Hierarchical splines and efficient quasi-interpolation

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Quasi-interpolation is a well-known technique for constructing accurate approximants to given data or functions by means of a local approach. Usually it consists in a linear combination of a given system of blending functions with suitable locally constructed coefficients.

Hierarchical B-splines have been introduced by [1] as an accumulation of tensor-product B-splines with nested knot vectors. They are designed to do local refinement, which is not possible in the case of classical tensor-product splines. Hierarchical spline spaces are of interest in different areas ranging from approximation theory to geometric modeling and isogeometric analysis, a recent paradigm for the numerical treatment of partial differential equations.

In this talk we discuss an alternative basis for the hierarchical spline space with an enhanced set of properties compared to the classical hierarchical B-spline basis. The so-called truncated hierarchical basis forms a nonnegative partition of unity, and is strongly stable with respect to the number of refinement levels [2, 3]. Then, we introduce a general framework for constructing quasi-interpolants in hierarchical spline spaces expressed in terms of the truncated hierarchical basis [4]. Quasi-interpolants of full approximation order and/or projections can be easily obtained. The main ingredient of the construction is the property of preservation of coefficients of the truncated hierarchical spline representation. Finally, we present local error estimations of such hierarchical quasi-interpolants and their derivatives in general L_q-norm [5].

References

[1] D.R. Forsey, R.H. Bartels. *Hierarchical B-spline refinement.* ACM Siggraph Computer Graphics, 22 (1988) 205–212.

[2] C. Giannelli, B. Jüttler, H. Speleers. *THB-splines: The truncated basis for hierarchical splines*, Computer Aided Geometric Design, 29 (2012) 485–498.

[3] C. Giannelli, B. Jüttler, H. Speleers. *Strongly stable bases for adaptively refined multilevel spline spaces*, Advances in Computational Mathematics, 40 (2014) 459–490.

[4] H. Speleers, C. Manni. *Effortless quasi-interpolation in hierarchical spaces*, Numerische Mathematik, 132 (2016) 155–184.

[5] H. Speleers. *Hierarchical spline spaces: Quasi-interpolants and local approximation estimates*, Advances in Computational Mathematics, 43 (2017) 235–255.