Energy density flux of space-time coupled wavepackets

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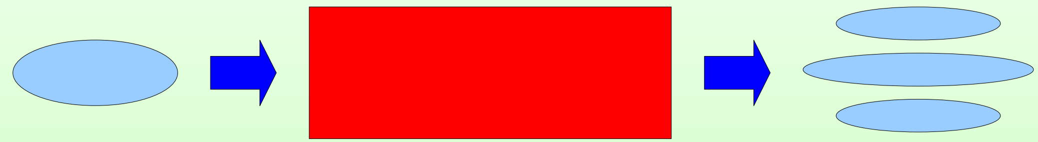
Pulse shaping

Transformation of an optical pulse propagating through a linear or nonlinear medium



Information about the medium is encoded in ***intensity*** and ***phase*** of the pulse.

Spatio-temporal shape of the pulse influence material response.

Pulse and beam shaping

Technological applications

Importance of retrieving the information "encoded" in an optical pulse

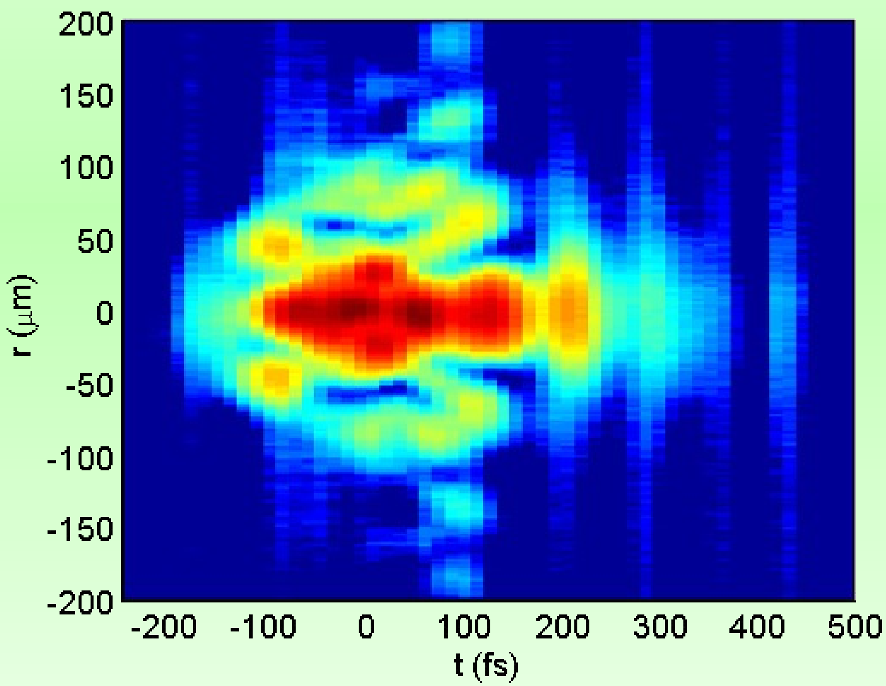
Phase Information

Amplitude / Intensity is the usual quantity experimentally accessible

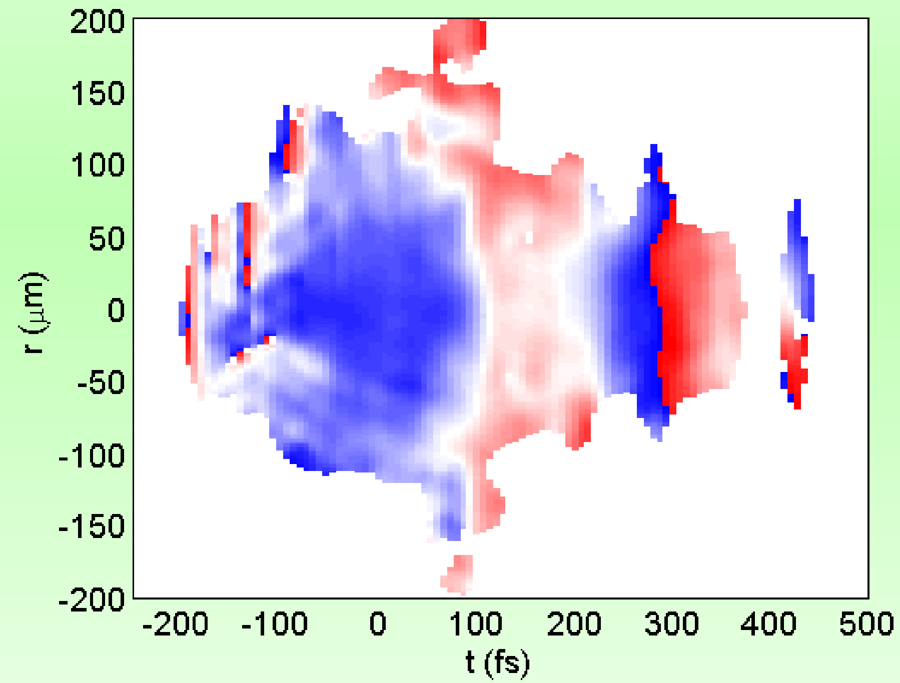
Phase is important:
- *evolution* during propagation of an optical pulse;
- nonlinear processes (phase-matching).

→ Development of techniques to retrieve amplitude and phase of optical pulses.

How to retrieve the information stored in the phase ?



Intensity



Phase

No clear interpretation of phase profiles

Energy density flux

NLSE equation

$$\frac{\partial \mathcal{E}}{\partial z} = \frac{i}{2k_0}\nabla_\perp^2 \mathcal{E} - i\frac{k_0''}{2}\frac{\partial^2 \mathcal{E}}{\partial \tau^2} + N(\mathcal{E})$$

$$N(\mathcal{E}) = i\frac{k_0 n_2}{n_0}T^2\left[(1-f_R)|\mathcal{E}|^2 + f_R\int_{-\infty}^{\tau} dt R(\tau - t)|\mathcal{E}(t)|^2\right]\mathcal{E} + -T\frac{\beta^{(K)}}{2}\left(1-\frac{\rho}{\rho_{at}}\right)|\mathcal{E}|^{2K-2} - \frac{\sigma}{2}(1+i\omega_0\tau_0)\rho\mathcal{E}$$

$$\frac{\partial |\mathcal{E}|^2}{\partial z} + \widetilde{\mathrm{div}}\mathbf{J} = -\beta^{(K)}\left(1-\frac{\rho}{\rho_{at}}\right)|\mathcal{E}|^{2K} - \sigma\rho|\mathcal{E}|^2$$

Loss terms

- Temporal coordinate = longitudinal coordinate
- *z* = evolution parameter
- Reference frame moving at the Gaussian group velocity

"Current" of energy density

$$\mathbf{J}_\perp = \frac{1}{2ik_0}\left[\mathcal{E}^*\nabla_\perp\mathcal{E} - \mathcal{E}\nabla_\perp\mathcal{E}^*\right]$$

$$J_\tau = -\frac{k_0''}{2i}\left[\mathcal{E}^*\frac{\partial \mathcal{E}}{\partial \tau} - \mathcal{E}\frac{\partial \mathcal{E}^*}{\partial \tau}\right]$$

Linked to the **intensity** and the **gradient of the phase** of the pulse

$$\mathcal{E} = |\mathcal{E}|\exp(i\phi) \quad \Longrightarrow \quad \mathbf{J} = \frac{1}{k_0}|\mathcal{E}|^2\left(\nabla_\perp\phi, -k_0 k_0''\frac{\partial \phi}{\partial \tau}\right)$$

Link to the Poynting vector

$$< \mathbf{S} > = < \mathbf{E} \times \mathbf{H} >$$

$$S_x = \Re\left\{ -\frac{i}{k_0} \mathcal{E}_x{}^* \partial_x \mathcal{E}_x \right\}$$

$$S_y = \Re\left\{ +\frac{i}{k_0} \mathcal{E}_x \partial_y \mathcal{E}_x{}^* \right\}$$

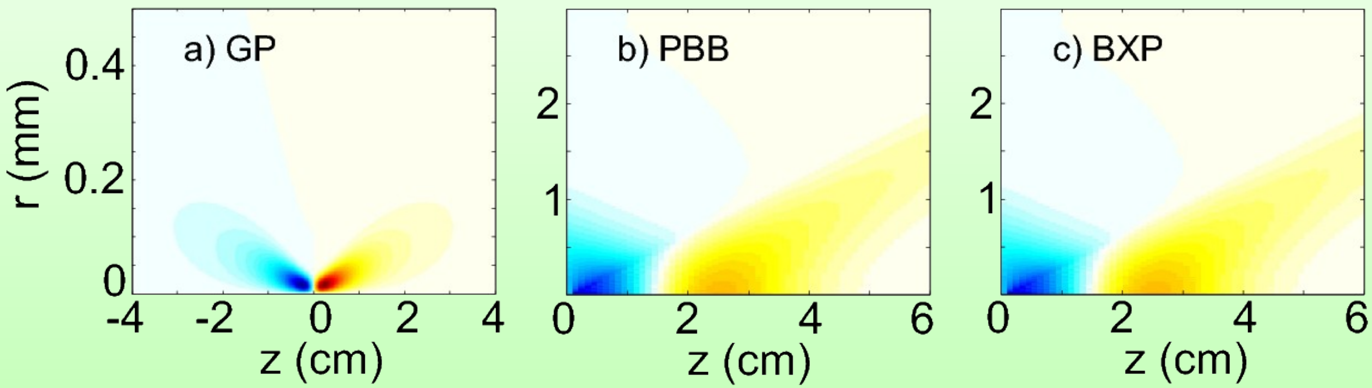
$$S_z = \Re\left\{ |\mathcal{E}_x|^2 + \frac{i}{\omega_0} \mathcal{E}_x \partial_t \mathcal{E}_x{}^* + \frac{i}{k_0} \mathcal{E}_x \partial_z \mathcal{E}_x{}^* \right\}$$

$$\mathbf{J}_\perp = \frac{1}{2ik_0} \left[\mathcal{E}^* \nabla_\perp \mathcal{E} - \mathcal{E} \nabla_\perp \mathcal{E}^* \right]$$

$$J_\tau = -\frac{k_0''}{2i} \left[\mathcal{E}^* \frac{\partial \mathcal{E}}{\partial \tau} - \mathcal{E} \frac{\partial \mathcal{E}^*}{\partial \tau} \right]$$

J = ***J****(x,y,t,z)* is not integrated over the temporal coordinate. Its longitudinal component refers to the local reference frame of the pulse.

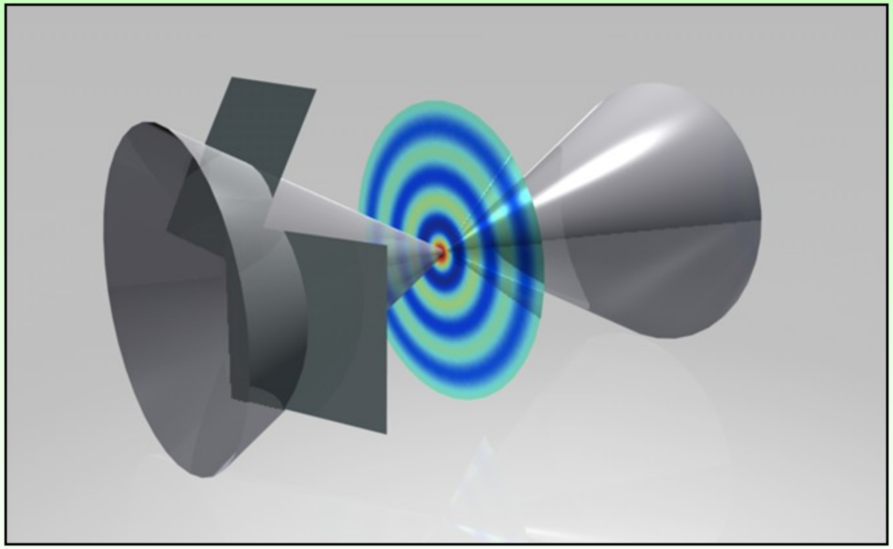
J_r = S: **Monochromatic** vs Polychromatic



Transverse flux integrated over τ

Our pulses are defined in (x,y,t)

No time integration !



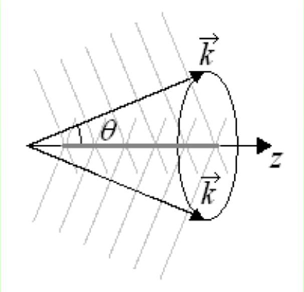
This is especially important for space-time coupled pulses.
As an example:
CONICAL WAVES

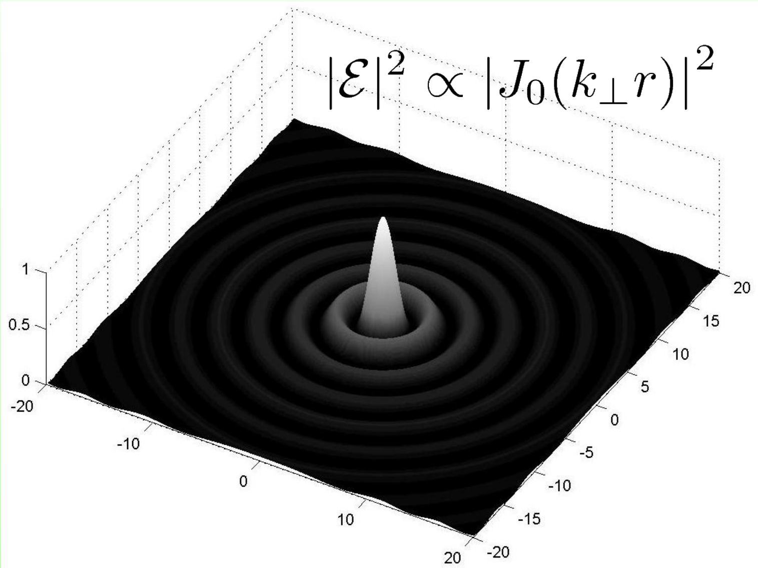
Conical waves

• Simplest conical wave: **Monochromatic Bessel Beam**: superposition of monochromatic plane waves propagating along a cone.

• Bessel profile is an interference pattern.

• *Central peak* propagates without changing shape.



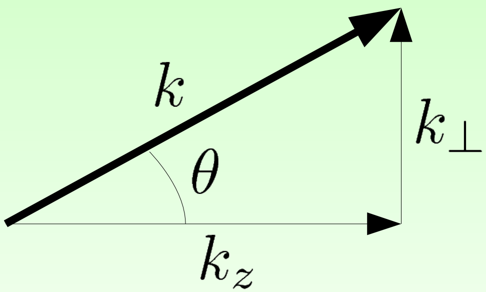


• **Polychromatic conical wave**: superposition of monochromatic Bessel beams

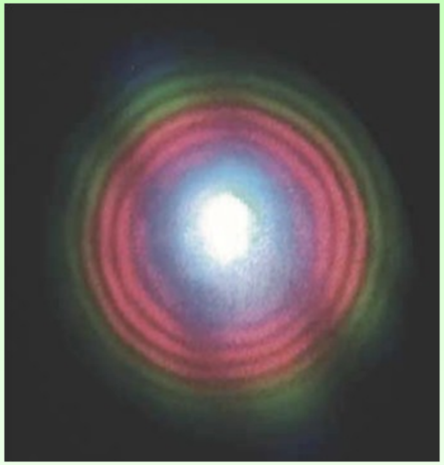
$$\mathcal{E}(r,t,z) = \int_0^\infty \mathcal{S}(\omega) J_0(k_\perp(\omega) r) \exp[i(k_z z - \omega t)] d\omega$$

$$k_\perp(\omega) = \sqrt{k^2(\omega) - k_z^2(\omega)}$$

Material dispersion



θ_y



θ_x

Conical waves – PBB, BXP

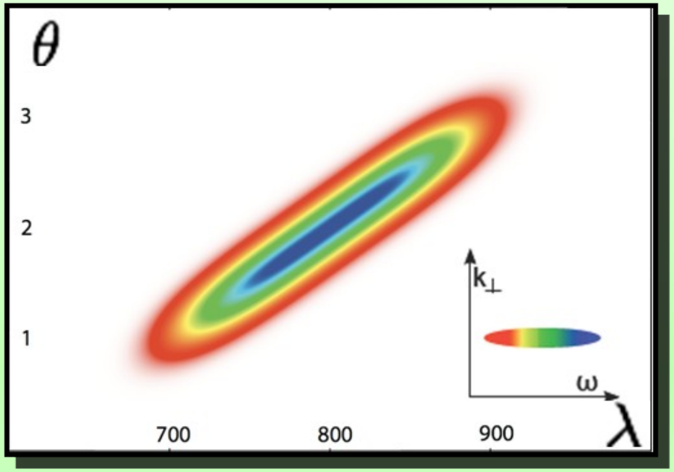
The main features of the wavepackets depend on the *($k_\perp$,ω)* spectral distribution.

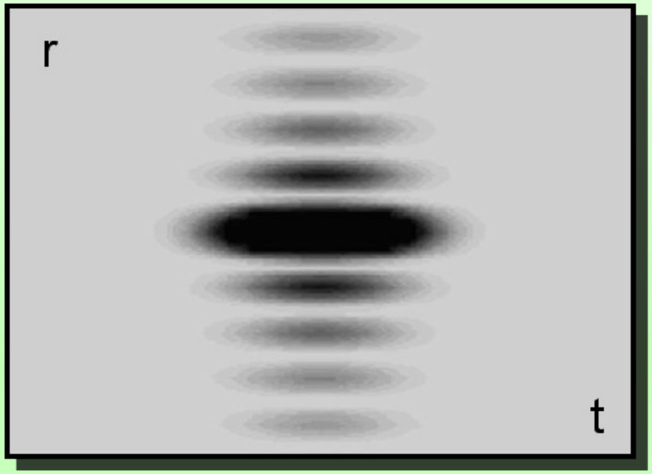
$$k_\perp(\omega) = \sqrt{k^2(\omega) - k_z^2(\omega)}$$

Pulsed Bessel Beam

$$k_\perp = \text{const.}$$

$$\theta = \arcsin\left(k_\perp / k(\omega)\right)$$

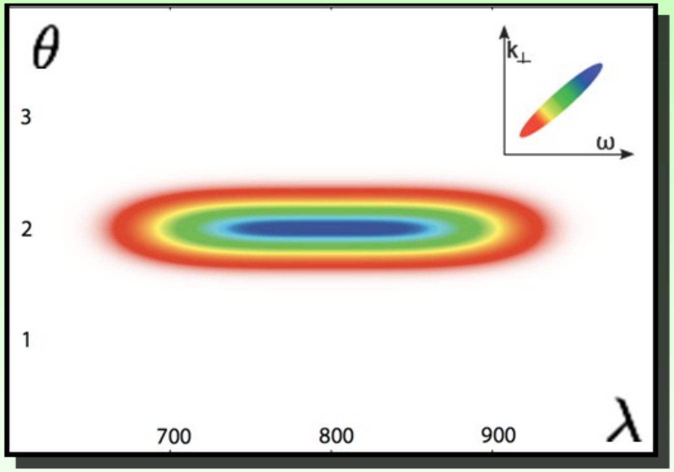


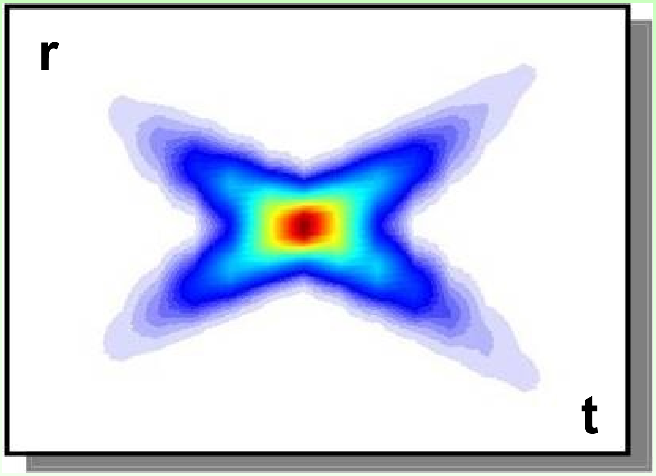


Bessel-X Pulse

$$\theta = \text{const.}$$

$$k_\perp = k(\omega)\sin\theta$$





Conical waves – Stationary waves

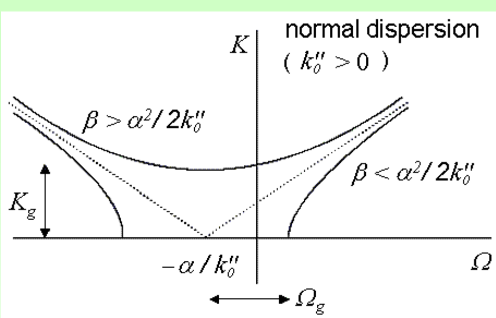
If $k_z(\omega)$ is linear in ω, the pulse is ***non-dispersive***

$$k_\perp(\omega) = \sqrt{k^2(\omega) - k_z^2(\omega)}$$

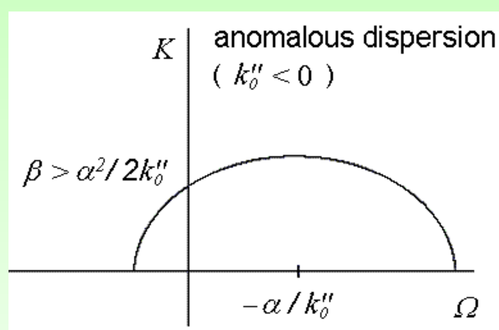
$$k_z(\omega) = k(\omega_0) + \frac{\omega - \omega_0}{V} \qquad V = \left(\frac{dk_z}{d\omega}\right)^{-1} = \text{const.}$$

The $(k_\perp,\omega)$ spectral shape is determined by the *Group Velocity Dispersion* of the medium

***k_0" > 0* normal GVD**
Hyperbolic profile
X-wave

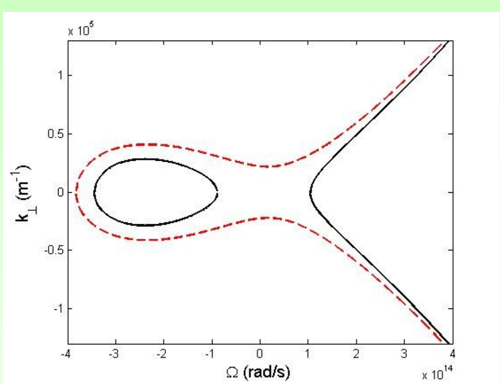


***k_0" < 0* anomalous GVD**
Elliptic profile
O-wave

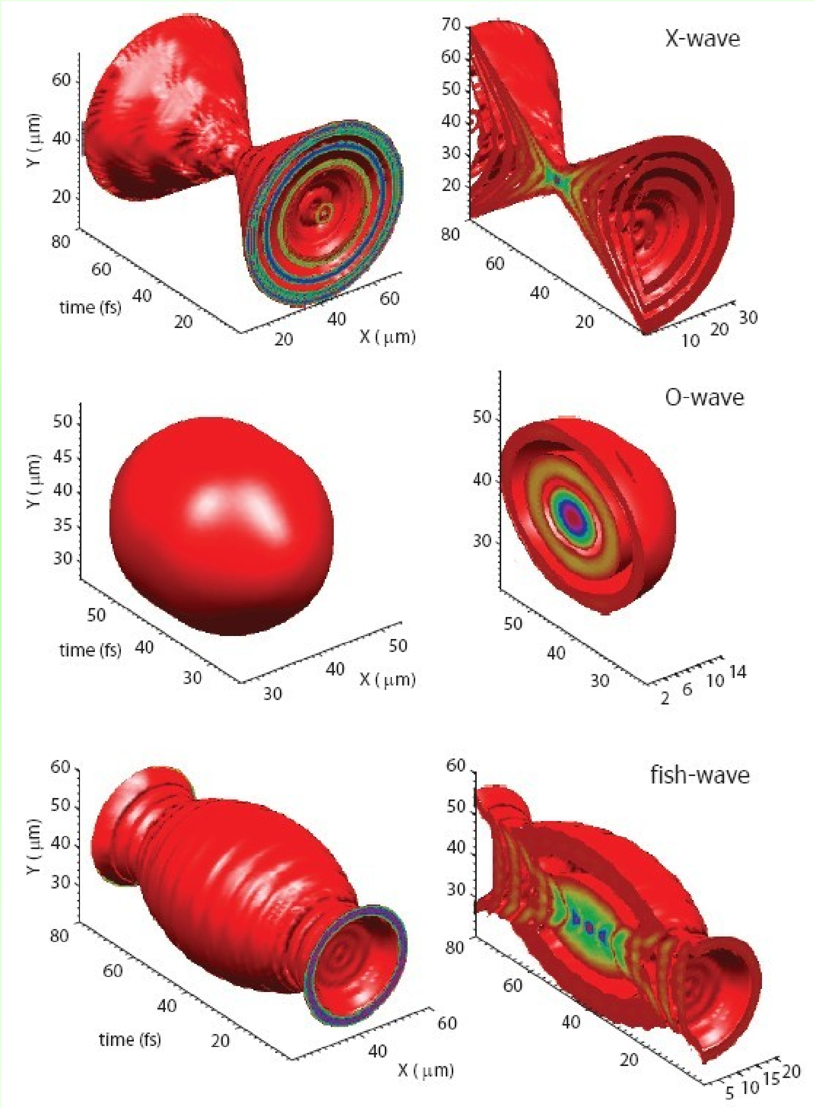


***k_0" ≈ 0* near-zero GVD**

Fish wave



Conical waves – Stationary waves



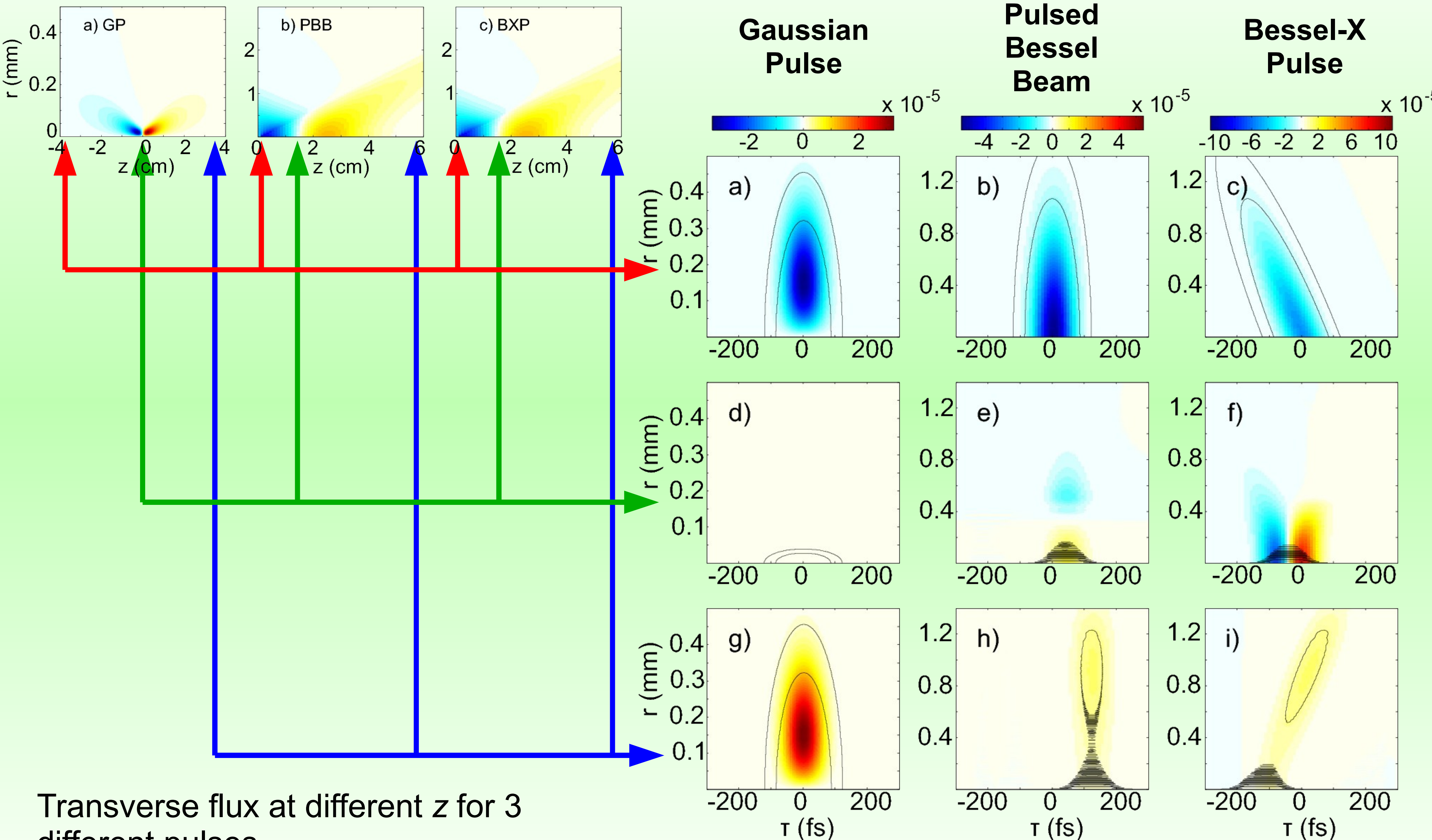
$$k_{\perp}(\omega) = \sqrt{k^2(\omega) - k_z^2(\omega)}$$

$$k_z(\omega) = k(\omega_0) + \frac{\omega - \omega_0}{V}$$

Superluminal propagation =
V > 1/k'$_0$ = Gaussian pulse group velocity

Subluminal propagation =
V < 1/k'$_0$ = Gaussian pulse group velocity

J_r : Monochromatic vs **Polychromatic**



Transverse flux at different *z* for 3 different pulses.

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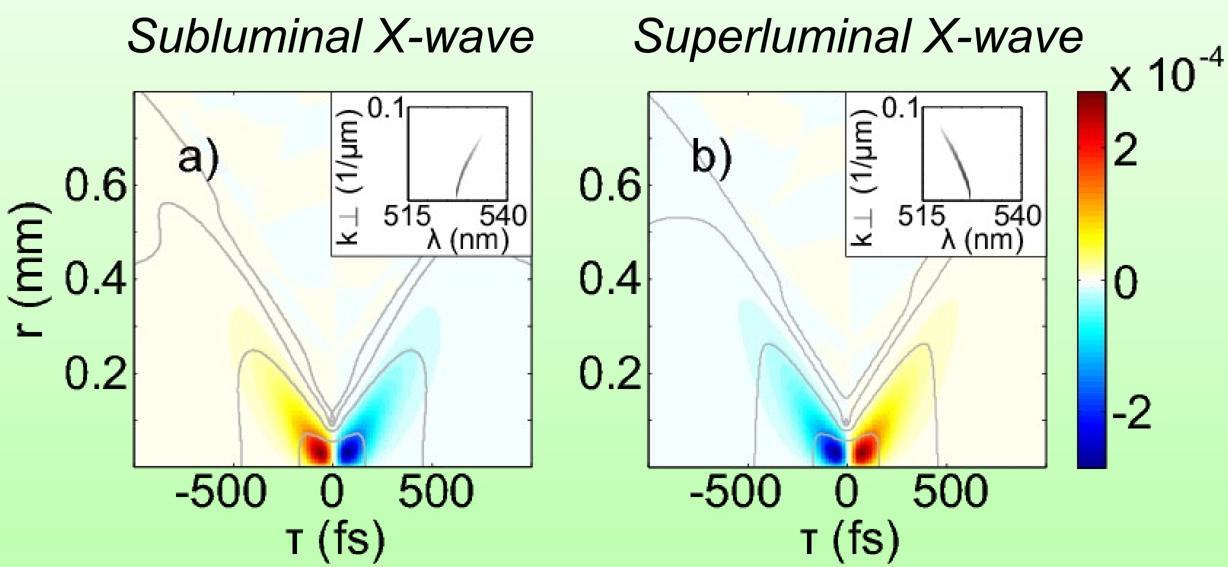
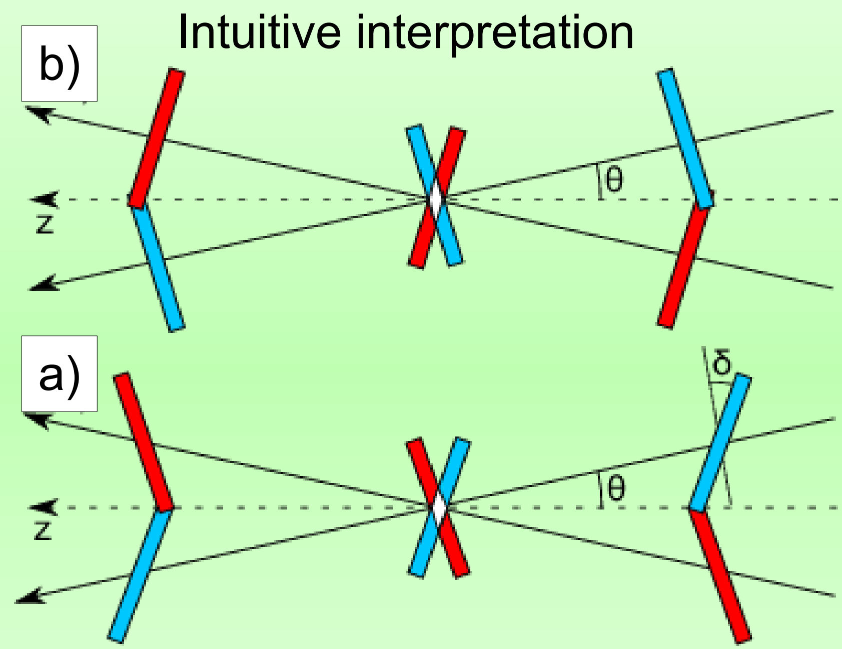
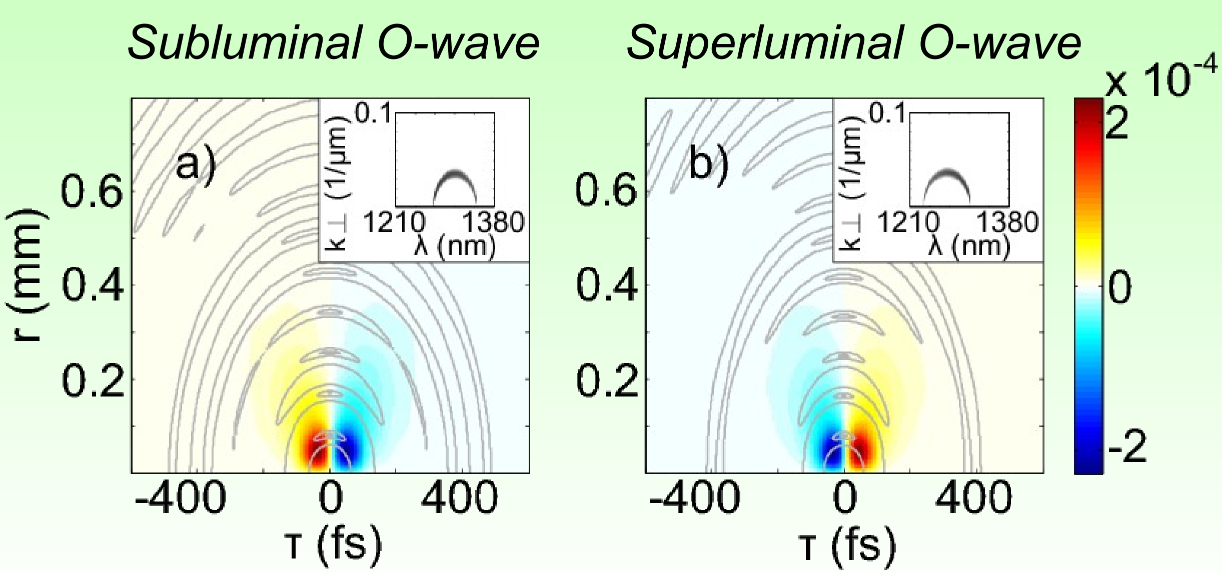
J_r : X-waves & O-waves

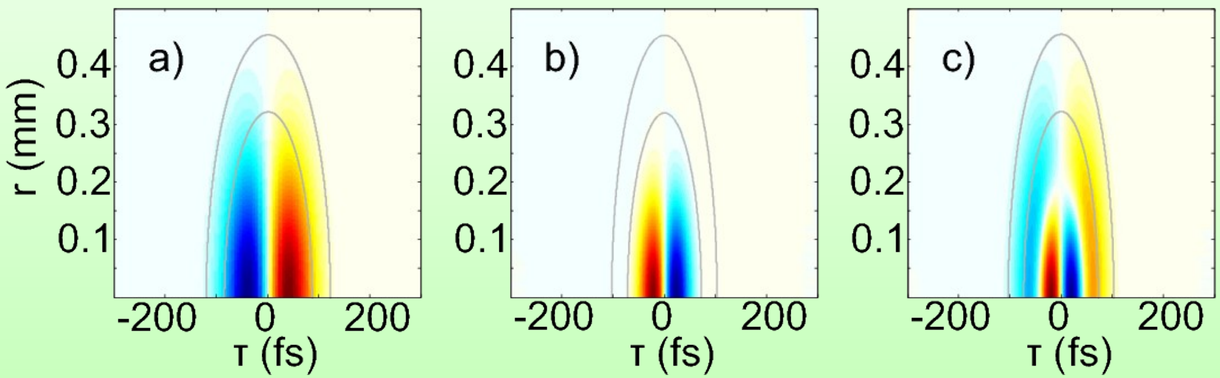






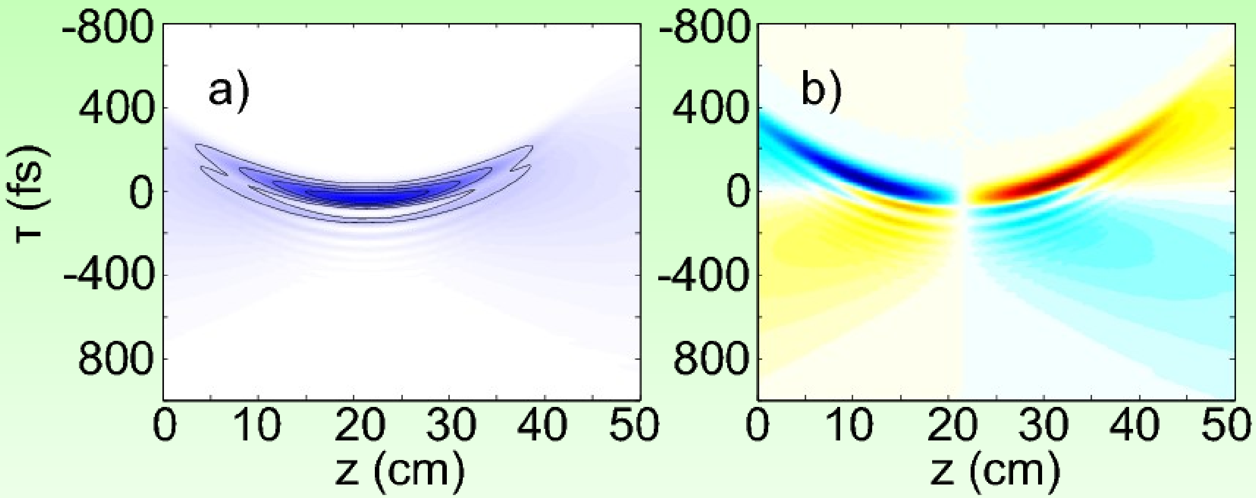
J_τ : Longitudinal component - Examples

Linear | *"strongly" Nonlinear + Anomalous GVD* | *"weakly" Nonlinear + Anomalous GVD*



Propagation of a Gaussian Pulse

Propagation of a Gaussian pulse with quadratic and cubic chirp



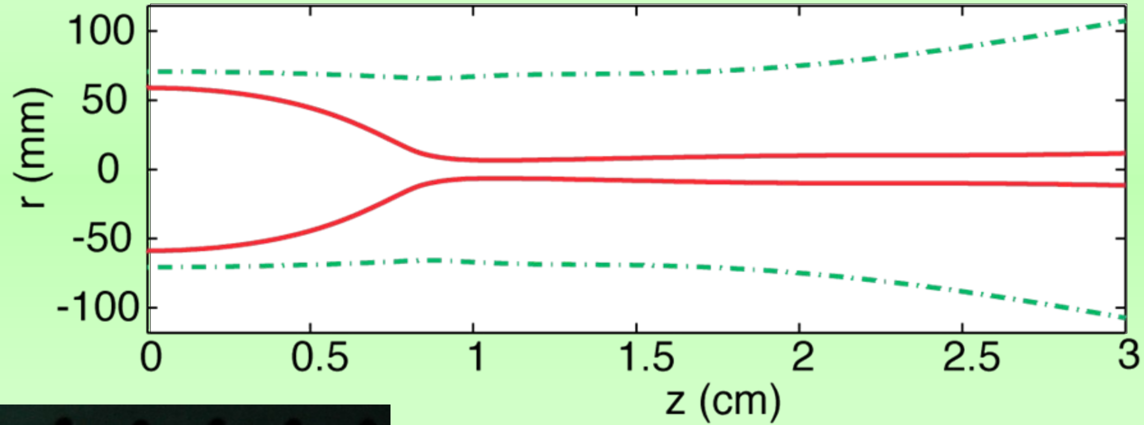
Intensity vs z | r-integrated J_τ vs z

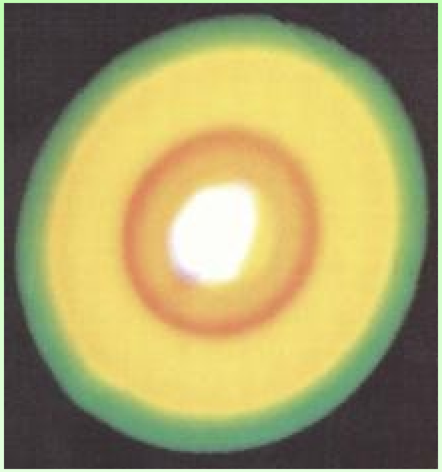
"Accelerating" intensity peak

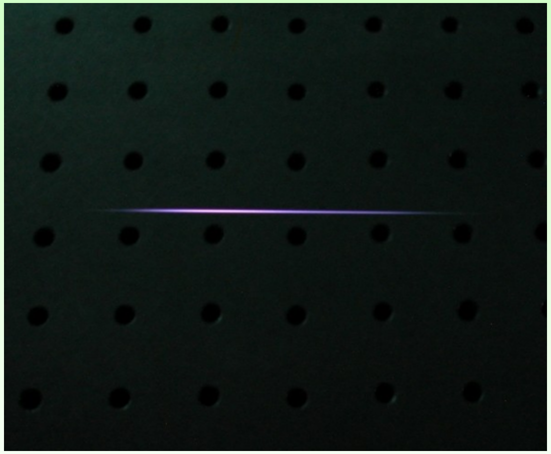
The Experiment : ultrashort laser pulse filamentation

"Dynamic regime in which an optical pulse propagating through a Kerr medium focuses and maintains a central narrow peak in the transverse plane for a length (much) longer than the length scale of diffraction."

$$P > P_{cr} = 3.77\frac{\lambda^2}{8\pi n_0 n_2}$$







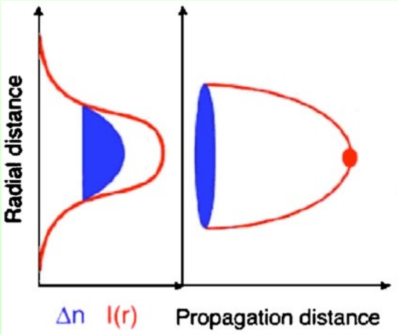
- *Self-healing* properties;
- Spectral broadening, *super-continuum* generation;
- *Conical emission*;
- *Plasma* channel;
- Temporal *splitting* & pulse compression.

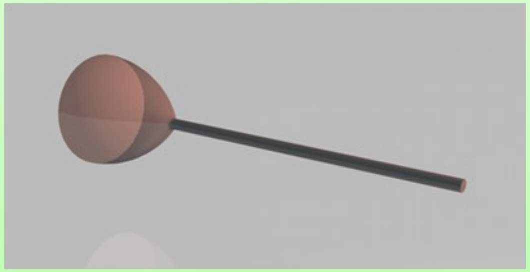
The Experiment : ultrashort laser pulse filamentation

Dynamical balancing between a series of linear and nonlinear effects.

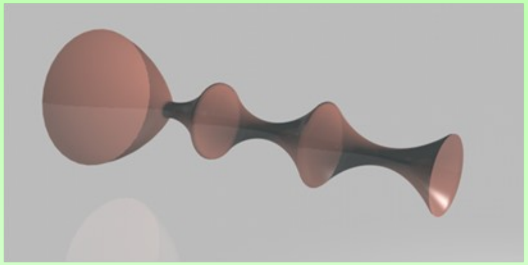
$$n = n(r,t) = n_0 + n_2 I(r,t)$$

Kerr self-focusing

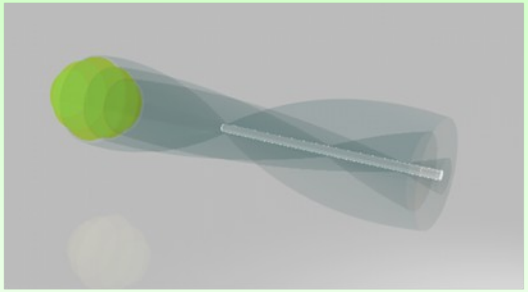




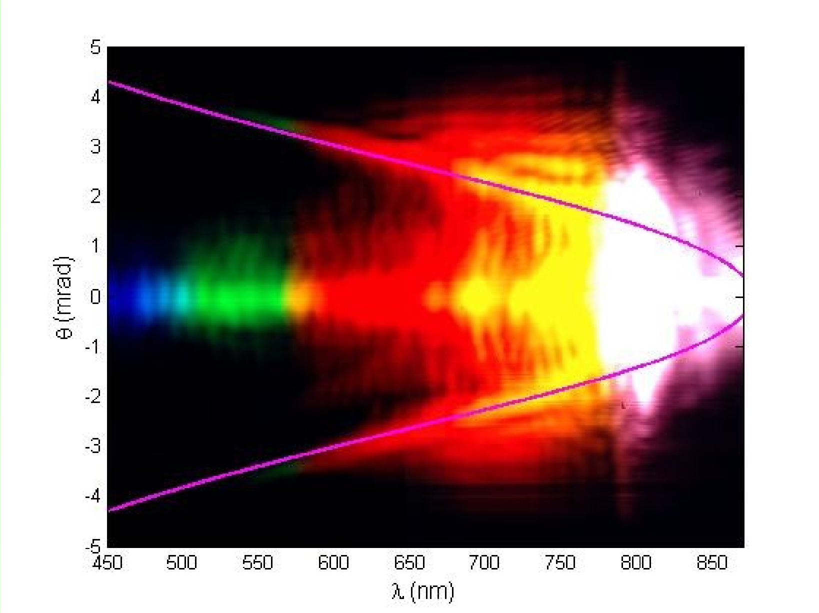
Townes soliton



Cyclic Kerr-focusing
& plasma defocusing



Moving focus

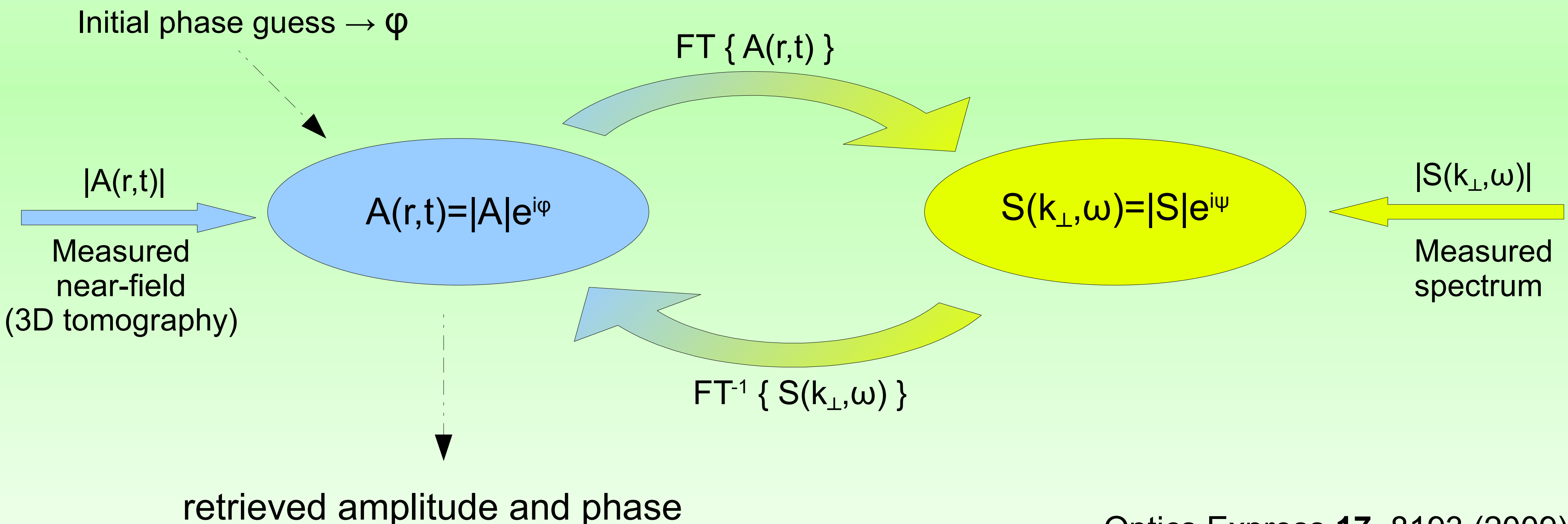


Spectral features interpreted as the spontaneous generation of effective X-waves.

Amplitude and phase retrieval

Error-reduction (Gerchberg-Saxton)
- Usually used to retrieve amplitude and phase in *(x,y)* spatial coordinates
- Generalization to *(x,t)* or *(r,t)* space-time coordinates

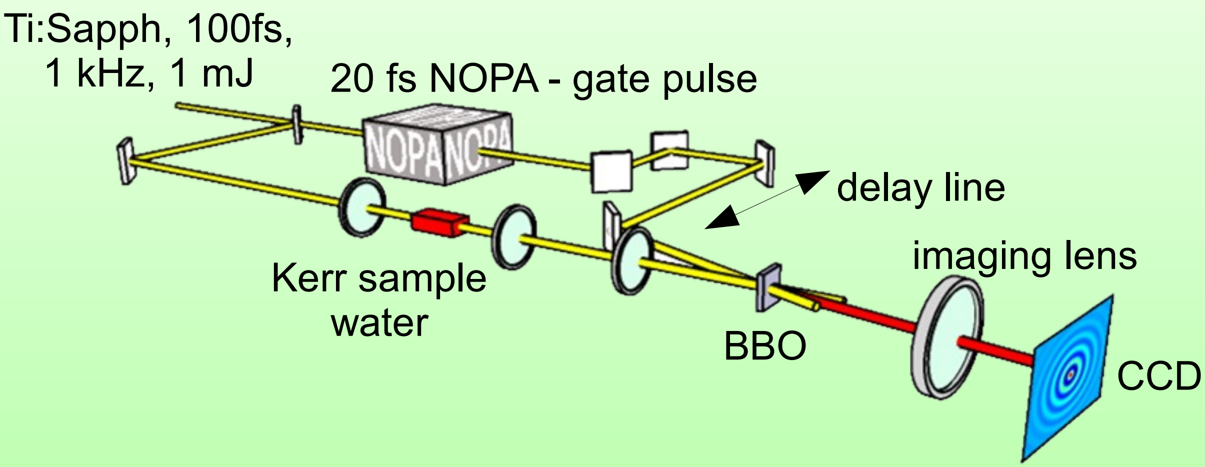
Input data - *Fourier pairs*:
- ***3D tomographic near-field (r,t) reconstruction***
- ***(θ,λ) spectral measurement***

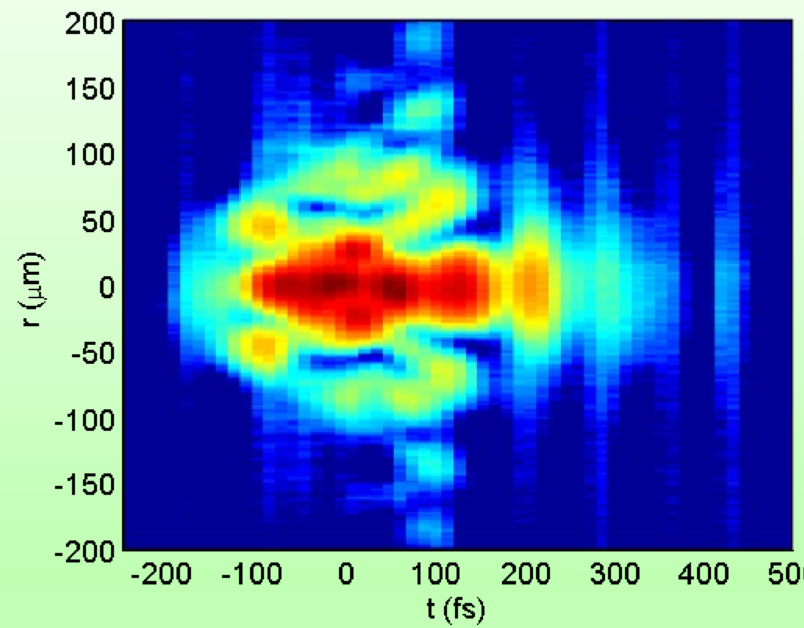


Optics Express **17**, 8193 (2009)

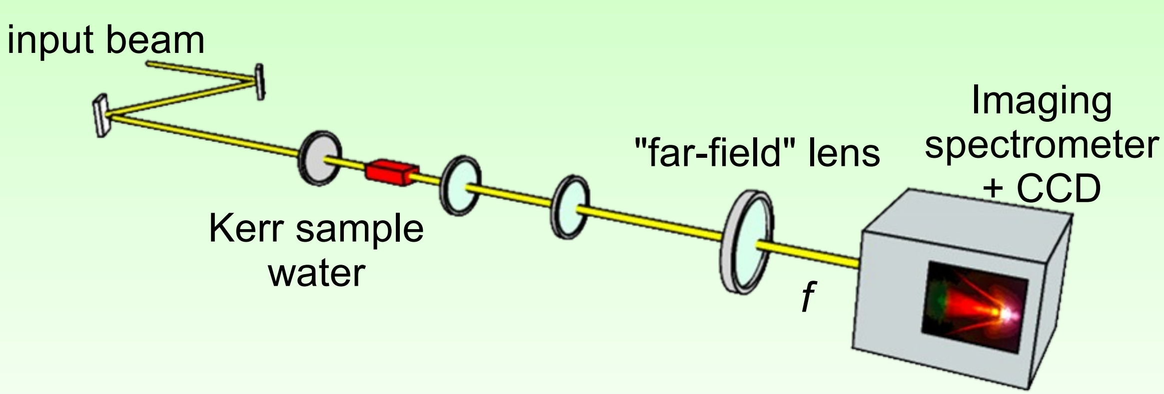
Measurements

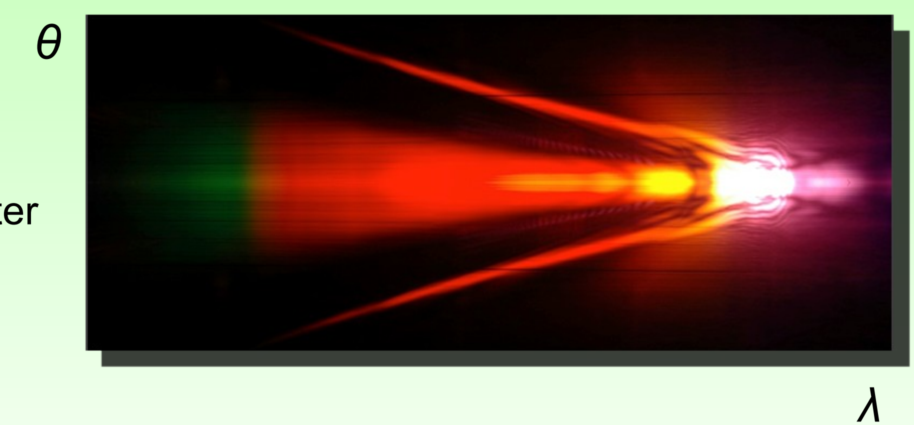
3D tomography





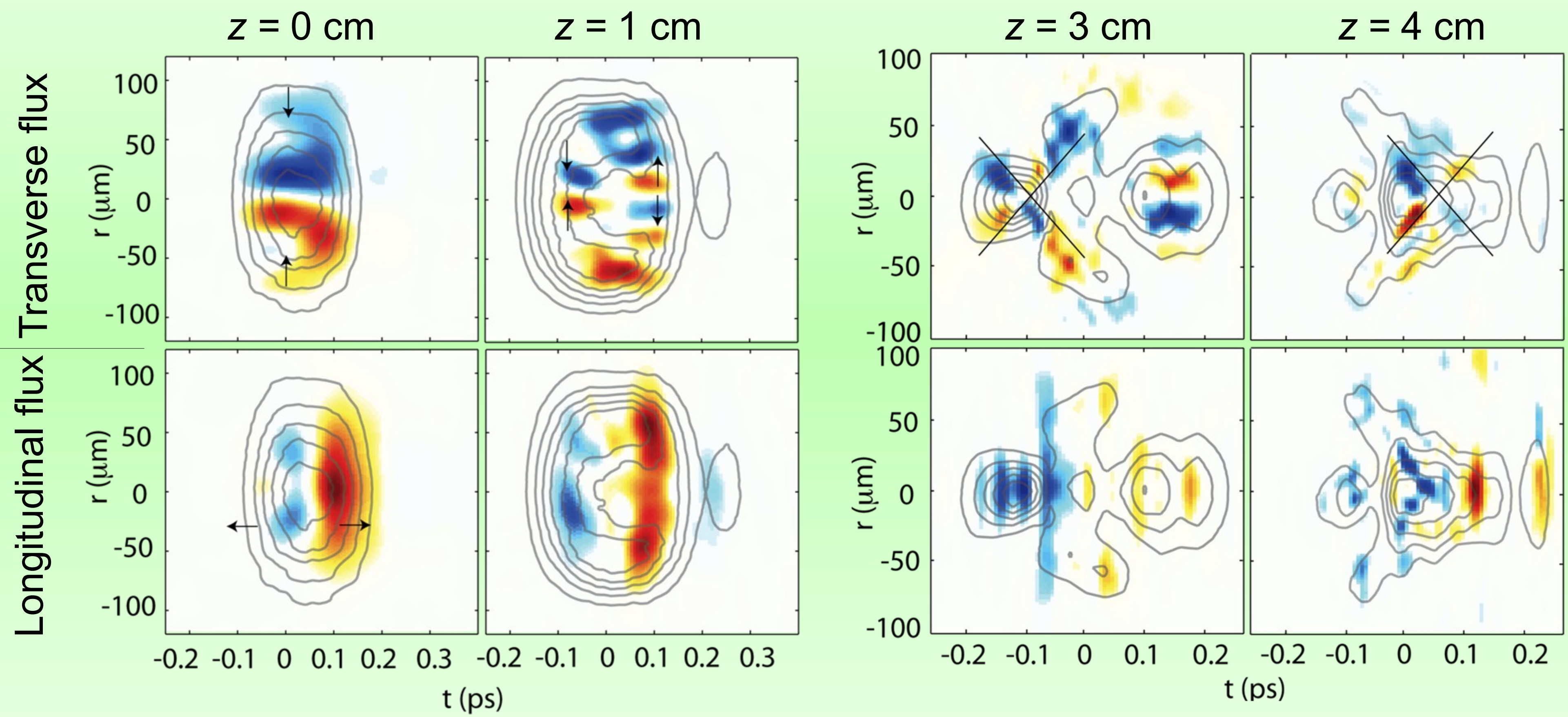
(θ,λ) Spectral Measurement





Experimental results

- Space-time amplitude and phase → energy density flux
  Kerr medium: water
  Laser pulse 800 nm, 3 μJ, 160 fs, focused with 50 cm lens

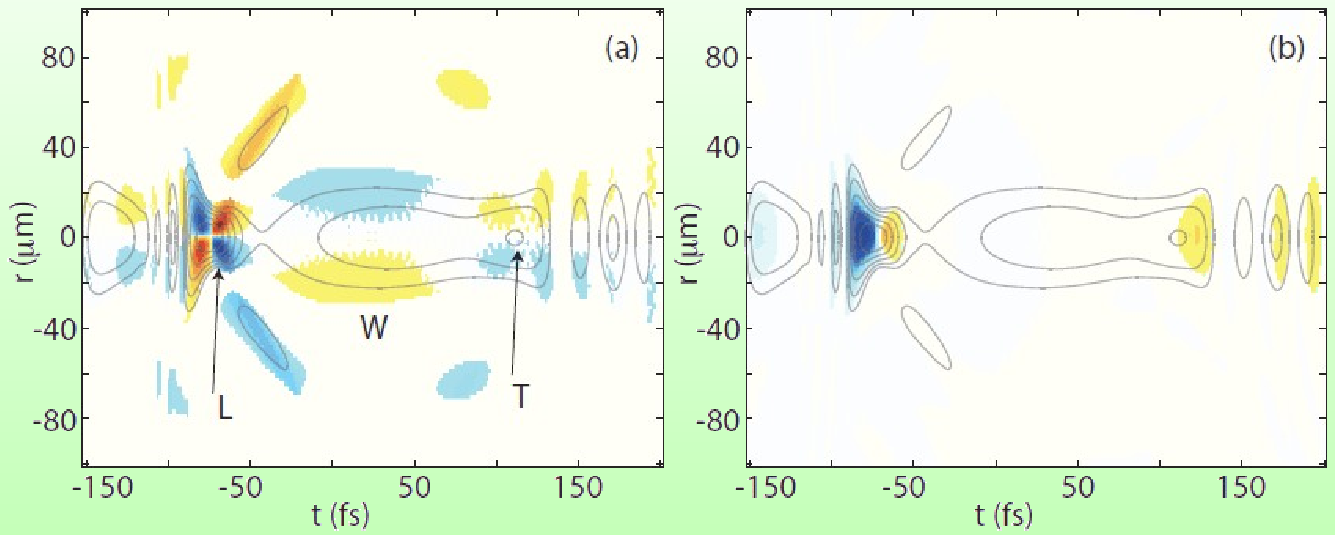


- All typical features – SF, plasma defocusing – of filamentation are clearly visible
- **Clear formation of a conical pulse with X-shaped flux**

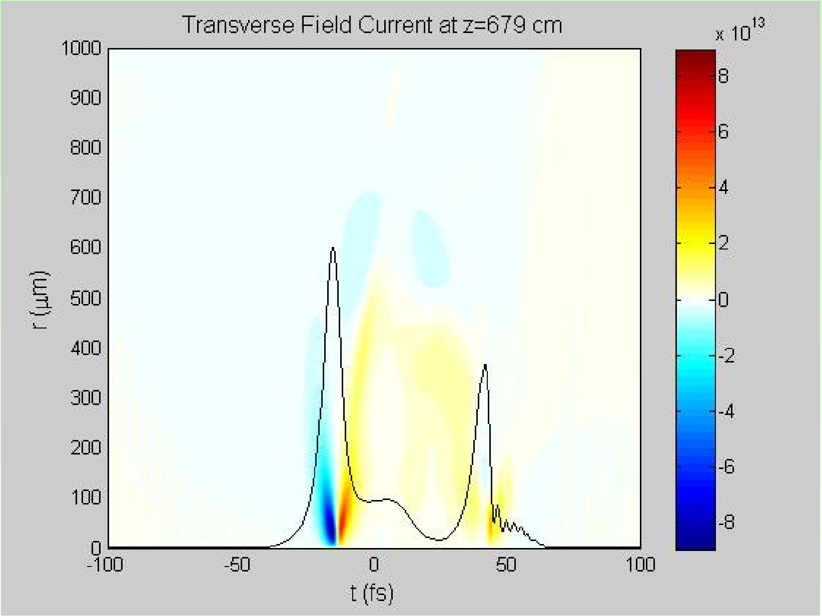
Numerical simulations

Using NLSE,
reproducing experimental conditions.

Water



Transverse flux: filamentation in *air*,
f = 10 m, 800 nm, 35 fs, 3 mJ



Conclusions – Perspectives

• Definition of ***space-time energy flux*** through the NLSE

• Valuable instrument to visualize ***phase information***

• ***Experimental analysis tool***, thanks to amplitude and phase retrieval techniques
(e.g. SEA TADPOLE, SEA SPIDER, STRIPED FISH, Shackled FROG,...)

• Continuity equation could be used for pulse characterization ?
3D tomography reconstruction of *I(r,t,z)* → find phase

$$\mathrm{div}\mathbf{J} = -\frac{dI}{dz}$$

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