire I3S UMR 7271, Sophia Antipolis, France
[b]Università degli Studi di Ferrara, Ferrara, Italy
[c]Université Côte d'Azur, UNS, Laboratoire J. A. Dieudonné UMR 7351, Nice, France

Abstract

We focus on the minimization of the least square loss function under a k sparse constraint. As proposed in [1], we investigate an equivalent reformulation of the L_0 norm as an augmented optimization problem. The new global problem is biconvex with respect to the primal and auxiliary (augmented) variable, under a linear constraint which links the two variables. We describe an alternate optimization procedure and show results on singular molecule localization application for super-resolution in fluorescence microscopy. Comparisons with minimization of the least square loss function with a L_0 penalization term using the CEL0 approach [2] will be given. [1] G. Yuan, B. Ghanem "Sparsity constrained minimization via mathematical programming with equilibrium constraints.

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Inexact variable metric proximal gradient methods with line-search for convex and nonconvex optimization

Silvia Bonettini[a]

[a]Università degli Studi di Modena e Reggio Emilia, Modena, Italy

Abstract

In this talk we focus on variable metric forward-backward algorithms, where the metric underlying the backward/proximal step may change at each iteration to better capture the local features of the objective function and constraints. The main theoretical convergence properties will be described in the convex and nonconvex case and practical implementation issues will be discussed.

$\ell^p - \ell^q$ minimization methods for image restoration

Alessandro Buccini[a], Lothar Reichel[a]

[a]Kent State University, Kent, Ohio (USA)

Abstract

Discrete ill-posed problems arise in many areas of science and engineering. Their solutions, if they exist, are very sensitive to perturbations in the data. Regularization aims to reduce this sensitivity. Many regularization methods replace the original problem with a minimization problem with a fidelity term and a regularization term. Recently, the use of a p-norm to measure the fidelity term and a q-norm to measure the regularization term has received considerable attention, see, e.g., [3, 4, 5] and references therein. The relative importance of these terms is determined by a regularization parameter. When the perturbation in the available data is made up of impulse noise it is often beneficial to let $0 < p < 1$, while, if the perturbation is made up of white Gaussian noise, $p = 2$ is a more appropriate choice. Moreover, the choice $q < 1$ is very effective in enforcing sparsity, which is often desired, on the reconstructed solution. When either $p < 1$ or $q < 1$ the minimized functional is in general non-convex. For the minimization of such non-convex functional we resort to the algorithm proposed in [5].

The choice of a suitable regularization parameter is crucial for the quality of the computed solution. In this talk we discuss approaches for determining the regularization parameter automatically, without user-interaction. Computed examples that illustrate the performance of these approaches.

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An adaptive backtracking strategy for non-smooth composite optimisation problems

Luca Calatroni[a], Antonin Chambolle[a]

[a]FMJH & CMAP, Ecole Polytechnique, Palaiseau Cedex, France

Abstract

In this talk we present a backtracking strategy for a variant of the Beck and Teboulle's Fast Iterative Shrinkage/Thresholding Algorithm (FISTA) which has been recently proposed by Chambolle and Pock (2016) for strongly convex objective functions. Differently from standard Armijo-type line searching, our backtracking rule allows for local increase and decrease of the Lipschitz constant estimate along the iterations, i.e. decrease/increase of the gradient step size. For such adaptive strategy we prove accelerated convergence rates showing in particular linear convergence in smooth cases. We validate the resulting algorithm on some exemplar image denoising problems where strong convexity appears typically after smoothing of the regularisation term.

Photometric Stereo under unknown lights position

Anna Concas[a], Riccardo Dessì[a], Caterina Fenu[a], Giuseppe Rodriguez[a], Massimo Vanzi[a]

[a]Università degli Studi di Cagliari, Cagliari, Italy

Abstract

A classical problem in Computer Vision consists in reconstructing the 3D shape of an object, starting from a set of images of the same surface. Photometric Stereo technique is used to extract shape and color information from an object which is observed from the same fixed point of view but under different lighting conditions.

We will describe an algorithm to approximate the framed object, treating, in particular, the case when the position of the light sources are unknown. Numerical experiments will be illustrated.

References

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Non-stationary structure-preserving preconditioning for image restoration

Pietro Dell'Acqua[a]

[a]Università degli Studi dell'Aquila, L'Aquila, Italy

Abstract

Non-stationary regularizing preconditioners have recently been proposed for the acceleration of classical iterative methods for discrete ill-posed problems. This paper explores how these preconditioners can be combined with the flexible GMRES iterative method. A new structure-respecting strategy to construct a sequence of regularizing preconditioners is proposed. We show that flexible GMRES applied with these preconditioners is able to restore images that have been contaminated by strongly non-symmetric blur, while several other iterative methods fail to do this.

Photometric 3D-reconstruction

Jean-Denis Durou [a]

[a]Institut de Recherche en Informatique de Toulouse, Toulouse, France

Abstract

Photometric 3D-reconstruction techniques aim at inferring 3D clues from 2D measurements, by analyzing luminous quantities in images. This is achieved by inverting a physics-based image formation model which describes the interactions between 3D-shape, surface reflectance, lighting and camera. In this talk, I will first describe the main characteristics of the shape-from-shading (SFS) problem, which uses a single view and is ill-posed. I will then describe some recent advances regarding modeling and resolution of two well-posed extensions, which use several views of a 3D-scene, taken either under varying lighting (photometric stereo) or from different points of view (multi-view SFS).

Variable exponent Lebesgue spaces for adaptive regularization

Claudio Estatico [a]

[a]Università degli Studi di Genova, Genova, Italy

Abstract

Within the framework of regularization in Banach spaces, we propose an iterative algorithm developed in the variable exponent Lebesgue spaces $L^{p}(.)$. The method is able to automatically set up different levels of filtering in different regions of the domain. Basically, modelling in $L^{p}(.)$ spaces allows setting pointwise regularization parameters, associated to different values of the function parameter $p(.)$. This is useful in image deblurring problems, where background, low intensity, and high intensity values of the image to restore often require different regularization levels. Some numerical evidences will be shown.

Adaptive Filtered Schemes for first order Hamilton-Jacobi equations and applications

Maurizio Falcone[a], Giulio Paolucci[a], Silvia Tozza[ab]

[a]Università La Sapienza, Roma, Italy
[b]Istituto Nazionale di Alta Matematica (INdAM)

Abstract

The accurate numerical solution of Hamilton-Jacobi equations is a challenging topic of growing importance in many fields of application including image processing. For example, these equations appear in the segmentation problem via the level set method and in 3D reconstruction based on Shape-from-Shading. Due to the lack of regularity of viscosity solutions the construction of high-order methods can be rather difficult. We consider a class of "filtered" schemes for first order evolutive Hamilton- Jacobi equations. These schemes are based on a mixture of a high-order (possibly unstable) scheme and a monotone scheme, according to a filter function F and a coupling parameter ε. This construction allows to have a scheme which is high- order accurate where the solution is smooth and monotone otherwise. This feature is crucial to prove that the scheme converges to the unique viscosity solutions. In this talk we present an improvement of the classical filtered scheme, introducing an adaptive and automatic choice of the parameter ε at every iteration. To this end, we use a smoothness indicator in order to select the regions where we can compute the regularity threshold epsilon. Our smoothness indicator is based on some ideas developed for the construction of the WENO schemes, but other indicators with similar properties can be used. We present a convergence result and error estimates for the new scheme, the proofs are based on the properties of the scheme and on the properties of the indicators. Some numerical tests will show the features of this adaptive method and some applications.

Rotation Invariance in Exemplar-based Image Inpainting

Martin Eller[a], Massimo Fornasier[a]

[a]Technische Universität München, Munich, Germany

Abstract

Due to their good performance on textured images, exemplar-based methods for image recovery have been subject of research in recent years. In this talk, the variational framework of exemplar-based inpainting is reviewed and enriched by rotation invariance as an additional degree of freedom in patch searching. For an efficient rotation invariant comparison of image patches we present methods based on sampled Circular Harmonics (CH) expansions, in particular, we also elaborate a method for gradient-based comparisons thanks to the property of CH of being eigenfunctions of the Laplacian. These new pattern matching methods allow for an extremely efficient implementation of the alternating optimization scheme of exemplar-based inpainting, also when rotation invariant patch matching is considered. The patch non-local means algorithm and its performance in the recovery of image structures and textures are described in detail and we demonstrate by numerical examples the significant improvement in recovering smooth edges, which is due to the additional rotation invariance.

On the steplength selection in Stochastic Gradient Methods

Giorgia Franchini[a]

[a]Università degli Studi di Modena e Reggio Emilia, Modena, Italy

Abstract

Many machine learning methodologies based on the minimization of the empirical risk lead to optimization problems in which the objective function is the sum of loss functions depending on the samples of a finite training set. These optimization problems are challenging in the case of large scale training sets because the computation of the objective function and its gradient is too expensive. In these cases, Stochastic Gradient Descent (SGD) methods [1] are the main approaches for solving this kind of optimization problem. Many SGD variants are available in literature [2], based on different strategies for reducing the adverse effect of noisy gradient estimates and for defining the steplength (learning rate) parameter. In this work, starting from recent advances on state of the art steplength rules for deterministic gradient schemes and noise reduction strategies for SGD methods [3], we investigate possible techniques for selecting the learning rate parameter in SGD approaches. Preliminary studies on the behavior of popular steplength selections, such as the Barzilai-Borwein rules, in the stochastic gradient framework have shown that many interesting questions need to be fixed before obtaining effective benfits [4, 5]. We discuss some of these open problems on the learning rate selection within widely used stochastic optimizers, exploiting momentum terms and adaptive variance techniques.

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Convex Non-Convex Segmentation of Scalar Fields over Arbitrary Triangulated Surfaces

Martin Huska[a]

[a]Università di Bologna – Alma Mater Studiorium A.D. 1088 , Bologna, Italy

Abstract

An extension of the Mumford-Shah model for image segmentation is introduced to segment real-valued functions having values on a complete, connected, 2-manifold embedded in $\mathbb{R}^3$. The proposed approach consists of three stages: first, a multi-phase piecewise smooth partition function is computed, then its values are clustered and, finally, the curve tracking is computed on the segmented boundaries. The first stage, which constitutes the key novelty behind our proposal, relies on a Convex Non-Convex variational model where an ad-hoc non-convex regularization term coupled with a space-variant regularization parameter allows to effectively deal with both the boundaries and the inner parts of the segments. The cost functional is minimized by means of an efficient numerical scheme based on the Alternating Directions Methods of Multipliers.

Space-variant generalized Gaussian regularization for image restoration

Monica Pragliola[a]

[a]Università di Bologna – Alma Mater Studiorium A.D. 1088 , Bologna, Italy

Abstract

We propose a new space-variant regularization term for variational image restoration based on the assumption that the gradient magnitudes of the target image distribute locally according to a half-Generalized Gaussian distribution. This leads to a highly flexible regularizer characterized by two per-pixel free parameters, which are automatically estimated from the observed image. The proposed regularizer is coupled with either the L_2 or the L_1 fidelity terms, in order to effectively deal with additive white Gaussian noise or impulsive noises such as, e.g, additive white Laplacian and salt and pepper noise. The restored image is efficiently computed by means of an iterative numerical algorithm based on the alternating direction method of multipliers. Numerical examples indicate that the proposed regularizer holds the potential for achieving high quality restorations for a wide range of target images characterized by different gradient distributions and for the different types of noise considered.

An alternating variable metric inexact linesearch based algorithm for nonconvex nonsmooth optimization

Simone Rebegoldi[a], Silvia Bonettini[b], Marco Prato[b]

[a]Università degli Studi di Ferrara, Ferrara, Italy
[b]Università degli Studi di Modena e Reggio Emilia Modena, Modena, Italy

Abstract

In this talk we propose a novel block coordinate proximal gradient method for minimizing the sum of a smooth (possibly nonconvex) term, plus a separable part given by the sum of two or more convex (possibly nonsmooth) functions, each depending on a single block of variables [3]. Our proposed approach cyclically performs a bounded number of steps of the Variable Metric Inexact Linesearch based Algorithm (VMILA) [2] on each block of variables, ensuring a suffcient decrease condition at each inner iteration, and then forces a further descent condition at each outer iteration. Unlike other existing methods in the literature [1, 4, 5], the parameters defining the variable metric in the proximal operator may be computed according to any desired adaptive rule, provided that they belong to compact sets. We show that each limit point of the iterates sequence is stationary and we prove convergence to the limit point when the objective function satisfies the Kurdyka–Łojasiewicz inequality, the gradient of the smooth part is locally Lipschitz continuous and the proximal operator is computed exactly. We provide some practical rules to adaptively select the parameters and we report a numerical experience on an image blind deconvolution problem, showing the benefits obtained by combining the variable metric of the proximal operator with a variable number of the inner iterations.

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Geometric Deep Learning

Emanuele Rodolà[a]

[a]Università La Sapienza, Rome, Italy

Abstract

The past decade in computer vision research has witnessed the re-emergence of "deep learning", and in particular convolutional neural network (CNN) techniques, allowing to learn powerful image feature representations from large collections of examples. CNNs achieve a breakthrough in performance in a wide range of applications such as image classification, segmentation, detection and annotation. Nevertheless, when attempting to apply the CNN paradigm to 3D shapes (feature-based description, similarity, correspondence, retrieval, etc.) one has to face fundamental differences between images and geometric objects. Shape analysis and geometry processing pose new challenges that are non-existent in image analysis, and deep learning methods have only recently started penetrating into our community. The purpose of this talk is to overview the foundations and the current state of the art on learning techniques for 3D shape analysis. Special focus will be put on deep learning techniques (CNN) applied to Euclidean and non-Euclidean manifolds for tasks of shape classification, retrieval and correspondence. The tutorial will present in a new light the problems of shape analysis and geometry processing, emphasizing the analogies and differences with the classical 2D setting, and showing how to adapt popular learning schemes in order to deal with 3D shapes.

New and not so new methods for estimating a regularization parameter

Giuseppe Rodriguez[a]

[a]Università degli Studi di Cagliari, Cagliari, Italy

Abstract

The definition of a regularization method involves the adoption of a parameter estimation strategy that ensures the convergence of the method. There are various The discrepancy principle, as well as the other a-posteriori parameter estimation methods, requires strong assumption about the noise, and in particular the knowledge of its standard deviation. When such information is unavailable, the noise level can be estimated from the data by heuristic techniques. We will review some classical and some recent parameter choice rules, and describe in detail a new one.

Morphing of Manifold-Valued Images and Regularization of Inverse Problems using Time Discrete Geodesics in Image Spaces

Gabriele Steidl[a], Sebastian Neumayer[a], Johannes Persch[a]

[a]Technische Universität Kaiserslautern, Kaiserslautern, Germany

Abstract

Smooth image transition, also known as image morphing, is a frequently addressed task in image processing and computer vision, and there are various approaches to tackle the problem. For example, in feature based morphing only specific features are mapped to each other and the whole deformation is then calculated by interpolation. This talk is related to a special kind of image morphing, the so-called metamorphosis introduced by Miller, Trouvé and Younes.The metamorphosis model can be considered as an extension of the flow of diffeomorphism model and its large deformation diffeomorphic metric mapping framework in which each image pixel is transported along a trajectory determined by a diffeomorphism path. As an extension the metamorphosis model allows the variation of image intensities along trajectories of the pixels. This talk builds up on a time discrete geodesic paths model by Berkels, Effland and Rumpf. We enlarge the model twofold:

i) for morphing manifold-valued images,

ii) for regularizing inverse problems.

We prove the existence of minimizers for the space continuous models and demonstrate the very good performance of the corresponding algorithms.

References

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Accurate image segmentation via a high-order scheme for level-set equations

Maurizio Falcone[a], Giulio Paolucci[a], Silvia Tozza[ab]

[a]Università La Sapienza, Roma, Italy
[b]Istituto Nazionale di Alta Matematica (INdAM)

Abstract

A classical problem in image processing is to detect the boundaries of the object(s) represented inside an image. There are several different ways to solve this image segmentation problem that appears in many application fields, e.g. biomedicine, security, astronomy.
In this talk we follow the level set approach [3], which consists in solving the following:

$$\begin{cases} u_t(t,x) + c(x)|\nabla u(t,x)| = 0, & (t,x) \in [0,T] \times \mathbb{R}^2, \\ u(0,x) = u_0(x), & x \in \mathbb{R}^2, \end{cases} \tag{1}$$

where c is a prescribed velocity, u_0 must be a representation function for the initial front $\partial\Omega_0$, with Ω_0 a given open bounded subset of $\mathbb{R}^2$. The velocity c is chosen in order to guarantee that the front stops at the boundaries of the object, so typically it depends on the value of the image $I(x,y)$ and in particular on the variations of I since the boundaries are perceived as the curve where I has jumps. We discretize that evolutive problem using a new adaptive "filtered" scheme for Hamilton-Jacobi equations recently proposed [2]. The major feature of this scheme is to mix a high-order scheme (a priori unstable) and a monotone scheme according to a filter function [1]. Moreover, we propose a modification of the velocity function c in order to improve the accuracy of the results and to stabilize the segmentation process. Numerical experiments on synthetic and real images will show the validity of the proposed filtered scheme and of the new choice of the velocity.

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