Solving Differential Equations on Graphs

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We study the numerical solution of boundary and initial value problems for differential equations posed on graphs or networks. The graphs of interest are *quantum graphs*, i.e., metric graphs endowed with a differential operator acting on functions defined on the graph's edges with suitable vertex conditions. We describe and analyze the use of linear finite elements to discretize the spatial derivatives for a class of linear elliptic model problems. The solution of the discrete equations is discussed in the context of a (non-overlapping) domain decomposition approach. For model elliptic problems and a wide class of graphs, we show that a combination of Schur complement reduction and diagonally preconditioned conjugate gradients results in optimal complexity. For problems of parabolic type, we consider the use of exponential integrators based on Krylov subspace methods. Numerical results are given for both simple and complex graph topologies.

References

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