Nonbacktracking Walk Centrality for Directed Networks

Francesca Arrigo
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We derive an analytic expression for the generating function of nonbactracking walks counts on a directed network. This generalizes previous results that apply to the undirected case. We show how the radius of convergence of the generating function is determined by the spectrum of a three-by-three block matrix involving the original adjacency matrix. The analysis leads naturally to a centrality measure that has the same computational complexity as the standard walk-based Katz version. By eliminating traversals around the network that are unlikely to be of relevance, we arrive at a larger range of choices for the attenuation parameter. The new centrality measure may also be interpreted as standard Katz on a modified network, where negative self-loops are added, and where nonreciprocal edges are augmented with negatively weights. We also show that the radius of convergence of the generating function is invariant to the removal of leaf nodes, which can lead to computational efficiencies. We illustrate the centrality measure on real networks.

Non-stationary regularizing preconditioners for ill-posed problems

Alessandro Buccini
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Many applications in science and engineering lead to *discrete ill-posed inverse problems*

$$A\mathbf{x} + \boldsymbol{\eta} = \mathbf{b}^{\delta}, \tag{1}$$

where $\mathbf{x} \in \mathbb{R}^n$, $\boldsymbol{\eta}, \mathbf{b}^{\delta} \in \mathbb{R}^m$, and $A \in \mathbb{R}^{m\times n}$. A is severely ill-conditioned, i.e., its singular values rapidly decreases to zero with no significant gap, and $\boldsymbol{\eta}$ represents the inevitable noise present on the data. We assume that a bound on the noise $\|\boldsymbol{\eta}\| \leq \delta$ is known.

The combination of the ill-conditioning of the operator and the presence of noise implies that it is meaningless to solve (1) directly and that we need to resort to regularization methods.
One of the most popular regularization method is Tikhonov regularization, whose *general form* is

$$\mathbf{x}_{\alpha} = \arg\min_{\mathbf{x}\in\mathbb{R}^n} \left\| A\mathbf{x} - \mathbf{b}^{\delta} \right\|^2 + \alpha \left\| L\mathbf{x} \right\|^2, \tag{2}$$

where $\alpha > 0$ is the *regularization parameter* and $L \in \mathbb{R}^{p\times n}$ is the *regularization operator* such that $\mathcal{N}(L)\cap\mathcal{N}(A) = \{\mathbf{0}\}$, where with $\mathcal{N}(\cdot)$ we denote the null space of $\cdot$. Tikhonov regularization is said to be in *standard form* when $L = I_n$. The parameter α balances the trade-off between the two terms, while the operator L weights the norm in the penalty term.

To improve the quality of the reconstructions obtained with (2) the Iterated Tikhonov (IT) method was developed. The IT algorithm is obtained by solving the error equation in a refinement technique using Tikhonov in standard form

$$\mathbf{x}_{k+1} = \mathbf{x}_k + (A^tA + \alpha I_n)^{-1}A^t(\mathbf{b}^{\delta} - A\mathbf{x}_k),$$

where A^t denotes the transpose of A. The role of α is crucial in this case also, and an imprudent choice can provide poor reconstructions. A fast convergent method is obtained when a non-stationary succession is chosen, i.e., when α changes at each iteration.

Starting from the IT method, in [4] an algorithm for inverse problems, that is able to achieve very high precision while keeping under control the computational effort, was developed. Let C be an approximation of A. The proposed method is obtained by replacing A with C in the error equation. If a clever choice of C is performed, we can lower the computational effort by using its properties for speeding up the computations. In order to construct a stable and robust method a non-stationary choice of the regularization parameter that exploits the knowledge of δ was used, thus obtaining the following iteration

$$\mathbf{x}_{k+1} = \mathbf{x}_k + C^t(CC^t + \alpha_k I_n)^{-1}(\mathbf{b}^{\delta} - A\mathbf{x}_k).$$

We will refer to this algorithm as *Approximated Iterated Tikhonov* (AIT), since the error equation is solved using an approximated version of the IT method.

In this talk we propose three extensions of the previously described AIT method that were proposed in [1].

In the first one we generalize the AIT method to the case in which Tikhonov in general form is used for the solution of the error equation, we refer to this method as *Approximated Iterated Tikhonov with General Penalty term* (AIT-GP). The second extension is constructed by projecting in the set Ω at each step the iterations of the AIT method. We call this method *Approximated Projected Iterated Tikhonov* (APIT). The third extension, called *Approximated Projected Iterated Tikhonov with General Penalty term* (APIT-GP), combines both the usage of the regularization operator L with the projection into Ω.

All the proposed algorithms inherit the good properties of the original method AIT; in particular, they do not need the estimation of any parameter and, they have a very low computational cost, if C is appropriately selected.

We show the performances of the proposed methods on image deblurring problems. We show that not only the proposed extensions are able to outperform the original method, but also are competitive with state of the art methods.

In recent years multigrid has been successfully used for the solution of ill-posed inverse problems (see, e.g., [5]). In [3] a multigrid method with regularization properties was developed; it combined framelet soft-thresholding with the CGLS algorithm for image deblurring. Similarly, we construct a multigrid method that combines APIT with framelet soft-thresholding. This combination let us develop an efficient and highly performing algorithm that is able to produce high quality restorations with a reasonable computational effort. We rely on the knowledge of δ for the estimation of all the parameters involved, thus we do not need to tune any constant obtaining a method that is both stable and robust [2].

This is a joint work with Marco Donatelli.

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Convergenza numerica e proprietà del metodo Filtraggio Iterativo per lo studio di segnali nonlineari e nonstazionari

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La scomposizione ed analisi di segnali nonstazionari e nonlineari è di grande interesse sia teorico che applicativo.

Tra le possibili applicazioni si annoverano, a titolo esemplificativo, l'individuazione non distruttiva di difetti in strutture di ingegneria civile o meccanica, quali ponti, edifici e meccanismi in generale, l'identificazione di quasiperiodicità celate ed andamenti di lungo periodo in serie storiche che possono andare dalla temperatura media della troposfera terrestre ad indici del mercato finanziario passando per misure del campo magnetico del vento solare. Tecniche comunemente usate nello studio dei segnali, quali la trasformata di Fourier o quella wavelet, non sono in grado di cogliere in modo opportuno fenomeni nonlineari e nonstazionari. Per questo motivo, negli ultimi anni, sono stati proposti in letteratura una serie di tecniche ad hoc tra cui il metodo del Filtraggio Iterativo. La convergenza di quest'ultimo, benché nota nel continuo, non è mai stata studiata in dettaglio da un punto di vista numerico.

In questo seminario, dopo aver fatto una breve carrellata sui vari metodi sviluppati fino ad ora, presenteremo risultati di convergenza numerica ed alcune proprietà del metodo di Filtraggio Iterativo applicato a segnali finito dimensionali.

Lavoro in collaborazione con Haomin Zhou (School of Mathematics, Georgia Institute of Technology).

Matrix algorithms for the seriation problem

Anna Concas
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The seriation problem, a generalization of the consecutive ones problem, is an important ordering issue. It is frequently encountered in archaeology [3, 4] and has applications in many fields such as genetics, bioinformatics and graph theory.
We will show how seriation can be associated to a network problem, and we will introduce a spectral algorithm [1] which is based on the use of the Fiedler vector of the graph for finding the correct ordering of the nodes, and of PQ-trees [2] for storing the admissible permutations. Finally, we will illustrate some numerical experiments and we will discuss other possible applications of complex networks theory to the seriation problem.

This work is a joint research with Caterina Fenu and Giuseppe Rodriguez.

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Using an Auction Algorithm in AMG based on Maximum Weighted Matching in Matrix Graphs

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Recently, we proposed the use of maximum weighted matching on the adjacency graph of SPD matrices as a reliable and completely automated way to coarsen sparse matrices in Algebraic Multigrid Methods (AMG) [1, 2].

The algorithm, named *coarsening based on compatible weighted matching*, exploits a maximum product matching in the original matrix graph to enhance matrix diagonal dominance, reflecting the convergence properties of an appropriately defined compatible relaxation scheme.

The matched nodes are aggregated to form coarse index spaces and standard, piecewise constant or smoothed, interpolation operators are applied for the construction of a multigrid hierarchy, without referring to any a priori knowledge of matrix origin and/or any assumed strength of connection definition. Instead, information about the smooth error is generated and used to define edge weights assigned to the original matrix graph.

A key issue in this approach is to find an efficient yet accurate enough computation of a maximum product matching; this accounts for the largest part of the computational time needed to build the AMG solver. The most widely used algorithm for maximum weighted matching [3] generally exhibits a superlinear computational complexity; for our purposes, we needed a linear cost algorithm, thus we resorted to an approximate algorithm producing matchings whose weight is at least 1/2 of the optimum [4].

Currently, we are considering the use of an auction algorithm for solving the maximum product maximum cardinality matching problem. Auction algorithms have been demonstrated to compute high-quality matchings for scaling sparse matrices in Sparse Direct Linear Algebra Computations [5] and they are also readily parallelizable. We show that they can also improve the quality of the coarsening with respect to the previously studied half-approximate algorithm, giving results which are comparable to the ones obtained by using exact maximum product matching at a reasonable computational cost.

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BFGS-like updates of constraint preconditioners for KKT linear systems

Valentina De Simone
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In this talk we focus on efficient preconditioning techniques for sequences of KKT linear systems arising from the interior point solution of large convex quadratic programming problems. Constraint preconditioners are very effective in accelerating Krylov methods in the solution of KKT systems; nevertheless, their computation may be very expensive for large-scale problems. We overcome this problem by computing the CP from scratch only at selected interior point iterations and by updating the last computed CP at the remaining iterations, via suitable low-rank modifications based on a BFGS-like formula. The proposed technique is supported by a spectral analysis of the preconditioned matrix and by numerical results illustrating its performance.

This is a joint work with L. Bergamaschi, D. di Serafino and A. Martínez.

Problemi aperti nella simulazione numerica di magneti superconduttori

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Il *quench* è un fenomeno osservabile di frequente nei magneti superconduttori, in cui avviene una transizione locale da uno stato superconduttivo (in cui la resistività è nulla) a uno stato normale. Tale processo si manifesta al superamento di soglie critiche in temperatura, campo magnetico e corrente nel cavo: in tali casi l'energia magnetica si trasforma in calore. L'innalzamento repentino della temperatura provoca forti stress locali (dovuti alla dilatazione termica di differenti materiali a contatto) che possono rompere il magnete.

La simulazione numerica del comportamento in caso di transizione allo stato resistivo è quindi un aspetto fondamentale della progettazione di magneti superconduttivi, che deve prevedere un'adeguata protezione dal quench.

In questo intervento vengono discussi gli aspetti numerici anche legati all'algebra lineare, soffermandosi sulle difficoltà che lo rendono sostanzialmente un problema aperto; gli stessi software commerciali di simulazione adottano forti semplificazioni.

Il modello matematico del problema è infatti legato alla propagazione del calore, alle leggi di Ohm e al calcolo del campo magnetico; l'equazione di diffusione è in particolare caratterizzata da una marcata non linearità dei coefficienti e dall'ordine di grandezza fortemente variabile delle proprietà dei diversi materiali (aspetto trascurato dai software commerciali). L'impatto sul problema discretizzato si traduce in un malcondizionamento delle matrici coinvolte, che per problemi reali possono raggiungere dimensioni proibitive.

Lavoro in collaborazione con Valerio Calvelli (INFN, sede di Genova) e Giuseppe Ciaccio (DIBRIS, Università di Genova).

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Reconstructing the electrical conductivity and the magnetic permeability of the soil by a multi-frequency FDEM data inversion

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Following some previous papers [1, 2], this work aims to identify, by non destructive investigation of soil properties, inhomogeneities in the ground or the presence of particular conductive substances such as metals, minerals and other geological structures.

One of the most important prospecting techniques in applied geophysics is the one that analyze the propagation of an electromagnetic field into the ground. The *frequency domain electromagnetic* (FDEM) induction analysis utilizes a constant frequency signal and measures the deflection of the EM field produced by eddy currents into the ground.

The main device used for this kind of techniques is the *Ground Conductivity Meter* (GCM). It contains two coils (a transmitter and a receiver) placed at a fixed distance. An alternating sinusoidal current in the transmitter produces a primary magnetic field, which induces small eddy currents in the subsurface. These currents produce a secondary magnetic field, which is sensed by the receiver. The ratio of the secondary to the primary magnetic fields is then used, along with the instrumental parameters, to estimate electrical properties of the subsurface. It can be used on the ground or at heights above the ground. The orientation of the coils also affects the response of the instrument to variations in the soil electrical conductivity profile.

We use the following nonlinear model derived from Maxwell's equations, which has been described in [3], to model the interaction of an electromagnetic field with the soil,

$$M_1(\sigma,\mu;h;\omega) = -r^3 \int_0^\infty \lambda^2 e^{-2h\lambda} R_{\omega,0}(\lambda) J_0(r\lambda) d\lambda,$$
$$M_2(\sigma,\mu;h;\omega) = -r^2 \int_0^\infty \lambda e^{-2h\lambda} R_{\omega,0}(\lambda) J_1(r\lambda) d\lambda.$$

The expression M_1 reproduces the measurements for the coils in the vertical orientation, and M_2 does the same for the horizontal orientation. In the model above, σ and μ are the electrical conductivity and the magnetic permeability of the soil layers, respectively, while h and ω are the height and operating frequency of the device; J_0 and J_1 denote the Bessel functions of the first kind of order 0 and 1, respectively, and $R_{\omega,0}$ is the reflection factor, which can be computed by a recursion described in [1].

Starting from electromagnetic data collected by a multi-frequency Ground Conductivity Meter (GCM), we try to reconstruct both the electrical conductivity (see [2]) and the magnetic permeability of the soil with respect to depth [3].

The inverse problem consists of fitting the model to the data, that is determine the conductivity vector σ and the permeability vector μ which best approximate the measurements. The proposed regularized inversion procedure is based on the damped Gauss-Newton method. At each step the Jacobian matrix, for which we computed the exact analytical formulae, is approximated via either the truncated singular value decomposition (TSVD) or the truncated generalized singular value decomposition (TGSVD).

Various methods are implemented for the automatic estimations of both the truncation parameter and the relaxation parameter, which ensures the convergence of the method and the

positivity of the solution.

Numerical experiments on synthetic data sets illustrate the effectiveness of the method.

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Solving mixed classical and fractional partial differential equations using short-memory principle and approximate inverses

Fabio Durastante
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We consider an innovative efficient numerical solution algorithm for large linear systems of fractional differential equations . The key tool used is the *short–memory principle*. The latter ensures the decay of the entries of the inverse of the discretized operator, whose inverses are approximated here on a GPU device by a sequence of sparse matrices. In our talk, we solve the underlying linear systems by these approximations or by iterative solvers using sequence of preconditioners based on the above mentioned inverses.

This is a joint work with D. Bertaccini.

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Iterative regularization in variable exponent Lebesgue spaces

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Let us consider a functional equation $Af = g$ characterized by an ill-posed linear operator $A : X \longrightarrow Y$ between two Banach spaces X and Y. In this talk, we propose an extension of the Tikhonov regularization approach to the (unconventional) setting where X and Y are both two variable exponent Lebesgue spaces. Basically, a variable exponent Lebesgue space $L^{p(\cdot)}$ is a (non- Hilbertian) Banach space where the exponent $1 \leq p(\cdot) \leq +\infty$ used in the definition of the norm is not constant, but rather is a function $p(\cdot)$ of the domain [1]. This way, instead of $\int |f(x)|^p dx$ with p constant, to measure a function f in such a $L^{p(\cdot)}$ Banach spaces we have to compute

$$\int |f(x)|^{p(x)} dx.$$

Inside the general framework of the regularization theory in Banach spaces [2], we develop an iterative regularization method in variable exponent Lebesgue spaces $L^{p(\cdot)}$ based on duality maps, which is able to adaptively and automatically set up pointwise different regularization levels. Indeed, the formulation of the ill-posed problem in a variable exponent Lebesgue space $L^{p(\cdot)}$ allows us to assign different regularization parameters, related to different values of the function parameter $p(\cdot)$, on different regions of the domain.

In the case of image deblurring problems, different pointwise regularization is useful because background, low intensity, and high intensity values of the image to restore require different filtering (i.e., regularization) levels [3]. This way, the proposed iterative algorithm represents a natural and continuous extension of early procedures based on image segmentation techniques, developed to vary the "amount" of regularization depending on the "local" signal to noise ratios in all the different portions of the image domain. A numerical evidence of the proposal will be also discussed.

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Multiprecision algorithms for the matrix logarithm

Massimiliano Fasi
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We present two multiprecision algorithms to compute the principal logarithm of a matrix to arbitrary precision, given arbitrary precision floating-point arithmetic. Our approach combines the well established inverse scaling and squaring method with an improved version of the classical bound for the error of Padé approximation of Kenney and Laub, which is sharper for highly nonnormal matrices. We compare the behaviour in high precision of truncated Taylor series and diagonal Padé approximants.

This is a joint work with Nicholas. J. Higham (University of Manchester).

Un metodo iterativo di tipo Gauss-Newton per la risoluzione del problema TLS

Antonio Fazzi
Gran Sasso Science Institute

Il problema ai minimi quadrati totali (Total Least Squares) è una tecnica nota per la risoluzione di sistemi lineari sovradeterminati, nei quali si ammette la presenza di perturbazioni sia nella matrice che nel termine noto. Presentiamo un algoritmo innovativo per la risoluzione numerica di tale problema basato su un'iterazione di tipo Gauss-Newton con un controllo del passo che ne assicura la convergenza globale. Ogni iterazione richiede la risoluzione di un problema standard ai minimi quadrati in cui la matrice data è modificata da un termine di rango uno. Pertanto, questo metodo può essere implementato in modo transpose-free, avendo come nucleo computazionale il prodotto matrice-vettore con la matrice data, e può risultare conveniente in certi casi in cui i metodi classici basati sulla SVD risultano troppo onerosi.

Ricerca in collaborazione con D. Fasino (Udine).

A numerical method for Volterra integral equations basic to the solution of the KdV equation

Luisa Fermo
University of Cagliari

In this talk we present a numerical method to approximate the solution of the following Volterra integral equations:

$$K(x,x+y)-\frac{1}{2}\int_0^y\int_{x+\frac{1}{2}(y-x)}^{\infty} q(t)K(t,t+s)dtds=\frac{1}{2}\int_{x+\frac{1}{2}y}^{\infty} q(t)dt,$$

$$x\in\mathbb{R},\quad y\geq 0$$

$$M(x,x-y)-\frac{1}{2}\int_0^y\int_{-\infty}^{x-\frac{1}{2}(y-s)} q(t)M(t,t-s)dtds=\frac{1}{2}\int_{-\infty}^{x-\frac{1}{2}y} q(z)dz,$$

where $q\in L^1(\mathbb{R};(1+|x|)dx)$ and K and M are the bivariate unknown functions.

The above equations arise in the solution of the initial value problem of the Korteweg-de Vries (KdV) equation which governs the propagation of waves in shallow water

$$\begin{cases}\dfrac{\partial q(t,x)}{\partial t}-6q(t,x)\dfrac{\partial q(t,x)}{\partial x}+\dfrac{\partial^3 q(t,x)}{\partial x^3}=0, & x\in\mathbb{R}, \quad t\in\mathbb{R}^+\\ q(0,x)=q(x).\end{cases}$$

The numerical procedure is based on some basic properties of the unknown functions that we prove. These properties allow us to known K and M in each point of their unbounded domain by solving the equations in a bounded triangle. For this reason, we developed a particular technique to solve them in a computational triangle. The results give a numerical evidence of its effectiveness.

This is a joint work with Cornelis van der Mee and Sebastiano Seatzu.

A robust multilevel approximate inverse preconditioner for symmetric positive definite matrices

Andrea Franceschini
Università di Padova

The use of factorized sparse approximate inverse (FSAI) preconditioners in a standard multilevel framework for symmetric positive definite (SPD) matrices may pose a number of issues as to the definiteness of the Schur complement at each level.

The present work introduces a robust multilevel approach for SPD problems based on FSAI preconditioning, which eliminates the chance of algorithmic breakdowns independently of the preconditioner sparsity. The Multilevel FSAI algorithm is further enhanced by introducing Descending and Ascending Low-Rank corrections, thus giving rise to the MFLR preconditioner. The proposed algorithm is investigated in a number of test problems. The numerical results show that the MFLR preconditioner is a robust approach that can significantly accelerate the solver convergence rate preserving a good degree of parallelism. The possibly large set-up cost, mainly due to the computation of the eigenpairs needed by Low-Rank corrections, makes its use attractive in applications where the preconditioner can be recycled along a number of linear solves.

Staggered Discontinuous Galerkin methods for the incompressible Navier-Stokes equations: spectral analysis and computational results

Isabella Furci
University of Insubria - Como

We consider the specific problem of incompressible Navier-Stokes equations, solved by the novel numerical technique of high order accurate Discontinuous Galerkin methods on staggered meshes. We use as tools the theory of Toeplitz matrices generated by a function (in the most general block, multi-level form) and the more recent theory of Generalized Locally Toeplitz matrix sequences to arrive at a quite complete picture of the spectral features of the underlying matrices. This information is employed for giving a forecast of the convergence history of the preconditoned conjugate gradient method and several numerical tests are provided and illustrated in order to show the validity and the potential of the analysis.

IgA vs. FEA in the Spectral Approximation: Symbol-based Analysis

Carlo Garoni
Università della Svizzera Italiana

Isogeometric Analysis (IgA) and Finite Element Analysis (FEA) are two distinguished numerical methods for the numerical solution of differential problems. While FEA is a very popular technique, which is known since the past century, IgA has been introduced only recently by Thomas J. R. Hughes and his research team [1, 11]. However, due to its capability to enhance the connection between numerical simulation and Computer-Aided Design (CAD) systems, IgA is gaining more and more attention over time.

In this presentation, we focus on the numerical solution of a model eigenvalue problem by means of IgA and FEA [5]. We show that IgA is superior to FEA in the spectral approximation, because, while the numerical eigenvalues computed by IgA reproduce (approximately) all the exact eigenvalues, only a small portion of the numerical eigenvalues computed by FEA can be considered as approximations of the exact eigenvalues. Our analysis is based on an asymptotic matrix theory known as the theory of Generalized Locally Toeplitz (GLT) sequences [9, 15, 16] and on the related notion of (spectral) symbol [3, 4, 5, 6, 7, 8, 10], which will be introduced in the talk along with IgA and FEA. We will also show that this symbol-based analysis allows one to derive (and extend) the analytical spectral results which have already appeared in the relevant engineering literature [2, 12, 13, 14].

Let us see a very simple example which well illustrates the aim of this talk. Consider the unidimensional Laplacian eigenvalue problem

$$\begin{cases} -u_j''(x) = \lambda_j u_j(x), & x \in (0,1), \\ u_j(0) = u_j(1) = 0, \end{cases}$$

whose solution consists of the eigenpairs (λ_j, u_j) with

$$\lambda_j = (j\pi)^2, \qquad u_j(x) = \sin(j\pi x), \qquad j = 1, 2, \ldots$$

Compute the numerical approximations $\tilde{\lambda}_j$ of the first 800 eigenvalues λ_j by IgA (based on B-splines of degree 2) and FEA (based on piecewise polynomial functions of degree 2). In both cases, compute the eigenvalue errors $E_j = \tilde{\lambda}_j/\lambda_j$ for $j = 1, \ldots, 800$. The result is that, while for IgA we have $E_j \approx 1$ for all j, for FEA we have $E_j \approx 1$ for $j < 400$ and $E_j \gg 1$ for $j > 400$. Why this? The present talk will provide an answer to this question, as well as to other related and more general issues.

This is a joint work with Thomas J. R. Hughes, Carla Manni, Alessandro Reali, Stefano Serra-Capizzano and Hendrik Speleers.

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Platonic Solids, Restrictions Matrices and 3D Space-Time Energetic Galerkin BEM

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Engineering systems and many computational problems exhibit various kinds of symmetries, which may be exploited in order to improve related numerical algorithms. Usually symmetry stems from the domain or body on which the problem is considered. In these cases, the invariance of the domain can give important advantages in the numerical treatment of boundary value problems [1, 2, 7].
In this context, group representation theory can be employed for taking into account equivariance properties in the numerical treatment of space-time boundary integral equations related to 3D wave propagation problems which are invariant under a finite group $\mathcal{G}$ of congruences of $\mathbf{R}^3$. The presented technique is based upon suitable *restriction matrices* strictly related to a system of unitary, irreducible, pairwise not-equivalent matrix representations of $\mathcal{G}$ [1, 4]. In literature, restriction matrices have been widely used in the context of parallel multigrid algoritms [6], aggregation/disaggregation techniques and domain decomposition reduction methods for the numerical solution of elliptic boundary value problems [5].
Different techniques have been proposed in the Galerkin Boundary Element Method to reduce computational costs of matrix evaluation (e.g. panel clustering [8]) and linear system solution (e.g. $\mathcal{H}-$matrix method). Here we apply restriction matrices in the framework of energetic Galerkin Boundary Element Method applied to $3D$ Neumann exterior wave propagation problems, where the discretization matrices have a block lower triangular Toeplitz structure and the diagonal block, to be inverted at each time step, is typically dense [2, 3].
Several numerical results, obtained on Platonic solids, are presented.

This is a joint work with A. Aimi and M. Diligenti.

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Computations with semi-infinite quasi-Toeplitz matrices

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Denote by $\mathcal{W}_1$ the set of complex valued functions of the form $a(z) = \sum_{i=-\infty}^{+\infty} a_i z^i$ which are continuous on the unit circle, and such that $\sum_{i=-\infty}^{+\infty} |ia_i| < \infty$. We call CQT matrix a quasi-Toeplitz matrix A, associated with a continuous symbol $a(z) \in \mathcal{W}_1$, of the form $A = T(a) + E$, where $T(a) = (t_{i,j})_{i,j\in\mathbb{Z}^+}$ is the semi-infinite Toeplitz matrix such that $t_{i,j} = a_{j-i}$, for $i, j \in \mathbb{Z}^+$, and $E = (e_{i,j})_{i,j\in\mathbb{Z}^+}$ is a semi-infinite matrix such that $\sum_{i,j=1}^{+\infty} |e_{i,j}|$ is finite. We prove that the class of CQT matrices is a Banach algebra with a suitable sub-multiplicative matrix norm $\|\cdot\|$. We introduce a finite representation of CQT matrices together with algorithms which implement elementary matrix operations. Applications to the solution of matrix equations and to the computation of matrix functions are discussed.

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Spectral analysis and spectral symbol for the 2D curl-curl (stabilized) operator with applications to the related iterative solutions

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In this work, we develop and analyse the preconditioning of linear systems of equations arising from conforming finite element discretizations of $H(\mathrm{curl})$ elliptic variational problems. Such problems arise in Time Harmonic Maxwell and magnetostatic problems, as well in the preconditioning of MagnetoHydro- Dynamics equations.

We consider a Compatible B-Splines discretization based on a discrete De Rham sequence [1], and study the structure of the resulting matrices A_n. It turns out that we find a two-by-two pattern where each block is two-level banded and where the bandwidths grow linearly with the degree of the B-splines.

Looking at the coefficients in detail, we identify the symbol of each block. As expected, A_n has a two-by-two matrix-valued bivariate trigonometric polynomial symbol: in particular, there is a nice elegant connection with the continuous operator which has an infinite dimensional kernel, and in fact the symbol is a dyad having one eigenvalue like the one of the IgA Laplacian (whose spectral analysis can be found in [2]), and one identically zero eigenvalue. From the latter information, we are able to give a detailed spectral analysis of the matrices A_n; several numerical evidences are presented confirming the theoretical findings.

Finally, using the theory of the Generalized Locally Toeplitz (GLT) sequences (see [3]) we are able to suggest proper iterative solvers for the corresponding linear systems.

This is a joint work with Ahmed Ratnani (Max-Planck Institut für Plasmaphysik, Technische Universität München - Garching (Germany)) and Stefano Serra-Capizzano (Universty of Insubria - Como).

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The structured condition number of a differentiable map between matrix manifolds

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We discuss the structured condition number of differentiable maps between differentiable matrix manifolds, thus developing a theoretical framework that extends previous results on vector subspaces. Then, we derive a lower bound on the structured condition number, showing that the bound can be very cheap to compute. We present algorithms to compute both the structured condition number and the lower bound.

Finally, as an application, we provide numerical comparisons between the structured and unstructured condition numbers for matrices in automorphism groups and various maps: the matrix logarithm, the matrix square root, the polar decomposition, and the sign decomposition. We argue that the cheaply computable lower bound is often a good estimate of the structured condition number; moreover, we show experimentally that the structured and unstructured condition numbers can differ by several orders of magnitude, thus motivating the future development of more structure-preserving algorithms.

This talk is based on joint work with Bahar Arslan and Françoise Tisseur, both from the University of Manchester.

Large-scale matrix equations with banded right-hand side

Davide Palitta
Università di Bologna

We are interested in the numerical solution of the large-scale Lyapunov equation

$$AX + XA^T = D, \tag{1}$$

where $A, D \in \mathbb{R}^{n\times n}$ are both large, sparse and banded matrices with bandwidth β_A, β_D respectively. We suppose that A is symmetric and positive definite. If A is well-conditioned, the entries of the solution X to (1) decay in absolute value as their indeces move away from the sparsity pattern of D. This result relies on recent decay bounds for the entries of $(A \otimes I + I \otimes A)^{-1}$ where the Kronecker structure can be exploited. We show how such property can be employed to determine a memory-saving approximation to X.

More precisely, the matrix-oriented CG method can be employed in the numerical solution of (1). We show that all the iterates of the solution process are banded matrices with a bandwidth that linearly depends on the number of CG iterations. Therefore, for well-conditioned problems the memory requirements and computational costs of the algorithm remain of the order of n. Our description will be accompanied by numerical experiments to illustrate the performance of the proposed strategy.

This is a joint work with Valeria Simoncini.

Preconditioning PDE-constrained optimization with L^1-sparsity and control constraints

Margherita Porcelli
Università degli Studi di Firenze

PDE-constrained optimization aims at finding optimal setups for partial differential equations so that relevant quantities are minimized.

Including sparsity promoting terms in the formulation of such problems results in more practically relevant computed controls but adds more challenges to the numerical solution of these problems. The needed L^1-terms as well as additional inclusion of box control constraints require the use of semismooth Newton methods. We propose robust preconditioners for different formulations of the Newton's equation.

With the inclusion of a line-search strategy and an inexact approach for the solution of the linear systems, the resulting semismooth Newton's method is feasible for practical problems. We present results on the theoretical analysis of the preconditioned matrix and numerical experiments that illustrate the robustness of the proposed scheme.

This is a joint work with Valeria Simoncini and Martin Stoll.

Stability of network indexes defined trough matrix functions

Stefano Pozza
ISTI - Istituto di Scienza e Tecnologie dell'Informazione "A. Faedo" - CNR, Pisa

One of the major goals of network analysis is to identify important components in a network by exploiting the topological structure of connections between its nodes. To this end, recent years have seen the introduction of many new measures of importance of a node or a set of nodes, defined in terms of suitable entries of functions of matrices $f(A)$, for different choices of f and A [2]. Measures based on matrix functions are particularly relevant as they are able to capture the global structure of connections involving a node u, whereas most of other models rely on the local behavior around u. On the other hand, the matrix function approach requires a significant computational effort to address the entries of $f(A)$. This is particularly prohibitive when the network changes frequently and the important components have to be updated.

Given the adjacency matrix A of a graph $G = (V, E)$, in this work we address the problem of estimating the changes in the entries of $f(A)$ with respect to changes in the edge set E. Intuition suggests that, if the topology of connections in the new graph $\tilde{G} = (V, \tilde{E})$ is not significantly distorted, relevant components in G maintain their leading role in $\tilde{G}$. By exploiting the literature on functions of banded matrices [4, 1], we propose a bound showing that the magnitude of the variation of the entry $f(A)_{u,v}$ decays exponentially with the distance in G that separates either u or v from the set of nodes touched by the edges that are perturbed.

The interests of this bounds relies on the fact that often one needs to know only "who are" the first few most important nodes in the graph, whereas the nodes whose edge connection tends to change more often are those having a marginal role in the graph [3], and typically the distance in G from important nodes and nodes having a marginal role is large.

This is a joint work with Francesco Tudisco (University of Padua, Department of Mathematics "Tullio Levi-Civita", Italy).

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Comparison of first-order methods for phase estimation in differential-interference-contrast microscopy

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In this talk we address the problem of estimating the phase from color images acquired with differential-interference-contrast microscopy. In particular, we consider the nonlinear optimization problem obtained by regularizing a least-squares-like discrepancy term with a smoothed total variation functional and we investigate several optimization methods able to address its minimization. In particular, we revisit the conjugate gradient method proposed in the literature for this problem and compare it with standard conjugate gradient approaches and a more recent first-order method whose steplengths are related to the eigenvalues of the Hessian matrix of the objective function.

`EPSfun`: a Matlab toolbox for implementing the simplified topological ε–algorithms and its applications

Michela Redivo Zaglia
University of Padua

Let $(\mathbf{S}_n)$ be a sequence of elements of a vector space E on a field $\mathbb{K}$ ($\mathbb{R}$ or $\mathbb{C}$) which converges to a limit $\mathbf{S}$. If the convergence is slow, it can be transformed, by a *sequence transformation*, into a new sequence or a set of new sequences which, under some assumptions, converges faster to the same limit. When E is $\mathbb{R}$ or $\mathbb{C}$, a well known such transformation is due to Shanks, and it can be implemented by the scalar ε–algorithm of Wynn.

This transformation was generalized to sequences of elements of a general vector space E by the *topological Shanks transformations* that can be recursively implemented by the topological ε–algorithms of Brezinski [1].

Recently, simplified versions of these algorithms were obtained and called the *simplified topological ε–algorithms* [2]. They have only one recursive rule instead of two, they require less storage than the initial algorithms, elements of the dual vector space E^* of E no longer have to be used in the recursive rules but only in their initializations, the numerical stability is improved, and it was possible to implement the transformation not only with vectors, but also with matrices and tensors.

In this talk, we present the Matlab package `EPSfun` [3] that will be available in the public domain library *netlib* for implementing and using the simplified topological epsilon-algorithms for accelerating the convergence of sequences of elements of a vector space. The functions for other similar algorithms (i.e. the vector ε-algorithm of Wynn) are also provided.

Several results obtained by using the package will be presented. They concern methods for solving problems of different kind: solution of linear and nonlinear systems of vector and matrix equations, computation of matrix functions, nonlinear Fredholm integral equations of the second kind, and computation of tensor l^p-eigenvalues.

This is a joint work with Claude Brezinski (University of Lille, Laboratoire Paul Painlevé, UMR CNRS 8524, UFR de Mathématiques, France).

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Decay bounds for the numerical quasiseparable preservation in matrix functions

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Given matrices A and B such that $B = f(A)$, where $f(z)$ is a holomorphic function, we analyze the relation between the singular values of the off-diagonal submatrices of A and B. We provide family of bounds which depend on the interplay between the spectrum of the argument A and the singularities of the function. In particular, these bounds guarantee the numerical preservation of quasiseparable structures under mild hypotheses. We extend the Dunford-Cauchy integral formula to the case in which some poles are contained inside the contour of integration. We use this tool together with the technology of hierarchical matrices ($\mathcal{H}$-matrices) for the effective computation of matrix functions with quasiseparable arguments.

Spectral analysis of matrices in isogeometric methods with GB-splines

Fabio Roman
Università di Torino

Generalized splines are smooth piecewise functions with sections in spaces of the form

$$\mathbb{P}_p^{U,V} := \langle 1, t, \ldots, t^{p-2}, U(t), V(t) \rangle, \quad t \in [a,b], \quad 2 \leq p \in \mathbb{N}.$$

Besides classical polynomial splines, interesting examples are trigonometric or hyperbolic generalized splines for which U, V are taken as $\cos(\alpha t), \sin(\alpha t)$, or $\cosh(\alpha t), \sinh(\alpha t)$, respectively. Under suitable assumptions on U, V, generalized splines possess all the desirable properties of polynomial splines. In particular, they admit a representation in terms of basis functions that are a natural extension and possess all the nice properties of the polynomial B-splines. Such basis functions are referred to as generalized B-splines.

Thanks to the above properties, tensor-product generalized B-splines are an interesting problem-dependent alternative to tensor-product (polynomial) B-splines and NURBS in isogeometric analysis (IgA). IgA is a well-established paradigm for the analysis of problems governed by partial differential equations (PDEs). It aims at improving the connection between numerical simulation and computer aided design (CAD) systems. The main idea of IgA is to use the functions adopted in CAD systems not only to describe the domain geometry, but also to represent the numerical solution of the differential problem, within an isoparametric framework. Both Galerkin and collocation methods have been intensively employed in this context.

Like for any Galerkin/collocation method, the IgA paradigm requires to solve large linear systems. Information about the spectral properties of the related matrices (such as non-singularity, conditioning, spectral distribution and clustering of the eigenvalues) is extremely important for the design of fast solvers for these linear systems, in order to pursue an applicative interest.

In this talk we focus on the numerical solution of second order elliptic problems by the IgA approach based on trigonometric or hyperbolic GB-splines. In particular, we prove that the corresponding stiffness matrices possess an asymptotic eigenvalue distribution when the fineness parameters tend to zero so that the related matrix-size tends to infinity. This asymptotic distribution is compactly described by a function, the so-called symbol of the given sequence of matrices. Our results extend those obtained in the recent literature for classical algebraic polynomial B-splines.

This is a joint work with Carla Manni and Hendrik Speleers (Dipartimento di Matematica, Università di Roma "Tor Vergata", manni@mat.uniroma2.it, speleers@mat.uniroma2.it).

A linear algebra approach to the analysis of non-stationary subdivision for meshes with arbitrary topology

Lucia Romani
University of Milano-Bicocca

Convergence and smoothness analysis of a bivariate non-stationary subdivision scheme for meshes with arbitrary topology is still an open issue. In this talk we only consider the problem of analyzing the tangent plane continuity of non-stationary subdivision surfaces and, exploiting a linear algebra approach, we derive sufficient conditions for establishing G^1-continuity of any bivariate non-stationary subdivision scheme at the limit points of extraordinary vertices and/or extraordinary faces.

This work is in collaboration with Costanza Conti (University of Florence), Marco Donatelli and Paola Novara (University of Insubria - Como).

On the stability of the Coco - Russo method for the Poissin equation

Santina Chiara Stissi
University of Catania

The aim of my seminar is to study the consistency of the Coco - Russo method in one and two space dimensions.
The method is the finite difference method applied to the Dirichlet problem:

$$\begin{cases} -\Delta u = f & \text{in } \Omega, \\ u = g & \text{in } \partial\Omega \end{cases} \tag{1}$$

where Ω is an arbitrary domain, $f : \Omega \to \mathbb{R}$, $g : \Omega \to \mathbb{R}$ assigned functions and $u : \Omega \to \mathbb{R}$ unknown function.

In one dimension we solve numerically (1) in $\Omega = [a, 1] \subset [0, 1]$ (with $a > 0$).
We descritize the computational domain $[0, 1]$ on a uniform grid, with mesh size h, where $h = \frac{1}{N}$ so that the grid is formed by $N + 1$ points.
In the inside points we use standard finite difference method.
The left boundary a is located between x_0 and x_1.
The right boundary is located at $x_N = 1$.
The Dirichlet boundary conditions are imposed at a (through a linear interpolation procedure on the nodes x_0 and x_1) and at x_N.
Put $p = 1, 2, \infty$, the numerical experiments show that:

$$\|\tau_h\|_{L^p} \approx O(h^2),\ \|e_h\|_{L^p} \approx O(h^2),\ \|A_h^{-1}\|_p \approx O(h^{-\frac{1}{p}}) \text{ for all } p \geq 1,$$
$$\rho(A_h^{-1}) \approx O(1),$$

where τ_h, e_h denote, respectively, truncation error and error, while ρ is the spectral radius.
A possible explanation of such anomalous behavior is proposed.

In two dimensions we solve numerically (1) in $\Omega = C((\bar{x}, \bar{y}), R) \subset [0, 1]^2$.
We descritize the computational domain $[0, 1]^2$ on a uniform grid.
In the inside points we use standard finite difference method.
The Dirichlet boundary conditions are imposed in the projection of the ghost points on the circumference.
Put $p = 1, 2, \infty$, the numerical examples show that:

$$\|\tau_h\|_{L^p} \approx O(h^2),\ \|e_h\|_{L^p} \approx O(h^2),\ \text{for all } p \geq 1,$$
$$\|A_h^{-1}\|_p \approx O(h^0) \text{ with } p = 2, \infty,\ \text{and } \|A_h^{-1}\|_p \approx O(h^{-\frac{1}{2}}) \text{ with } p = 1.$$

Justification is made in progress.

A nonlinear Krylov-type method for mixed subordinate matrix norms

Francesco Tudisco
University of Padua

The $\beta \to \alpha$ norm of a complex matrix $A \in \mathbb{C}^{m\times n}$ is defined by

$$\|A\|_{\beta\to\alpha} = \max_{x\neq 0} f_A(x) = \max_{x\neq 0} \frac{\|Ax\|_\alpha}{\|x\|_\beta}$$

where $\|\cdot\|_\alpha$ and $\|\cdot\|_\beta$ are arbitrary vector norms. The computation of $\|A\|_{\beta\to\alpha}$ has several applications and arises in contexts such as approximation theory, estimation of the condition number, eigenvalue problems for tensors, manifold learning and graph clustering (see e.g. [3, 4, 5]). Although some closed-form expressions of $\|A\|_{\beta\to\alpha}$ are known for some ℓ^p vector norms, the computation of $\|A\|_{\beta\to\alpha}$ is generally NP-hard. The state-of-the-art method for its estimation is a Nonlinear Power Method (NPM), essentially introduced by Boyd in [1]. If $\|\cdot\|_{\alpha'}$ denotes the dual norm of $\|\cdot\|_\alpha$ and $J_\alpha(x) = \partial\|x\|_\alpha$ is its Clarke's subdifferential [2], then the sequence generated by the NPM is obtained by evaluating f_A on the following vectors:

$$x_0, \quad x_1 = \mathcal{S}_{\beta\to\alpha}(x_0), \quad x_2 = \mathcal{S}_{\beta\to\alpha}(x_1), \quad x_3 = \mathcal{S}_{\beta\to\alpha}(x_2), \quad \dots\ ,$$

where $\mathcal{S}_{\beta\to\alpha}(x) = J_{\beta'}(A^* J_\alpha(Ax))$. Let $\mathcal{V}_k = \operatorname{span}\{x_0, x_1, x_2, \dots, x_k\}$ and define $\lambda_k = \max_{x\in\mathcal{V}_k} f_A(x)$. Evidently the quantity λ_k is a better approximation of $\|A\|_{\beta\to\alpha}$ than $f_A(x_k)$. In the linear case, this choice for λ_k defines the Krylov subspace method and the computation of λ_k in that case can be efficiently done. When the norms are not Euclidean the computation of λ_k is more challenging and requires a different strategy.

In this talk we discuss a technique involving matrices of small rank, based on a modification of the NPM applied to f_A, but restricted to $\mathcal{V}_k$. Thus the overall method can be seen as a Nested modified NPM (NNPM) which is then well-suited for large matrices. We discuss the local and global convergence of the method and show several numerical tests where the performances of the NPM and the new nested scheme are compared.

This is a joint work with Antoine Gautier and Matthias Hein (Saarland University, Department of Mathematics and Computer Science, Saarbrücken, Germany).

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Multigrid methods and Subdivision schemes

Valentina Turati
University of Insubria - Como

Multigrid is an iterative method for solving linear systems of equations whose coefficient matrices are symmetric and positive definite. Each iteration consists of two steps: the so-called smoother and the coarse grid correction. The convergence rate of multigrid depends on the properties of the smoother and the so-called grid transfer operator appearing at the coarse grid correction step.

In this talk, we present the link between the properties of multigrid methods and of subdivision schemes. We define a new family of bivariate interpolatory subdivision schemes of dilation matrix (2,3)I and we construct new grid transfer operators from their symbols. We show that the polynomial generation property and stability of the corresponding basic limit functions are crucial for the fast convergence of the corresponding multigrid method.

Our numerical results illustrate the behaviour of the new grid transfer operators when applied to linear systems arising from anisotropic Laplacian problem with finite differences discretization.

This work is in collaboration with Maria Charina (University of Vienna), Marco Donatelli (University of Insubria - Como) and Lucia Romani (University of Milano-Bicocca).