

# Fast and stable iterative regularization methods for image deblurring

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Current image deconvolution problems involve images of large dimensions. To handle the related computational issues, filtering techniques based on the fast Fourier transform are widely used, although they do not always provide an accurate image restoration [1]. When considering the classical approach based on least-squares, structured linear algebra tools allow to define reliable and efficient numerical methods, like the nonstationary preconditioned iteration in [2].

On the other hand, nonlinear models able to preserve image edges and possible constraints on the solution, like nonnegativity and sparsity of the solution, are normally computationally expensive. Recently, combinations of structured numerical linear algebra tools and optimization techniques have been proposed in order to simultaneously exploit the advantages of both the considered strategies. In particular, we consider image deblurring problems with a sparsity constraint on the wavelet coefficients of the image using a preconditioned linearized Bregman algorithm [3], which combines soft-thresholding with the iteration in [2]. If further informations on the operator are available, then the preconditioner can be defined by preserving the structure of the underlying coefficient matrix in [4]. Moreover, a fast and stable iterative method can be obtained by combining the soft-thresholding and the preconditioned iteration in a multilevel framework, that preserves also the non-negativity of the solution without requiring any a priori parameter estimation [5].

We prove that all the proposed iterations are regularizing and convergent methods. A large number of numerical experiments show that our methods provide accurate and fast restorations, when compared with the state of the art.

The aim of the talk is to show that specific information on the problem can be used for defining preconditioners and multilevel strategies in order to improve the convergence of iterative methods for nonlinear functionals as well.

## References

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