Hidden structures of stochastic numerical methods

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Modern Numerical Analysis is not only devoted to accurately approximating the solutions of various problems through efficient and robust schemes, but also to retaining qualitative properties of the continuous problem over long times. Sometimes such conservation properties naturally characterize the numerical schemes, while in more complex situations preservation issues have to be conveyed into the numerical approximations.

In this talk we aim to analyze *hidden* conservation properties that can be translated, as a matter of fact, into conditional stability properties of numerical schemes. Special emphasis is given to selected stochastic differential equations (SDEs) that exhibit a-priori known qualitative behaviours.

First of all, we analyze the numerical approximation of nonlinear SDEs of Itô type with exponential mean-square contractive solutions, i.e. given any two solutions $X(t)$ and $Y(t)$ of SDE with $\mathbb{E}|X_0|^2 < \infty$ and $\mathbb{E}|Y_0|^2 < \infty$, the following inequality holds true

$$\mathbb{E}|X(t) - Y(t)|^2 \leq \mathbb{E}|X_0 - Y_0|^2 e^{\alpha t},$$

with $\alpha < 0$. We aim to understand if such an inequality still holds true when the solutions are computed by stochastic linear multistep methods, in order to provide stepsize restrictions ensuring analogous exponential mean-square properties also numerically, without adding further constraints on the numerical method itself [1, 5].

We next consider second order stochastic differential equations modeling damped stochastic oscillators, i.e. the motion of a particle subject to the deterministic forcing and a random forcing, providing damped oscillations along the dynamics. We aim to analyze long-term properties of two-step stochastic methods, in order to study their long-term preservation of invariance laws [2, 3].

We finally consider stochastic Hamiltonian problems of Itô type, for which an energy drift is visible along the exact dynamics (differently from the deterministic case, for which the energy is preserved). We aim to study the behaviour of stochastic Runge-Kutta methods in order to understand their natural ability in retaining the same energy behaviour also along the numerical solutions [4].

References

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