Multiple Nested Structures: the Curse (or Blessing?) of Applied Mathematics

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Research in applied mathematics is often characterized by the need of finding a nontrivial trade-off between model accuracy and solvability. Solving mathematical models for the sizes required by actual applications requires careful exploitation of their *structure*. Extensive amounts of research and ingenuity are devoted to developing sophisticated specialized approaches capable of (as much as possible) efficiently solving models with a given structure. Due to the amount of research effort required to tackle any individual kind of structure, it is customary for applied mathematic researchers to focus to only one of these at a time. However, real-world applications typically entail *several different* forms of structure, often times nested and/or combined into complex and irregular patterns. Arguably, the currently available modelling and solving tools do not offer adequate support for fully exploiting this "rich" structure. Indeed, they either (in particular on the solution side) focus on one single—or a very small set—of very specific structures, or (in particular on the modelling side) focus on expressing a very large class of models, but then offer no other option than resorting to correspondingly very-general-purpose solvers, which provide little avenues for exploiting specialized solution methods for the specific sub-structures of the model at hand. Using some real-world applications of mathematical optimization as motivation, we discuss an on-going attempt at developing a flexible and extendable modelling system providing explicit support for the notion of multiple nested forms of structure, for the possibility to adding specialized solvers to specific parts of the model, and for the concept of *reformulation* that is oftentimes crucial in order to exploit all valuable forms of structure, but that (to the best of our knowledge) so far no modelling system has ever explicitly addressed. Since this system is still in the initial phases of development, we hope to gather feedback on what other features may be required. Although the system hopefully provides better support than currently available ones for the kind of approaches that are capable of exploiting multiple nested structures, it still requires considerable expertise and ingenuity for properly choosing, and implementing, the most appropriate formulation of the model at hand. We hope that the proposed system will provide the foundation for a more ambitious one that may help in automating these last steps as well, but such a system would require several other breakthroughs to be realised.