Spectral analysis and preconditioners for 2D Riesz distributed-order space-fractional diffusion equations on convex domains

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When dealing with the discretization of differential equations on non-rectangular domains, a careful treatment of the boundary is mandatory and may result in implementation difficulties and in coefficient matrices without a prescribed structure. Here we examine the numerical solution of a two-dimensional constant coefficient distributed-order space-fractional diffusion equation on a convex domain. To avoid the aforementioned inconvenience, we resort to the volume-penalization method used in [2]. This technique consists in embedding the domain into a rectangle and in adding a penalization term to the original equation that annihilates the regularized solution in the region outside the original domain.

Thanks to the volume-penalization, methods designed for problems in rectangular domains are available for those in convex domains and by applying an implicit finite difference scheme we obtain coefficient matrices with a 2-level Toeplitz structure plus a diagonal matrix which arises from the penalty term. Then, by using the Generalized Locally Toeplitz tools described in [1] and by building on the information already available in [3] for the one-dimensional case, we compute the corresponding symbol and describe the asymptotic eigenvalue distribution as the matrix size diverges. The spectral analysis is performed under different assumptions, with the aim of estimating the intrinsic asymptotic ill-conditioning of the involved matrices and of proposing ad hoc fast preconditioners for Krylov procedures. Numerical experiments with circulant and τ-preconditioners are conducted and critically discussed.

This is a joint work with M. Mazza and S. Serra Capizzano (Università degli Studi dell'Insubria).

References

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