Spectral properties in gradient-based optimization methods: a review and perspective

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It is well-known that the practical performance of gradient-based methods for both unconstrained and constrained optimization has been dramatically increased by using suitable adaptive steplength selection rules [1, 2, 7]. Many of these rules are aimed at capturing some low-cost second-order information, as emphasized by recent studies on the crucial role of the relationship between the steplengths and the spectrum of the Hessian of the objective function [3, 5, 6]. In this talk, starting from a review of the spectral properties exhibited by state of the art steplength rules in case of unconstrained optimization problems [4], we discuss how these properties generalize to constrained problems. In case of box-constrained quadratic programs, the proposed analysis gives rise to new steplength updating rules that provide improved versions of widely used gradient projection approaches. The numerical behaviour of the new methods is evaluated on both randomly generated test problems and constrained optimization problems arising in large-scale applications.

References

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