Multigrid finite-difference ghost-point methods for elliptic problems with arbitrary boundary conditions and discontinuous coefficients


ABSTRACT:

Elliptic equations with jumping coefficients across a (possibly moving) one-codimensional interface arise in several applications, such as the steady-state diffusion problem in two materials with different diffusion coefficients, the incompressible Navier-Stokes equation for fluids with different density or viscosity, porous-media equations, electrostatic problems, charge transport in semiconductors, crystal growth, and many others. Cartesian grid methods are an effective alternative to boundary-fitted grid methods, since the complex task of the grid generation process at each time step is avoided.

In this talk we present a finite difference numerical method on a Cartesian grid to solve the elliptic equation with discontinuous coefficient in an arbitrary domain, identified by a level set function. The method is based on ghost-point technique and the linear system arising from the discretization is solved by a multigrid approach. The whole problem is relaxed by introducing a fictitious time, which is chosen in such a way the relaxation results in a good smoother and compatible with the CFL condition. The interpolation and restriction operators are suitably modified close to the interface/boundary.

The convergence factor of the multigrid is close to the one predicted by the Local Fourier Analysis for inner relaxations, and it does not depend on the geometry nor on the jump in the coefficient. Preliminary study on adaptive grids and applications to volcanology problems are presented.