Next-to-Leading Order QCD Corrections to Single Inclusive Hadron Production in Transversely Polarized pp and $\bar{p}p$ Collisions

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- Transversity and its measurement
- Problems with NLO calculation involving transverse polarization
- Recently proposed technique
- Application to single inclusive hadron production
- Summary and outlook

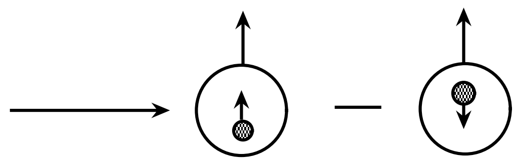
Transversity 2005

In collaboration with M. Stratmann (Regensburg) and W. Vogelsang (Brookhaven and Riken BNL)

What is Transversity ?

• Parton language : nucleon moving with (infinite) momentum along $\hat{z}$ direction but polarized in the transverse direction

Transversity $\delta q_a(x, Q^2) \rightarrow$ the number of partons of flavor a and momentum fraction x with spin parallel to the spin of the nucleon minus the number antiparallel



$$\delta f(x,\mu) = f_{\uparrow\uparrow}(x,\mu) - f_{\uparrow\downarrow}(x,\mu)$$

• Can be probed in

(1) Single transverse spin asymmetries in pp or ep scattering; azimuthal asymmetries..

(2) Double transverse spin asymmetries

Candidate processes $p^\uparrow p^\uparrow \rightarrow l^+ l^- X, p^\uparrow p^\uparrow \rightarrow \gamma X, p^\uparrow p^\uparrow \rightarrow jetX$...

Polarized Cross sections

- Factorization of polarized cross section :

$$\begin{aligned} d\Delta\sigma &= \frac{1}{4}\sum_{a,b}(f^{+}_{a,+} - f^{+}_{a,-}) \otimes (f^{+}_{b,+} - f^{+}_{b,-}) \\ &\otimes[d\hat{\sigma}(++) - d\hat{\sigma}(+-) - d\hat{\sigma}(-+) + d\hat{\sigma}(--)] \\ &= \sum_{a,b} \Delta f_a \otimes \Delta f_b \otimes d\Delta\hat{\sigma}_{ab} \end{aligned}$$

- Defines the spin-dependent cross section measurable in experiment
- We have used $f^{+}_{+} = f^{-}_{-}$ and $f^{+}_{-} = f^{-}_{+}$

Strong interactions invariant under parity and parity flips helicity

- QCD : Parity is conserved ; further simplification

$$d\Delta\sigma = \frac{1}{2}[d\sigma(++) - d\sigma(+-)]$$

$$d\Delta\hat{\sigma} = \frac{1}{2}[d\hat{\sigma}(++) - d\hat{\sigma}(+-)]$$

Double Transverse Spin Asymmetry

A_{TT} defined as :

$$A_{TT} = \frac{(\sigma_{++} + \sigma_{--}) - (\sigma_{+-} + \sigma_{-+})}{(\sigma_{++} + \sigma_{--}) + (\sigma_{+-} + \sigma_{-+})}$$

$+$ and $-$ $\rightarrow$ transverse spin directions of the beam proton.

• Chirality flipped twice at two soft distributions

✓ A_{TT} depends only on transversity (quadratically); not on any other unknown distributions
Cleanest possible extraction of transversity

$\times$ Small, because gluon initiated subprocesses contribute to the denominator but not to the numerator

Jaffe, Saito 96; Soffer, Stratmann, Vogelsang 02

Double Transverse Spin Asymmetry

✓ Only exception : Drell-Yan lepton pair production
At LO : $q\bar{q}$ annihilation; no gluon contribution to unpol. cross section

× RHIC : limited muon acceptance

× $\delta\bar{q}$ small

Martin, Schäfer, Stratmann, Vogelsang 98

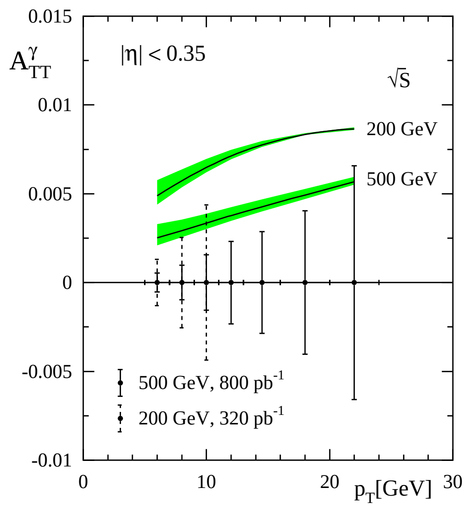
• Recent proposal : A_{TT} in $p^{\uparrow}\bar{p}^{\uparrow}$ processes at at GSI-FAIR facility
Theoretical work so far considered Drell-Yan

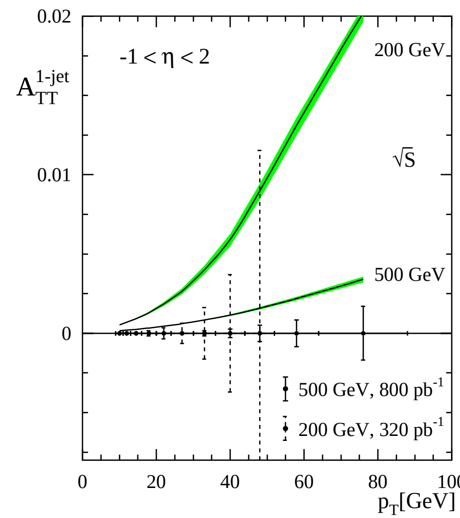
Anselmino, Barone, Drago, Nikolaev; Efremov, Goeke, Schweitzer 04;
Shimizu, Sterman, Vogelsang, Yokoya 05

× Much lower energy, higher twist effects...

✓ A_{TT} large

A_{TT} for other processes at RHIC :





Soffer, Stratmann, Vogelsang 02

- LO estimate by saturating Soffer's inequality at low input scale

$$f(x) + \Delta f(x) \geq 2 \mid \delta f(x) \mid$$

at higher scales transversity is obtained by solving the evolution eqn.

- Hard to detect : good control over systematic and statistical errors necessary
- Higher order correction a must : reduction of scale dependency ..
- Further motivation : technical challenge

Problem with Transverse Polarization Beyond LO

- Technical challenge for NLO calculation of cross sections involving transversely polarized particles in the initial state
- Spin vectors introduce extra spatial directions : nontrivial Φ dependence
- Assuming both initial spin vectors in $\pm x$ direction in cm frame of the initial hadrons; for a parity conserving theory with vector couplings

$$\frac{d^3\delta\sigma}{dp_T d\eta d\Phi} \equiv \cos(2\Phi) \left\langle \frac{d^2\delta\sigma}{dp_T d\eta} \right\rangle$$

- Φ cannot be integrated out
- Difficult to use standard tools for doing phase space integrations at NLO (especially for dimensional reg.)
- Need : A general technique to perform calculations at NLO with transverse polarization

Projection Technique for Azimuthal Dependence

Consider prompt photon production as an example $A + B \to \gamma + X$

$A, B \to$ Transversely polarized protons

Integrate with $cos 2\Phi$ weight :

$$\left\langle \frac{d^2\delta\sigma}{dp_T d\eta} \right\rangle = \frac{1}{\pi} \int_0^{2\pi} d\Phi \cos(2\Phi) \, \frac{d^3\delta\sigma}{dp_T d\eta d\Phi}$$

LO $\to q\bar{q} \to \gamma g$

Polarization for initial quark projected out by

$$u(p_a, s_a)\, \bar{u}(p_a, s_a) = \frac{1}{2} \not{p}_a \left[1 + \gamma_5 \not{s}_a\right]$$

Note : Covariant expression below give $cos 2\Phi$ in the c. m. frame of initial hadrons

$$\mathcal{F}(p_\gamma, s_a, s_b) = \frac{s}{\pi t u} \left[2\,(p_\gamma \cdot s_a)\,(p_\gamma \cdot s_b) + \frac{tu}{s}\,(s_a \cdot s_b) \right]$$

AM, Stratmann, Vogelsang 03

Projection Technique (Continued)

At LO for $q\bar{q} \to \gamma g$ we have

$$\frac{d\delta^2 \hat{\sigma}^{(0)}_{q\bar{q}\to\gamma g}}{dtd\Phi} = \frac{1}{32\pi^2 s^2}\,\delta|M(q\bar{q} \to \gamma g)|^2 \ ,$$

$$\delta|M(q\bar{q} \to \gamma g)|^2 = (ee_q g)^2\,\frac{4C_F}{N_C}\,\frac{s}{tu}\,\left[2\,(p_\gamma \cdot s_a)\,(p_\gamma \cdot s_b)\ +\ \frac{tu}{s}\,(s_a \cdot s_b)\right]$$

- Multiply $\delta|M|^2$ by $\mathcal{F}(p_\gamma, s_a, s_b)$

- Dependence on spin vectors : $(p_\gamma \cdot s_a)^2(p_\gamma \cdot s_b)^2$, $(p_\gamma \cdot s_a)(p_\gamma \cdot s_b)(s_a \cdot s_b)$, and $(s_a \cdot s_b)^2$

- Expand tensors $p_\gamma^\mu p_\gamma^\nu p_\gamma^\rho p_\gamma^\sigma$ and $p_\gamma^\mu p_\gamma^\nu$ into all possible tensors made up of the metric tensor and the incoming partonic momenta

Projection Technique (Continued)

$$\int d\Omega_\gamma \, (p_\gamma \cdot s_a)^2 (p_\gamma \cdot s_b)^2 = \int d\Omega_\gamma \, \frac{t^2 u^2}{8 s^2} \left(2(s_a \cdot s_b)^2 + s_a^2 s_b^2\right) \; = \; \int d\Omega_\gamma \, \frac{3 t^2 u^2}{8 s^2} \; ,$$

$$\int d\Omega_\gamma \, (p_\gamma \cdot s_a)(p_\gamma \cdot s_b)(s_a \cdot s_b) = - \int d\Omega_\gamma \, \frac{tu}{2s} (s_a \cdot s_b)^2 \; = \; - \int d\Omega_\gamma \, \frac{tu}{2s} \; ,$$

- $s_i \cdot p_a = s_i \cdot p_b = 0$ ($i = a, b$) and $s_a^2 = s_b^2 = -1$
- Now integrate phase space over p_γ covariantly including the (now trivial) azimuthal part
- Particularly suitable at NLO
- At NLO we have $ab \to \gamma cd$; one has to do

$$\int d\Omega_\gamma \int d\Omega_c \, \mathcal{F}(p_\gamma, s_a, s_b) \, \delta |M(ab \to \gamma cd)|^2$$

- Momentum of particle d fixed by momentum conservation
- Dimensional regularization : $d = 4 - 2\epsilon$ dimension
- Multiply by the projector and use similar tensor decompositions to remove terms containing s_i, then do the phase space integrations

Projection Technique : Continued

- Scalar products of s_i $(i = a, b)$ with p_c : $\propto (s_a \cdot p_c)(s_b \cdot p_c)$ and $\propto (s_i \cdot p_c)$ $\to$ removed by expanding the ensuing tensor and vector integrals in terms of the available tensors

- We are left with terms containing $(p_\gamma \cdot s_i)$: they are of the form $(p_\gamma \cdot s_a)^2 (p_\gamma \cdot s_b)^2$ and $(p_\gamma \cdot s_a)(p_\gamma \cdot s_b)$.

In d dimensions

$$\int \quad d\Omega_\gamma \quad (p_\gamma \cdot s_a)^2 (p_\gamma \cdot s_b)^2 = \int d\Omega_\gamma \, \frac{t^2 u^2}{4(1-\varepsilon)(2-\varepsilon)s^2} \left[2(s_a \cdot s_b)^2 + s_a^2 s_b^2 \right]$$

$$\int d\Omega_\gamma \, (p_\gamma \cdot s_a)(p_\gamma \cdot s_b)(s_a \cdot s_b) = - \int d\Omega_\gamma \, \frac{tu}{2(1-\varepsilon)s} (s_a \cdot s_b)^2$$

We can now integrate over all phase space using known techniques from unpolarized and longitudinally polarized cases

- Check : reproduces known NLO result for Drell-Yan transversity cross section

Application to Single Inclusive Pion Production

AM, Stratmann, Vogelsang 05

$A + B \to \pi + X$

$A, B \to$ Transversely polarized pp or $\bar{p}p$

Differential cross section

$$d\delta\sigma \quad = \quad \sum_{a,b,c} \delta f_a \otimes \delta f_b \otimes D_c^{\pi} \otimes d\delta\hat{\sigma}_{ab\to cX}$$

D_c^{π} : parton to pion fragmentation function
Five subprocesses contribute at NLO :

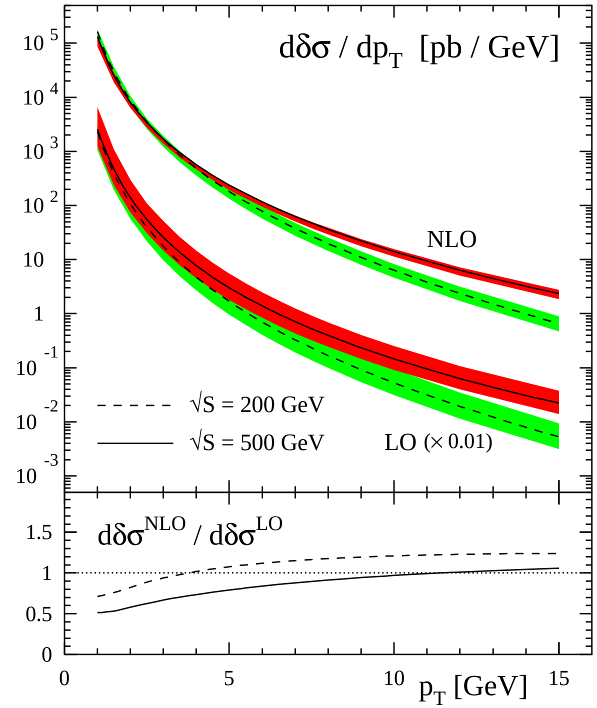
$$qq \;\to\; qX; \;\; q\bar{q} \to qX; \;\; q\bar{q} \;\to\; q'X; \;\; q\bar{q} \;\to\; gX; \;\; qq \to gX.$$

At NLO, X : one or two parton final state
First 4 already present at LO.

Used dimensional regularization, calculation done in $d = 4 - 2\epsilon$ dimension

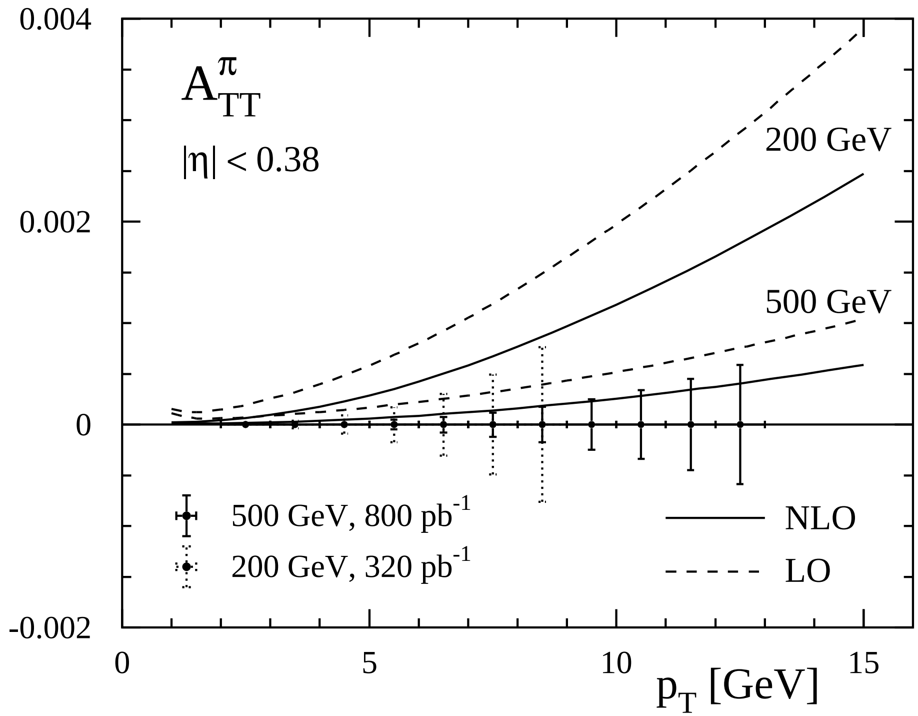
HVBM prescription of γ_5, projection technique

Numerical Results : Single Inclusive Pion Production at RHIC



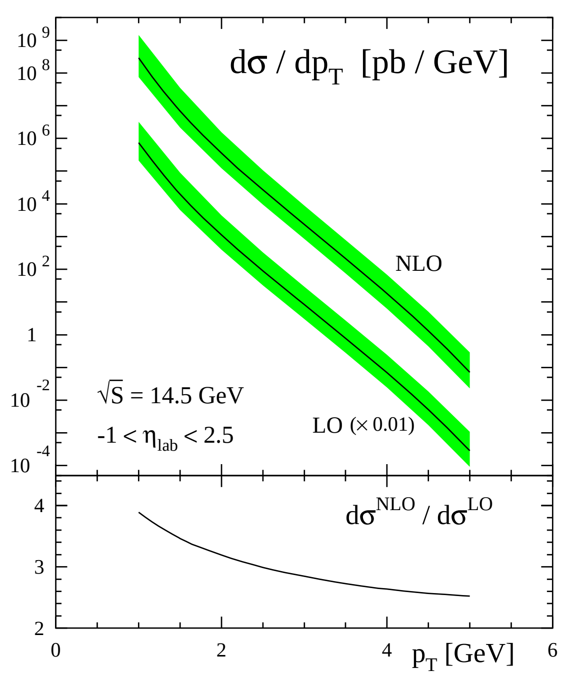
- PHENIX detector at RHIC : pseudorapidity $|\eta| \leq 0.38$; $-\pi/4 < \Phi < \pi/4$ and $3\pi/4 < \Phi < 5\pi/4$
- Saturate Soffer's inequality at a low input scale $\mu_0 \simeq 0.6\,\mathrm{GeV}$ using GRV and GRSV, for higher scale transversity density is obtained by solving the evolution eq.
- Scale : p_T to $4p_T$; K-factor at scale $\mu = 2p_T$
- Substantial reduction of scale dependency at NLO

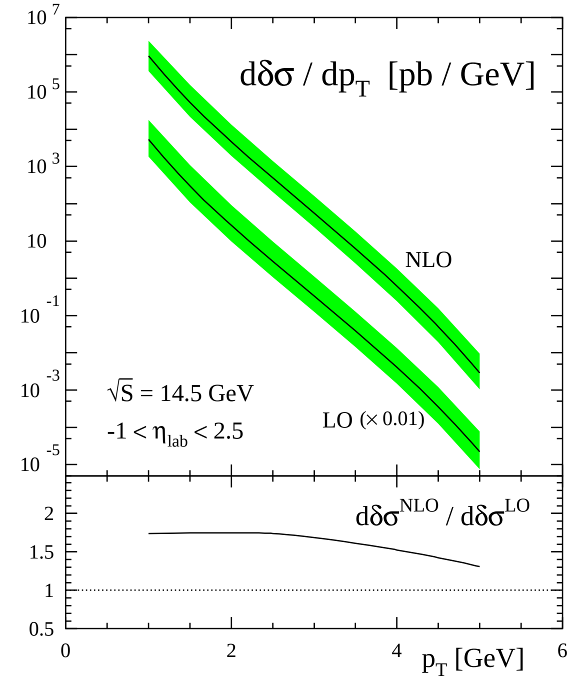
Numerical Results : Single Inclusive Pion Production at RHIC



- CTEQ6M pdfs, scale $\mu = p_T$ (largest cross sections)
- Statistical error : $\delta A^{\pi}_{\mathrm{TT}} \simeq \frac{1}{P^2\sqrt{\mathcal{L}\sigma_{\mathrm{bin}}}}$; Beam polarization 70%
- A^{π}_{TT} very small; technically challenging for measurement at RHIC : expected systematic error 10^{-3}

Numerical Results : Single Inclusive Pion Production at GSI



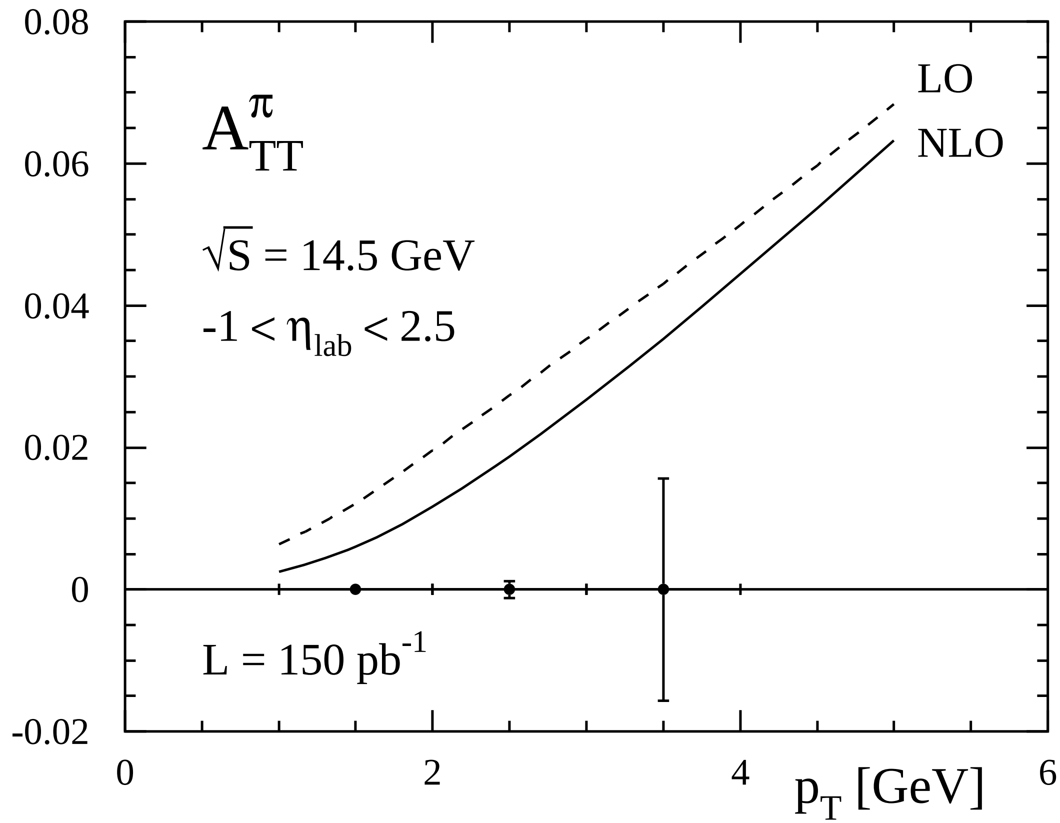


- $\bar{p}p$ collider at $\sqrt{S} = 14.5$ GeV ; $E_p = 3.5$ GeV (pol. 50 %), $E_{\bar{p}} = 15$ GeV (pol. 30 %)

$\eta_{lab} = \eta + \frac{1}{2} Log \frac{E_{\bar{p}}}{E_p}$ gives $| \, \eta \, | \leq 1.75$.

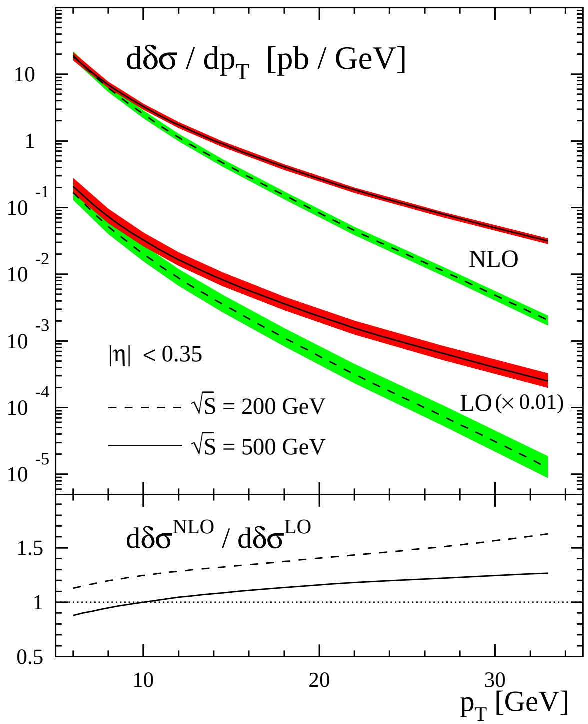
- Unpol : GRV, pol : saturating Soffer's inequality as before with GRV and GRSV
- Scale : p_T to $4p_T$: no improvement in scale dependence at NLO; resummation of higher logs and higher twist effects need to be investigated

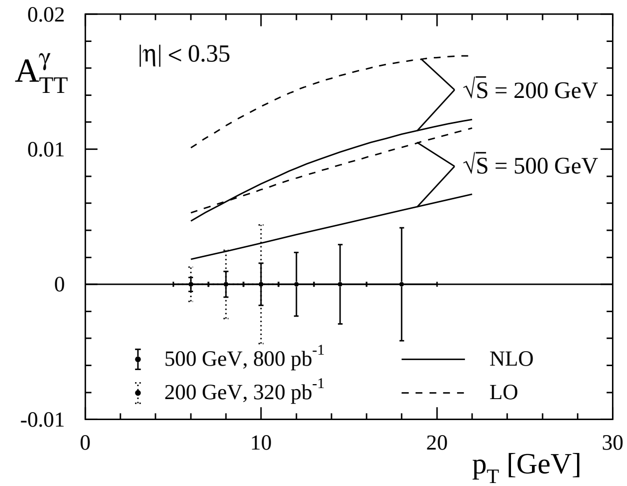
Numerical Results : Single Inclusive Pion Production at GSI



- GRV pdfs
- Statistical error : $\delta A^{\pi}_{\mathrm{TT}} \simeq \frac{1}{P_1\, P_2 \sqrt{\mathcal{L}\sigma_{\mathrm{bin}}}}$
- All scales set to p_T

Numerical Results : Isolated Prompt Photon Production at RHIC





- Isolation cuts to separate the photon signal from hadronic background
- PHENIX detector at RHIC : pseudorapidity $|\eta| \leq 0.35$; $-\pi/4 < \Phi < \pi/4$ and $3\pi/4 < \Phi < 5\pi/4$
- Scale : $p_T/2$ to $2p_T$, reduction of scale dependency

Summary

• Measurement of transversity by measuring double transverse spin asymmetries : depend only on transversity, no unknown functions; cross section can be factorized

• Presented first calculation of cross sections and spin asymmetries for single inclusive pion production in transversely polarized pp and $p\bar{p}$ collisions at NLO using recently proposed projection technique

• Upper bound of the asymmetry assuming the saturation of Soffer's bound :A_{TT}^{π} very small at RHIC, measurement is challenging.

• A_{TT}^{γ} slightly larger, good control over systematic error necessary

•Substantial reduction of scale dependency at NLO

• A_{TT}^{π} large in $\bar{p}p$ collisions at GSI ($\sqrt{S} = 14.5$ GeV); no reduction of scale dependency at NLO, needs further study.

• Important : also need precise measurement of the unpolarized cross section at GSI to test the theoretical framework

•Combined information from Drell-Yan and single inclusive hadron production at GSI use-

ful -> probe transversity in different kinematical regions