The Cosmic Microwave Background radiation and the Integrated Sachs-Wolfe effect as a test bed for cosmological models

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Outline

The Standard FLRW Cosmology

The FLRW metric

Friedmann, 1922, Lemaître 1927, Robertson 1929, Walker 1935

Cosmological principle = Isotropy and homogeneity ⇒ FLRW metric

$$ds^2 = dt^2 - a(t)^2\gamma_{ij}dx^i dx^j = a(\eta)^2\left(d\eta^2 - \gamma_{ij}dx^i dx^j\right)$$

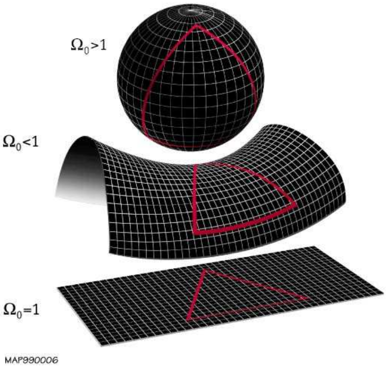
a(t) is the scale factor, it measures the relative expansion of the Universe

$$\gamma_{ij} = \delta_{ij}\frac{1}{\left(1 + \frac{1}{4}Kr^2\right)^2}$$

is the metric of the spatial hypersurfaces (constant time t), which can be closed, flat or open ($K = 1, 0, -1$)

The Standard FLRW Cosmology

The spatial FLRW metric



Spatial metric:

$$\gamma_{ij} = \delta_{ij} \frac{1}{\left(1 + \frac{1}{4} K r^2\right)^2}$$

with $K = \Omega_0 - 1$
Ω_0 present time density parameter
Present time (Today) $= t_0$ such that $a(t_0) = 1$
From WMAP, $\Omega_0 = 1.02 \pm 0.02$

The Standard FLRW Cosmology

The evolution equations

Einstein's equations $G_{\mu\nu} = 8\pi G T_{\mu\nu}$ computed from the FLRW metric give Friedmann equation

$$H^2 = \frac{8\pi G}{3} T^0_0 - \frac{K}{a^2}$$

and Raychaudhuri equation

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} T$$

where $H = \frac{\dot{a}}{a}$ is the Hubble parameter, T^0_0 is the energy density and $T = T^\mu_\mu$

The Hubble constant is $H(t_0) = H_0$

$H_0 \approx 70 (km/s)/Mpc \Rightarrow 1/H_0 \approx 13 Gyr$ and $c/H_0 \approx 4000\ Mpc$

The Standard FLRW Cosmology

The perfect fluid stress-energy tensor

The stress-energy tensor may have numerous different origins. The standard one is that of a perfect fluid

$$T_{\mu\nu} = (\rho + p)\, u_\mu u_\nu - p g_{\mu\nu}$$

where $g_{\mu\nu}$ is the FLRW metric.
As a matrix, in the comoving reference frame
$T_\mu{}^\nu = diag\,(\rho, -p, -p, -p)$
Friedmann and Raychaudhuri equations become:

$$H^2 = \frac{8\pi G}{3}\rho - \frac{K}{a^2}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\,(\rho + 3p)$$

When $\rho + 3p \geqslant 0$ the expansion of the Universe accelerates

The Standard FLRW Cosmology

Fluid types

From the energy conservation equation

$$\dot{\rho} = -3\frac{\dot{a}}{a}\rho\,(1+w)$$

When w is constant

$$\rho = \rho_0 a^{-3(1+w)}$$

$w \equiv p/\rho$	Fluid type	$a(t)$
0	Dust (Baryons)	$\propto t^{3/2}$
$1/3$	Radiation	$\propto t^{1/2}$
-1	Cosmological constant	$\propto e^{H_0 t}$

The curvature K can be regarded as a $w = -1/3$ fluid
In general, for a generic $w \neq -1$ constant fluid one has

$$a(t) \propto t^{\frac{2}{3(1+w)}}$$

Cosmological perturbation theory

The Lifshitz's approach

Lifshitz, 1946

Consider a small perturbation $\delta g_{\mu\nu}$ of the FLRW metric $g_{\mu\nu}$

$$ds^2 = (g_{\mu\nu} + \delta g_{\mu\nu})\, dx^\mu dx^\nu$$

The evolution equations are again the Einstein's ones

$$\delta G_{\mu\nu} = 8\pi G \delta T_{\mu\nu}$$

computed for the perturbed metric $\delta g_{\mu\nu}$

Initial conditions *Amplitude* A_S and the *spectral index* n_S of the primordial power spectrum $P(k) = A_S k^{n_S}$

When $n_S = 1$ we get the scale-invariant, Harrison-Zel'dovich spectrum

The Standard cosmological model

The ΛCDM model

The most famous and widely studied model

$$\frac{H^2}{H_0^2} = \frac{\Omega_r}{a^4} + \frac{\Omega_m}{a^3} + \frac{\Omega_K}{a^2} + \Omega_\Lambda$$

where

$$\Omega_r + \Omega_m + \Omega_K + \Omega_\Lambda = 1$$

The density parameter for the X component is defined as

$$\Omega_X = \frac{\rho_X}{\rho_{cr}}$$

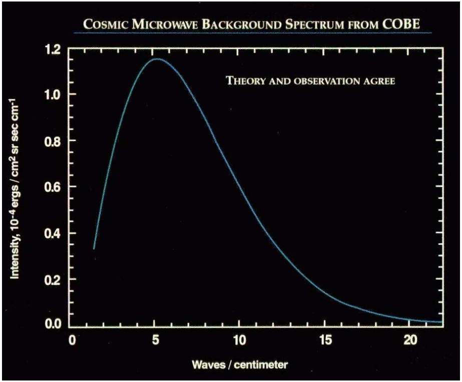
where the critical density

$$\rho_{cr} = \frac{3H_0^2}{8\pi G} = 1.9h^2 \cdot 10^{-29} g\, cm^{-3}$$

$\approx$ 10 protons per m^3 or $\approx$ 100 solar masses per kpc^3

The Cosmic Background radiation

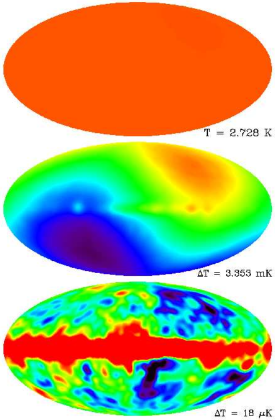
The COBE FIRAS spectrum



- CMB radiation discovered in 1964 by Penzias and Wilson
- COsmic Background Explorer launched in 1989
- Black-body spectrum with temperature $T \approx 2.726$ K
- $\Omega_R h^2 = 4.2 \cdot 10^{-5}$
- Confirmation of the hot Big Bang Model

The Cosmic Background radiation

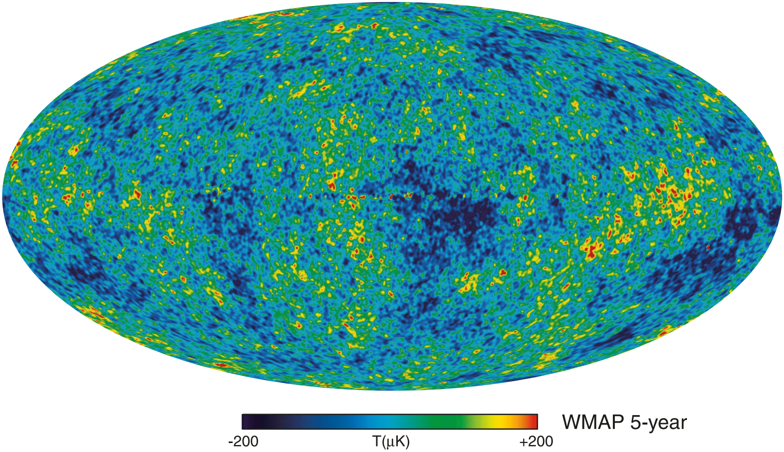
The COBE DMR experiment and the anisotropies



- Black-body temperature $T = 2.726 \pm 0.002$ K
- Dipole anisotropy caused by relative motion $\Delta T = 3.353 \pm 0.024$ mK
- Primordial fluctuations $\Delta T_{RMS} = 15.3^{+3.8}_{-2.8}\ \mu K$

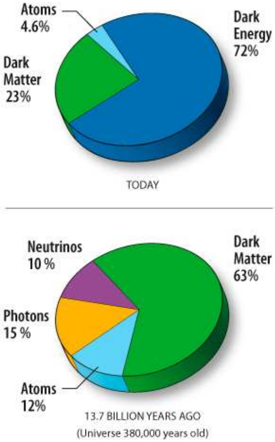
The Cosmic Background radiation

The WMAP skymap



The Cosmic Background radiation

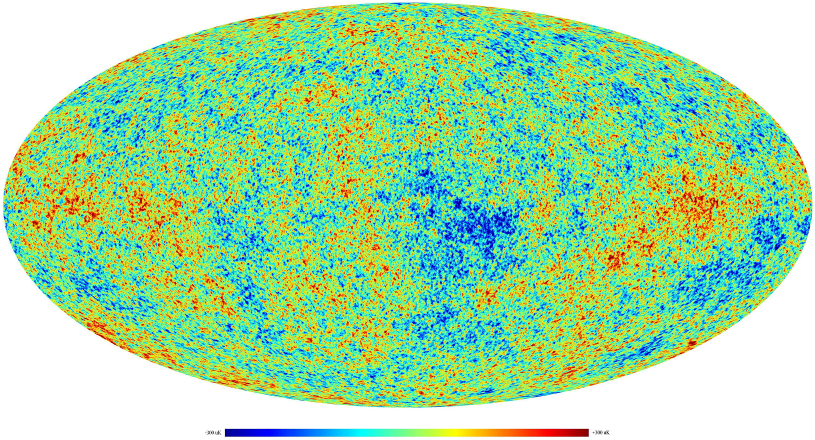
The WMAP5 results



- $H_0 = 71.9^{+2.6}_{-2.7}$ $(km/s)/Mpc$
- $t_0 = 13.69 \pm 0.13 \ Gyr$
- $\Omega_\Lambda = 0.742 \pm 0.030$
- $\Omega_{CDM} = 0.214 \pm 0.027$
- $\Omega_b = 0.0441 \pm 0.0030$
- $\Omega_{TOT} = 1.02 \pm 0.02 \Rightarrow$ Flat Universe ($K \approx 0$)
- $1 - n_S = 0.037^{+0.015}_{-0.014}$

The Cosmic Background radiation

The Planck mission: October 31, 2008



The Cosmic Background radiation

The Angular power spectrum

Radiation and temperature perturbations

$$\rho_\gamma \propto T^4 \Rightarrow \delta\rho_\gamma/\rho_\gamma = 4\Delta T/T \equiv 4\Theta$$

Expand the relative temperature fluctuations in spherical harmonics

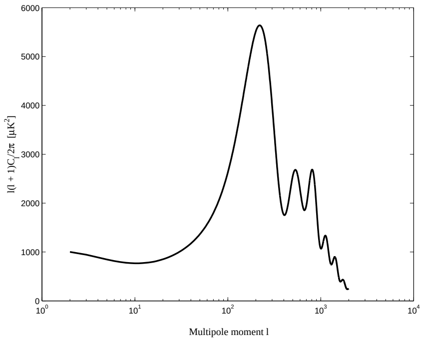
$$\Theta(\theta,\phi) = \sum_{l=2}^{\infty}\sum_{m=-l}^{l} a_{lm} Y_{lm}(\theta,\phi)$$

For a gaussian distribution $\langle \bar{a}_{lm} a_{l'm'} \rangle = \delta_{ll'}\delta_{mm'} C_l$ the correlation function

$$\langle \Theta(\mathbf{n})\Theta(\mathbf{n'})\rangle = \frac{1}{4\pi}\sum_{l=2}^{\infty}(2l+1)C_l P_l(\cos\theta)$$

The Cosmic Background radiation

The CMB angular power spectrum



Main features:

- The Sachs-Wolfe plateau
- Growth for small l: late times ISW effect
- Peaks: Acoustic oscillations
- Different peaks height: Baryon Drag
- Global fall-off: Silk Damping

CMB Physics

Multipole decomposition

Hu and Sugiyama, 1994
Consider the multipole decomposition

$$\Theta(\theta, \eta, k) = \sum_{l=0}^{\infty} (-1)^l \Theta_l(\eta, k) P_l(\cos\theta)$$

Relation between C_l and Θ_l

$$\frac{2l+1}{4\pi} C_l = \frac{1}{2\pi^2} \int_0^{\infty} \frac{dk}{k} k^3 \frac{|\Theta_l(\eta, k)|^2}{2l+1}$$

CMB Physics

General equations I: Collisional brightness equation

Boltzmann equation with a collisional part given by Thomson scattering (Newtonian approximation)

$$\dot{\Theta} + ik\mu\left(\Theta + \Psi\right) = -\dot{\Phi} + \dot{\tau}\left[\Theta_0 - \Theta - \frac{1}{10}\Theta_2 P_2(\mu) - i\mu V_b\right]$$

with $\mu = \cos\theta$, $\dot{\tau} = x_e n_e \sigma_T a$ differential optical depth to Thomson scattering, P_2 quadrupole moment of energy distribution, V_b baryons' velocity, Ψ and Φ gravitational potentials

CMB Physics

General equations II: Continuity and Euler equations

Equations for baryons in the total matter rest frame
Continuity equation

$$\dot{\Delta}_b = -k(V_b - \Theta_1) + \frac{3}{4}\dot{\Delta}_\gamma$$

and Euler equation

$$\dot{V}_b = -\frac{\dot{a}}{a}V_b + k\Psi + \dot{\tau}(\Theta_1 - V_b)/R$$

where

$$R = 3\rho_b/4\rho_\gamma = 3.0 \cdot 10^4(1+z)^{-1}\Omega_b h^2$$

$\Omega_b h^2 \approx 0.01 - 0.02$

The CMB primary anisotropies

The tight coupling limit

Baryon-photon fluid: tight coupling limit $\dot{\tau} \gg 1 \Rightarrow V_b = \Theta_1$ and $\Theta_l = 0$ for $l \geq 2$

Fluctuations are then described by

$$\ddot{\Theta}_0 + \frac{\dot{a}}{a}\frac{R}{1+R}\dot{\Theta}_0 + k^2 c_s^2 \Theta_0 = F(\eta)$$

where

$$F(\eta) = -\ddot{\Phi} - \frac{\dot{a}}{a}\frac{R}{1+R}\dot{\Phi} - \frac{k^2}{3}\Psi$$

and $c_s^2 = 1/3(1+R)$

Suppose $R \ll 1$ and a constant driving force $F = -k^2\Psi/3$

$$\ddot{\Theta}_0 + \frac{k^2}{3}\left(\Theta_0 + \Psi\right) = 0$$

The CMB primary anisotropies

The acoustic oscillations

With adiabatic initial conditions, $\Theta_0 \neq 0$ and $\dot{\Theta}_0 = 0$, the solution of the equation is

$$\Theta_0 + \Psi = \frac{\Psi}{3} \cos\left(\frac{k}{\sqrt{3}}\eta\right)$$

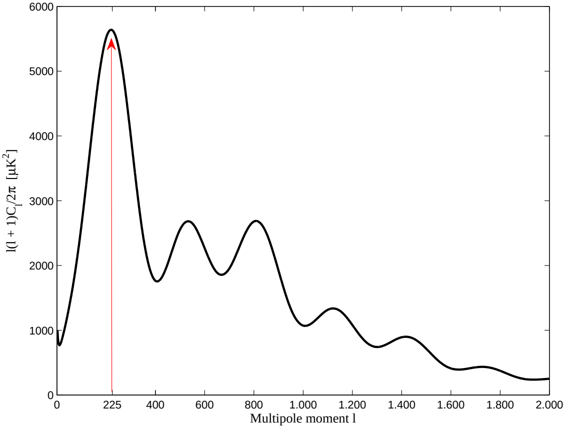
The effective temperature is $\Theta_0 + \Psi$ (Sachs-Wolfe effect) and it oscillates between $-\Psi/3$ and $\Psi/3$
The peaks are located at

$$k_n = \frac{n\pi\sqrt{3}}{\eta_*}$$

For a flat Universe, the first peak is located at $l = 225$

The CMB primary anisotropies

The acoustic oscillations



The CMB primary anisotropies

The baryon drag I

The baryons "drag" the baryon-photon fluid into the potential wells, making them deeper

Suppose R constant but not negligible

$$\ddot{\Theta}_0 + \frac{k^2}{3}\left(\frac{\Theta_0}{1+R} + \Psi\right) = 0$$

The solution becomes

$$\Theta_0 + \Psi = \frac{1}{3}\Psi(1+3R)\cos\left(\frac{k}{\sqrt{3(1+R)}}\eta\right) - R\Psi$$

The bottom of the well is now at $-\frac{\Psi}{3}(1+6R)$ and therefore peaks and throats are no more symmetric

The CMB primary anisotropies

The baryon drag II

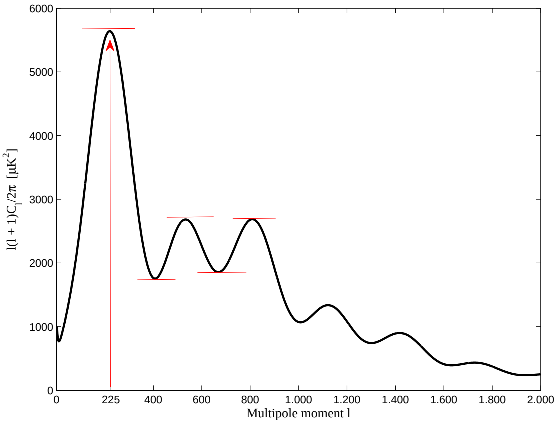
If we do not neglect the damping term

$$\ddot{\Theta}_0 + \frac{\dot{a}}{a}\frac{R}{1+R}\dot{\Theta}_0 + \frac{k^2}{3}\left(\frac{\Theta_0}{1+R} + \Psi\right) = 0$$

The amplitude of the oscillations decays as $(1+R)^{-1/4}$

The CMB primary anisotropies

The baryon drag effect



The CMB primary anisotropies

Silk Damping

Silk, 1968

Despite tight coupling, photons have a non zero mean free path λ_D in the baryon-photon fluid $\Rightarrow$ hot photons and cold photons are mixed for $\lambda < \lambda_D$ $\Rightarrow$ suppression of the correlation

$$(\Theta_0 + \Psi) \rightarrow (\Theta_0 + \Psi)e^{-(k/k_D)^2}$$

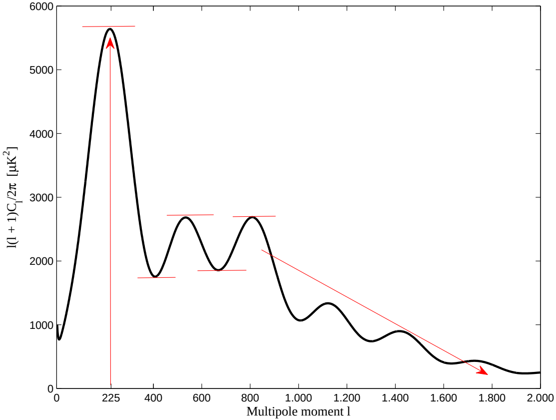
The diffusion scale is given by

$$k_D^{-1}(\eta) = \frac{1}{6}\int_0^\eta d\eta \frac{1}{\dot{\tau}} \frac{R^2 + 4(1+R)/5}{(1+R)^2}$$

Diffusion scale (Silk scale) $\lambda_D \lesssim 3\ Mpc \Rightarrow l \gtrsim 800$

The CMB primary anisotropies

The Silk Damping



The ISW effect

Intro

Sachs and Wolfe, 1967

The gravitational potentials occurring in the driving force

$$F(\eta) = -\ddot{\Phi} - \frac{\dot{a}}{a}\frac{R}{1+R}\dot{\Phi} - \frac{k^2}{3}\Psi$$

are constant only in the matter dominated era $\Rightarrow$ the potential wells which photons pass through have a time-independent shape $\Rightarrow$ no Integrated Sachs-Wolfe effect $\Rightarrow$ Sachs-Wolfe plateau

Another form of energy dominates $\Rightarrow$ the potential wells change their shape $\Rightarrow$ one has to integrate along the photon pattern $\Rightarrow$ Integrated Sachs-Wolfe effect

- Radiation $\rightarrow$ Early times ISW effect
- Dark energy $\rightarrow$ Late times ISW effect

The ISW effect

The early times and late times ISW effect

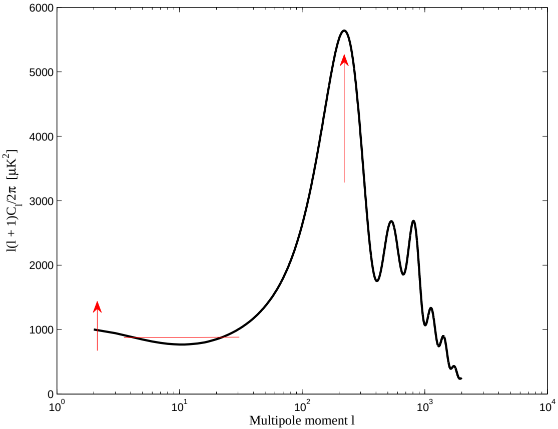
Early times ISW effect

The potential decays on large scales (small l's) ⇒ photons experience a kick ⇒ height of the peak in the angular power spectrum

Late times ISW effect

Again the potential decays. The wells are sweeped out by the acceleration, but this happens on larger scales ⇒ growth of the angular power spectrum for very small l's

The ISW effect



The ISW effect

An analytic approach

Bertacca and Bartolo, 2007

One fluid model with adiabatic perturbations $\Rightarrow \Phi = -\Psi$

Evolution equation for the potential

$$u'' - c_s^2 \Delta u - \frac{\theta''}{\theta} u = 0$$

where $u \equiv \frac{\Phi}{\sqrt{\rho+p}}$ and $\theta = \frac{1}{a\sqrt{1+w}}$ The ISW effect contribution to the angular power spectrum

$$\frac{2l+1}{4\pi} C_l = \frac{1}{2\pi^2} \int_0^\infty \frac{dk}{k} k^3 \frac{|\Theta_l(\eta_0, k)|^2}{2l+1}$$

with

$$\frac{\Theta_l(\eta_0, k)}{2l+1} = 2 \int_{\eta_*}^{\eta_0} \Phi'(\tau, k) j_l[k(\eta_0 - \tau)] d\tau$$

The ISW effect

An analytic approach: two regimes

Consider the conformal time $\eta_{1/3}$ at which acceleration takes place $w(\eta_{1/3}) = -1/3$
There are two regimes

$$\frac{\Theta_l(\eta_0, k)}{2l+1} = \begin{cases} 2\Delta\Phi_k j_l[k(\eta_0 - \eta_{1/3})] & k\eta_{1/3} \ll 1 \\ 2\Phi'_k(\eta_k) I_l/k & k\eta_{1/3} \gg 1 \end{cases}$$

where $\eta_k = \eta_0 - \dfrac{l+1/2}{k}$ and $I_l = \dfrac{\sqrt{\pi}}{2}\dfrac{\Gamma[(l+1)/2]}{\Gamma[(l+2)/2]}$
From the peak of the Bessel function at a given k
$k(\eta_0 - \eta_k) = l + \frac{1}{2}$
$\Delta\Phi_k$ photon kick on large scales

The ISW effect

An analytic approach: the contribution to the C_l's for $k\eta_{1/3} \ll 1$

From equation

$$u'' - c_s^2 \Delta u - \frac{\theta''}{\theta} u = 0$$

If $k\eta_{1/3} \ll 1$ then $c_s^2 k^2 \ll |\theta''/\theta|^2 \Rightarrow$ the contibution is

$$C_l \propto A_S \left| \frac{1}{10} - H(\eta) \int_{\eta_*}^{\eta_0} a^2(\tau) d\tau \right|^2 \int_0^{1/\eta_{1/3}} \frac{dk}{k} k^{n_S - 1} j_l^2[k(\eta_0 - \eta_{1/3})]$$

The same happens when $k\eta_{1/3} \gg 1$ with $c_s^2 k^2 \ll |\theta''/\theta|^2$. In this case the angular power spectrum slopes as $1/l$

Starobinsky and Kofman, 1985

No stringent discrimination among cosmological models

The ISW effect

An analytic approach: the contribution to the C_l's for $k\eta_{1/3} \gg 1$

Consider on the other hand $k\eta_{1/3} \gg 1$ and $c_s^2 k^2 \gg |\theta''/\theta|^2$

The sound velocity plays a fundamental role

Among the various contributions, the most important is

$$C_l \propto 4c_s^4(\eta_0)(l + 1/2) \int_{\frac{l+1/2}{\eta_0 - \eta_{1/3}}}^{\infty} \frac{dk}{k} k^{n_S - 1} \cos^2(D_0 k)$$

It makes the angular power spectrum to grow as l^3 until $l \approx 25$ and therefore sensibly reduces the peak-to-plateau ratio

$l \approx 25$ comes from $l \approx k_{eq}(\eta_0 - \eta_{1/3})$ and $1/k_{eq}$ Hubble scale at equivalence. Meszaros effect.

$D_0 = \int_{\eta_{1/3}}^{\eta_0} c_s(\tau) d\tau$ sound horizon

Strong constraint on cosmological models!! $c_s^2 k_{eq}^2 < |\theta''/\theta|$

The ISW effect and the generalized Chaplygin gas

The generalized Chaplygin gas

Kamenshchik, Moschella and Pasquier, 2001

A fluid with an equation of state

$$p = -A\rho^{-\alpha}$$

which interpolates the dust-dominated era and the cosmological-dominated era

$$\rho = \left(A + \frac{B}{a^{3(\alpha+1)}}\right)^{\frac{1}{\alpha+1}}$$

It contains all-in-one the properties of dark matter and dark energy

- Mild constraints from Supernovae Ia
- Severe constraints from large scale structure observation
- Severe constraints from CMB

The ISW effect and the generalized Chaplygin gas

ISW constraint and superduperluminal gCg

The sound velocity in the gCg model is

$$c_s^2 = \frac{\alpha}{1 + \frac{B}{A} a^{-3(\alpha+1)}}$$

Its value at present time is

$$c_s^2 = \bar{A}\alpha$$

with $\bar{A} \approx 0.7$
Asking for $c_s^2 k_{eq}^2 < |\theta''/\theta|$ we find the constraint $\alpha < 10^{-4}$
However, giving up the conservative limit $\alpha < 1$ is abandoned another constraint can be found $\alpha > 740$

Conclusions

What to do now?

This analytic approach seems to completely rule out the gCg model, unless $\alpha > 740$

We now search for numerical full solutions $\Rightarrow$ add also the baryon component and all the other primary anisotropies

Numerical codes

- CMBFAST *Seljak and Zaldarriaga, 1996*
- CROSS-CMBFAST *Corasaniti, 2005*
- CAMB *Lewis and Challinor, 2002*

These codes are devised for the ΛCDM model, quintessence and the simple model $w(a) = w_0 + w_a(1 - a)$ $\Rightarrow$ we have to modify them in order to treat a general $w(a)$ fluid