Helicity formalism and spin asymmetries in hadronic processes

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More details and a complete list of references can be found here:

1. U. D'Alesio, FM: PRD70 (2004) 074009;
2. M. Anselmino, M. Boglione, U. D'Alesio, E. Leader, FM: PRD71 (2005) 041002;
3. **M. Anselmino, M. Boglione, U. D'Alesio, E. Leader, S. Melis, FM: hep-ph/0509035;**

Summary

- Basics of the approach and master formula for the $AB \to C+X$ process
- Helicity formalism:
  - Helicity density matrices for partons and hadrons
  - Description of quark and gluon polarization states
  - Helicity amplitudes for the soft $A(B) \to a(b) + X$, $c \to C + X$ processes
  - Helicity amplitudes for the hard elementary scattering $ab \to cd$
  - Polarized TMD parton distribution (PDF) and fragmentation functions (FF)
- General expression of kernels for the process $A(S_A)+B(S_B) \to C+X$
- Application: transverse SSA for $pp \to \pi + X$ process:
  - Numerical estimates of all (maximized) contributions to the numerator and the denominator of the SSA
  - Results for FNAL-E704, STAR-RHIC and PAX-GSI kinematical configurations

Conclusions and outlook

Basics of the approach and master equation

The cross section for the polarized inclusive process $A(S_A) + B(S_B) \to C + X$ can be given as a LO (factorized) convolution of all possible hard elementary QCD processes, $ab \to cd$, with soft (leading-twist), spin and $k_\perp$ dependent, PDF and FF:

$$\frac{E_C\, d\sigma^{(A,S_A)+(B,S_B)\to C+X}}{d^3\boldsymbol{p}_C} = \sum_{a,b,c,d,\{\lambda\}} \int \frac{dx_a\, dx_b\, dz}{16\pi^2 x_a x_b z^2 s}\, d^2\boldsymbol{k}_{\perp a}\, d^2\boldsymbol{k}_{\perp b}\, d^3\boldsymbol{k}_{\perp C}\, \delta(\boldsymbol{k}_{\perp C}\cdot\hat{\boldsymbol{p}}_c)\, J(\boldsymbol{k}_{\perp C})$$
$$\times\ \rho^{a/A,S_A}_{\lambda_a,\lambda'_a}\, \hat{f}_{a/A,S_A}(x_a,\boldsymbol{k}_{\perp a})\, \rho^{b/B,S_B}_{\lambda_b,\lambda'_b}\, \hat{f}_{b/B,S_B}(x_b,\boldsymbol{k}_{\perp b})$$
$$\times\ \hat{M}_{\lambda_c,\lambda_d;\lambda_a,\lambda_b}\, \hat{M}^*_{\lambda'_c,\lambda_d;\lambda'_a,\lambda'_b}\, \delta(\hat{s}+\hat{t}+\hat{u})\, \hat{D}^{\lambda_C,\lambda_C}_{\lambda_c,\lambda'_c}(z,\boldsymbol{k}_{\perp C})$$

- A, B are initial spin ½ hadrons (typically two protons) in pure spin states S_A and S_B;
- C is the observed, unpolarized hadron, typically a pion [extension to spin ½ (hyperons) and spin 1 hadrons is in progress];
-$J(k_{\perp C})$ is a phase-space kinematical factor [$J(k_{\perp C}) \to 1$ for massless partons/hadrons and collinear configuration];
-We consider the process in the AB (hadronic) c.m. frame, assuming that hadron A moves along the $+Z_{cm}$ axis, and hadron C is produced in the $(XZ)_{cm}$ plane with $(p_C)_{X_{cm}} > 0$.

$\rho^{a/A,S_a}_{\lambda_a,\lambda'_a}\, \hat{f}_{a/A,S_A}(x_a,\boldsymbol{k}_{\perp a})$

Contains all information on parton a and its polarization state through its helicity density matrix and the leading-twist, spin and $k_\perp$ dependent PDF [analogously for parton b]

$\widehat{M}_{\lambda_c,\lambda_d;\lambda_a,\lambda_b}$ Helicity amplitudes for the (LO) elementary process ab $\to$ cd

$\widehat{D}^{\lambda_C,\lambda'_C}_{\lambda_c,\lambda'_c}(z,\boldsymbol{k}_{\perp C})$ Product of (soft) helicity fragmentation amplitudes for the c $\to$ C + X process

- All transverse motions of partons explicitely taken into account; exact non collinear kinematics in the elementary interactions
- Formal proof of factorization for non collinear kinematics missing [see, however, work by Collins, Ji and collab., for DIS, SIDIS and DY processes]
- Universality and evolution properties of the spin and TMD PDF and FF not well known
- Consistent higher-twist treatment of the process still missing [only enhanced higher-twist contributions included]
- Relation with calculations in the collinear approach including resummed higher-order perturbative corrections to be clarified [Vogelsang, de Florian]

$\hat{f}^a_{s_i/S_J}$ number density of partons a with spin along the i-axis (in the parton hel. frame) inside hadron A with spin along the J-axis (in the hadron hel. rest frame)

$$P^a_j\,\hat{f}_{a/A,S_A} = \hat{f}^a_{s_j/A,S_A} - \hat{f}^a_{-s_j/A,S_A} \equiv \Delta\hat{f}^a_{s_j/A,S_A}$$

Polarized TMD parton distribution functions (leading twist)

Helicity density matrix of quark a:

$$\rho^{a/A,S_A}_{\lambda_a,\lambda'_a} = \begin{pmatrix} \rho^a_{++} & \rho^a_{+-} \\ \rho^a_{-+} & \rho^a_{--} \end{pmatrix}_{A,S_A} = \frac{1}{2}\begin{pmatrix} 1+P^a_z & P^a_x - iP^a_y \\ P^a_x + iP^a_y & 1-P^a_z \end{pmatrix}_{A,S_A} = \frac{1}{2}\begin{pmatrix} 1+P^a_L & P^a_T\, e^{-i\phi_{s_a}} \\ P^a_T\, e^{i\phi_{s_a}} & 1-P^a_L \end{pmatrix}_{A,S_A}$$

Introducing the (nonperturbative) helicity amplitudes for the process A $\to$ a + X, we can write

$$\begin{aligned} \rho^{a/A,S_A}_{\lambda_a,\lambda'_a}\, \hat{f}_{a/A,S_A}(x_a,\boldsymbol{k}_{\perp a}) &= \sum_{\lambda_A,\lambda'_A} \rho^{A,S_A}_{\lambda_A,\lambda'_A} \sum\!\!\!\!\!\!\int_{X_A,\lambda_{X_A}} \hat{\mathcal{F}}_{\lambda_a,\lambda_{X_A};\lambda_A}\, \hat{\mathcal{F}}^*_{\lambda'_a,\lambda_{X_A};\lambda'_A} \\ &\equiv \sum_{\lambda_A,\lambda'_A} \rho^{A,S_A}_{\lambda_A,\lambda'_A}\, \hat{F}^{\lambda_a,\lambda'_a}_{\lambda_A,\lambda'_A} \end{aligned}$$

The **helicity density matrix of initial hadron A** is given by:

$$\rho^{A,S_A}_{\lambda_A,\lambda'_A} = \frac{1}{2}\begin{pmatrix} 1+P^A_Z & P^A_X - iP^A_Y \\ P^A_X + iP^A_Y & 1-P^A_Z \end{pmatrix}_{A,S_A} = \frac{1}{2}\begin{pmatrix} 1+P^A_L & P^A_T\, e^{-i\phi_{S_A}} \\ P^A_T\, e^{i\phi_{S_A}} & 1-P^A_L \end{pmatrix}_{A,S_A}$$

$$\hat{F}^{\lambda_a,\lambda'_a}_{\lambda_A,\lambda'_A} = \sum\!\!\!\!\!\!\!\int_{X_A,\lambda_{X_A}} \hat{\mathcal{F}}_{\lambda_a,\lambda_{X_A};\lambda_A} \hat{\mathcal{F}}^*_{\lambda'_a,\lambda_{X_A};\lambda'_A}$$

The $\hat{\mathcal{F}}_{\lambda_a,\lambda_{X_A};\lambda_A}$ are the (soft) helicity distribution amplitudes for the A $\to$ a + X process. From their general properties one can easily see that the following relations hold:

$$\hat{F}^{\lambda'_a,\lambda_a}_{\lambda'_A,\lambda_A} = \left(\hat{F}^{\lambda_a,\lambda'_a}_{\lambda_A,\lambda'_A}\right)^*$$

$$\hat{F}^{\lambda_a,\lambda'_a}_{\lambda_A,\lambda'_A}(x_a,\boldsymbol{k}_{\perp a}) = F^{\lambda_a,\lambda'_a}_{\lambda_A,\lambda'_A}(x_a,k_{\perp a})\exp[i(\lambda_A-\lambda'_A)\phi_a]$$

$$F^{-\lambda_a,-\lambda'_a}_{-\lambda_A,-\lambda'_A} = (-1)^{2(S_A-s_a)}(-1)^{(\lambda_A-\lambda_a)+(\lambda'_A-\lambda'_a)}F^{\lambda_a,\lambda'_a}_{\lambda_A,\lambda'_A}$$

This leaves eight independent functions, corresponding to the eight leading-twist TMD PDF:

$$F^{++}_{++} \,(\pm)\, F^{++}_{--}\,,\;\; F^{+-}_{+-} \,(\pm)\, F^{-+}_{+-}\,,\;\; F^{++}_{+-} \,(\pm)\, F^{--}_{+-}\,,\;\; F^{+-}_{++} \,(\pm)\, F^{+-}_{--}$$

$$\mathrm{Re}F^{++}_{+-}\,,\;\; \mathrm{Im}F^{++}_{+-}\,,\;\; \mathrm{Re}F^{+-}_{++}\,,\;\; \mathrm{Im}F^{+-}_{++}$$

By looking at the helicity indices it is easy to see to which of the PDF’s they are associated.

quark sector

$$
\begin{aligned}
\hat{f}_{a/A} &= \hat{f}_{a/A,S_L} = \left(F^{++}_{++} + F^{++}_{--}\right) \\
\hat{f}_{a/A,S_T} &= \hat{f}_{a/A} + \frac{1}{2}\,\Delta\hat{f}_{a/S_T} = \left(F^{++}_{++} + F^{++}_{--}\right) + 2\,\left[\mathrm{Im}F^{++}_{+-}\,\sin(\phi_{S_A} - \phi_a)\right] \\
P^a_x\,\hat{f}_{a/A,S_L} &= \Delta\hat{f}_{s_x/S_L} = 2\,\mathrm{Re}F^{+-}_{++} \\
P^a_x\,\hat{f}_{a/A,S_T} &= \Delta\hat{f}_{s_x/S_T} = \left[F^{+-}_{+-} + F^{-+}_{+-}\right]\,\cos(\phi_{S_A} - \phi_a) \\
P^a_y\,\hat{f}_{a/A,S_L} &= P^a_y\,\hat{f}_{a/A} = \Delta\hat{f}_{s_y/S_L} = -2\,\mathrm{Im}F^{+-}_{++} \\
P^a_y\,\hat{f}_{a/A,S_T} &= \Delta\hat{f}_{s_y/S_T} = -2\,\mathrm{Im}F^{+-}_{++} + \left[F^{+-}_{+-} - F^{-+}_{+-}\right]\,\sin(\phi_{S_A} - \phi_a) \\
P^a_z\,\hat{f}_{a/A,S_L} &= \Delta\hat{f}_{s_z/S_L} = \left(F^{++}_{++} - F^{++}_{--}\right) \\
P^a_z\,\hat{f}_{a/A,S_T} &= \Delta\hat{f}_{s_z/S_T} = 2\,\left[\mathrm{Re}F^{++}_{+-}\,\cos(\phi_{S_A} - \phi_a)\right]
\end{aligned}
$$

Amsterdam Group Notation

$$
\begin{aligned}
f_1(x_a, k_{\perp a}) &= F^{++}_{++} + F^{++}_{--} \\
\frac{k_{\perp a}}{M}\,f^{\perp}_{1T}(x_a, k_{\perp a}) &= -2\,\mathrm{Im}F^{++}_{+-} \\
g_{1L}(x_a, k_{\perp a}) &= F^{++}_{++} - F^{++}_{--} \\
\frac{k_{\perp a}}{M}\,g^{\perp}_{1T}(x_a, k_{\perp a}) &= 2\,\mathrm{Re}F^{++}_{+-}
\end{aligned}
$$

$$
\begin{aligned}
\frac{k_{\perp a}}{M}\,h^{\perp}_{1L}(x_a, k_{\perp a}) &= 2\,\mathrm{Re}F^{+-}_{++} \\
\frac{k_{\perp a}}{M}\,h^{\perp}_{1}(x_a, k_{\perp a}) &= 2\,\mathrm{Im}F^{+-}_{++} \\
h_1(x_a, k_{\perp a}) &= F^{+-}_{+-} \\
\frac{k^2_{\perp a}}{2M^2}\,h^{\perp}_{1T}(x_a, k_{\perp a}) &= F^{-+}_{+-}
\end{aligned}
$$

gluon sector

$$\rho^{g/A,S_A}_{\lambda_g,\lambda'_g} = \frac{1}{2}\begin{pmatrix} 1+P^g_z & \mathcal{T}^g_1 - i\mathcal{T}^g_2 \\ \mathcal{T}^g_1 + i\mathcal{T}^g_2 & 1-P^g_z \end{pmatrix}_{A,S_A} = \frac{1}{2}\begin{pmatrix} 1+P^g_{circ} & -P^g_{lin}\,e^{-2i\phi} \\ -P^g_{lin}\,e^{2i\phi} & 1-P^g_{circ} \end{pmatrix}_{A,S_A}$$

$$\begin{aligned}
\hat{f}_{g/A} &= \hat{f}_{g/A,S_L} = \left(F^{++}_{++} + F^{++}_{--}\right) \\
\hat{f}_{g/A,S_T} &= \hat{f}_{g/A} + \frac{1}{2}\Delta\hat{f}_{g/A,S_T} = \left(F^{++}_{++} + F^{++}_{--}\right) + 2\,\left[\mathrm{Im}F^{++}_{+-}\sin(\phi_{S_A} - \phi_a)\right] \\
\mathcal{T}^g_1\,\hat{f}_{g/A,S_L} &= \mathcal{T}^g_1\,\hat{f}_{g/A} = \Delta\hat{f}^g_{\mathcal{T}_1/S_L} = 2\,\mathrm{Re}F^{+-}_{++} \\
\mathcal{T}^g_1\,\hat{f}_{g/A,S_T} &= \Delta\hat{f}^g_{\mathcal{T}_1/S_T} = 2\,\mathrm{Re}F^{+-}_{++} + \mathrm{Im}\left[F^{+-}_{+-} + F^{-+}_{+-}\right]\sin(\phi_{S_A} - \phi_a) \\
\mathcal{T}^g_2\,\hat{f}_{g/A,S_L} &= \Delta\hat{f}^g_{\mathcal{T}_2/S_L} = -2\,\mathrm{Im}F^{+-}_{++} \\
\mathcal{T}^g_2\,\hat{f}_{g/A,S_T} &= \Delta\hat{f}^g_{\mathcal{T}_2/S_T} = -\,\mathrm{Im}\left[F^{+-}_{+-} - F^{-+}_{+-}\right]\cos(\phi_{S_A} - \phi_a) \\
P^g_z\,\hat{f}_{g/A,S_L} &= \Delta\hat{f}^g_{s_L/S_L} = \left(F^{++}_{++} - F^{++}_{--}\right) \\
P^g_z\,\hat{f}_{g/A,S_T} &= \Delta\hat{f}^g_{s_L/S_T} = 2\,\left[\mathrm{Re}F^{++}_{+-}\cos(\phi_{S_A} - \phi_a)\right]
\end{aligned}$$

Mulders, Rodrigues: PRD63 (2001) 094021

TMD fragmentation functions into unpol. hadrons (leading twist)

Introducing the soft, nonperturbative helicity fragmentation amplitudes for the process c $\to$ C + X, the following properties hold for their products:

$$\hat{D}^{\lambda_C,\lambda'_C}_{\lambda_c,\lambda'_c}(z,\boldsymbol{k}_{\perp C}) = \sum\!\!\!\!\!\!\!\int_{X,\lambda_X} \hat{\mathcal{D}}_{\lambda_C,\lambda_X;\lambda_c}(z,\boldsymbol{k}_{\perp C})\,\hat{\mathcal{D}}^*_{\lambda'_C,\lambda_X;\lambda'_c}(z,\boldsymbol{k}_{\perp C})$$

$$\hat{D}^{\lambda_C,\lambda'_C}_{\lambda_c,\lambda'_c}(z,\boldsymbol{k}_{\perp C}) = D^{\lambda_C,\lambda'_C}_{\lambda_c,\lambda'_c}(z,k_{\perp C})\,e^{i(\lambda_c-\lambda'_c)\phi^H_C}$$

$$\hat{D}^{C}_{\lambda_c,\lambda'_c}(z,\boldsymbol{k}_{\perp C}) = \sum_{\lambda_C,\lambda_C} \hat{D}^{\lambda_C,\lambda_C}_{\lambda_c,\lambda'_c}(z,\boldsymbol{k}_{\perp C}) = D^{C}_{\lambda_c,\lambda'_c}(z,k_{\perp C})e^{i(\lambda_c-\lambda'_c)\phi^H_C}$$

$$D^{C}_{-\lambda_c,-\lambda'_c}(z,k_{\perp C}) = (-1)^{2s_c}(-1)^{\lambda_c+\lambda'_c}D^{C}_{\lambda_c,\lambda'_c}(z,k_{\perp C})$$

Quark sector

$$\hat{D}^{C/q}_{++}(z,\boldsymbol{k}_{\perp C}) = D^{C/q}_{++}(z,k_{\perp C}) \equiv \hat{D}_{C/q}(z,k_{\perp C})$$

$$2\,\mathrm{Im}\,D^{C/q}_{+-}(z,k_{\perp C}) \equiv \Delta^N\hat{D}_{C/q^\uparrow}(z,k_{\perp C})$$

Gluon sector

$$\hat{D}^{C/g}_{++}(z,\boldsymbol{k}_{\perp C}) = D^{C/g}_{++}(z,k_{\perp C}) \equiv \hat{D}_{C/g}(z,k_{\perp C})$$

$$2\,\mathrm{Re}\,D^{C/g}_{+-}(z,k_{\perp C}) \equiv \Delta^N\hat{D}_{C/\mathcal{T}^g_1}(z,k_{\perp C})$$

Helicity amplitudes for the elementary partonic process ab → cd

All intrinsic parton motions are explicitely taken into account: all soft elementary processes, A(B) → a(b) + X and c → C + X, and the elementary QCD process ab → cd (which is not planar in the hadronic c.m. frame) take place out of the hadronic production plane (the XZ_{cm} plane). This introduces several non trivial azimuthal phases in the PDF and FF and in the helicity amplitudes for the elementary process. The helicity amplitudes in the hadronic c.m. frame and those in the canonical partonic c.m. frame (no azimuthal phases) are related in the following way [see PRD71 (2005) 041002 for details]:

$$\hat{M}_{\lambda_c,\lambda_d;\lambda_a,\lambda_b} = \hat{M}^0_{\lambda_c,\lambda_d;\lambda_a,\lambda_b}\, e^{-i(\lambda_a\xi_a+\lambda_b\xi_b-\lambda_c\xi_c-\lambda_d\xi_d)}\, e^{-i[(\lambda_a-\lambda_b)\tilde{\xi}_a-(\lambda_c-\lambda_d)\tilde{\xi}_c]}\, e^{i(\lambda_a-\lambda_b)\phi''_c}$$

Well-known parity properties hold for the canonical helicity amplitudes:

$$\hat{M}^0_{-\lambda_c,-\lambda_d;-\lambda_a,-\lambda_b} = \eta_a\,\eta_b\,\eta_c\,\eta_d\,(-1)^{s_a+s_b-s_c-s_d}(-1)^{(\lambda_a-\lambda_b)-(\lambda_c-\lambda_d)}\,\hat{M}^0_{\lambda_c,\lambda_d;\lambda_a,\lambda_b}$$

For massless partons there are only three independent helicity amplitudes:

$$\hat{M}_{++;++} \equiv \hat{M}^0_1\, e^{i\varphi_1}\,, \quad \hat{M}_{-+;-+} \equiv \hat{M}^0_2\, e^{i\varphi_2}\,, \quad \hat{M}_{-+;+-} \equiv \hat{M}^0_3\, e^{i\varphi_3}$$

At LO there are eight elementary contributions ab → cd which must be considered separately, since they get different PDF and FF weights in the phase-space integrations:

$$q_aq_b \to q_cq_d\,, \quad q\bar{q} \to g_cg_d\,, \quad g_ag_b \to q\bar{q}\,, \quad g_ag_b \to g_cg_d$$
$$qg \to qg\,, \quad gq \to gq\,, \quad qg \to gq\,, \quad gq \to qg$$

Canonical helicity amplitudes for processes which only differ by the exchange of the two initial particles, a $\leftrightarrow$ b, or of the two final partons, c $\leftrightarrow$ d, are related as follows:

$$\hat{M}^{0,\,ba\to cd}_{\lambda_c,\lambda_d;\lambda_b,\lambda_a}(\theta) = \hat{M}^{0,\,ab\to cd}_{\lambda_c,\lambda_d;\lambda_a,\lambda_b}(\pi-\theta)\,e^{-i\pi(\lambda_c-\lambda_d)}$$

$$\hat{M}^{0,\,ab\to dc}_{\lambda_d,\lambda_c;\lambda_a,\lambda_b}(\theta) = \hat{M}^{0,\,ab\to cd}_{\lambda_c,\lambda_d;\lambda_a,\lambda_b}(\pi-\theta)\,e^{-i\pi(\lambda_a-\lambda_b)}$$

up to an overall, helicity independent, phase, which is irrelevant in the expressions of the physical observables, where only bilinear combinations of the helicity amplitudes occurr.

Kernels for the process $A(S_A)+B(S_B) \to C + X$

$$\Sigma(S_A, S_B)^{ab\to cd} = \sum_{\{\lambda\}} \rho^{a/A,S_A}_{\lambda_a,\lambda'_a} \hat{f}_{a/A,S_A}(x_a, \boldsymbol{k}_{\perp a}) \, \rho^{b/B,S_B}_{\lambda_b,\lambda'_b} \hat{f}_{b/B,S_B}(x_b, \boldsymbol{k}_{\perp b}) \times \hat{M}_{\lambda_c,\lambda_d;\lambda_a,\lambda_b} \hat{M}^*_{\lambda'_c,\lambda_d;\lambda'_a,\lambda'_b} \hat{D}^{\lambda_C,\lambda_C}_{\lambda_c,\lambda'_c}(z, \boldsymbol{k}_{\perp C})$$

Two examples: $q_a\, q_b \to q_c\, q_d$ and $gq \to gq$ contributions

$$\Sigma(S_A, S_B)^{q_a q_b \to q_c q_d} = \frac{1}{2} \hat{D}_{C/c}(z, k_{\perp C}) \, \hat{f}_{a/S_A}(x_a, \boldsymbol{k}_{\perp a}) \, \hat{f}_{b/S_B}(x_b, \boldsymbol{k}_{\perp b}) \times$$
$$\Big\{ \left(|\hat{M}_1^0|^2 + |\hat{M}_2^0|^2 + |\hat{M}_3^0|^2\right) + P^a_z\, P^b_z \left(|\hat{M}_1^0|^2 - |\hat{M}_2^0|^2 - |\hat{M}_3^0|^2\right)$$
$$+ 2\hat{M}_2^0\, \hat{M}_3^0 \left[\left(P^a_x\, P^b_x + P^a_y\, P^b_y\right) \cos(\varphi_3 - \varphi_2) - \left(P^a_x\, P^b_y - P^a_y\, P^b_x\right) \sin(\varphi_3 - \varphi_2)\right] \Big\}$$
$$- \frac{1}{2} \Delta^N \hat{D}_{C/c^\uparrow}(z, k_{\perp C}) \, \hat{f}_{a/S_A}(x_a, \boldsymbol{k}_{\perp a}) \, \hat{f}_{b/S_B}(x_b, \boldsymbol{k}_{\perp b}) \times$$
$$\Big\{ \hat{M}_1^0\, \hat{M}_2^0 \left[P^a_x \sin(\varphi_1 - \varphi_2 + \phi^H_C) - P^a_y \cos(\varphi_1 - \varphi_2 + \phi^H_C)\right]$$
$$+ \hat{M}_1^0\, \hat{M}_3^0 \left[P^b_x \sin(\varphi_1 - \varphi_3 + \phi^H_C) - P^b_y \cos(\varphi_1 - \varphi_3 + \phi^H_C)\right] \Big\}$$

$$
\begin{aligned}
\Sigma(S_A, S_B)^{gq\to gq} = {} & \frac{1}{2}\, \hat{D}_{C/g}(z, k_{\perp C})\; \hat{f}_{g/S_A}(x_a, \boldsymbol{k}_{\perp a})\; \hat{f}_{q/S_B}(x_b, \boldsymbol{k}_{\perp b}) \times \\
& \left\{ \left(|\hat{M}_1^0|^2 + |\hat{M}_2^0|^2 \right) + P_z^g\, P_z^q \left(|\hat{M}_1^0|^2 - |\hat{M}_2^0|^2 \right) \right\} \\
& + \frac{1}{2}\, \Delta^N \hat{D}_{C/\mathcal{T}_1^g}(z, k_{\perp C})\; \hat{f}_{g/S_A}(x_a, \boldsymbol{k}_{\perp a})\; \hat{f}_{q/S_B}(x_b, \boldsymbol{k}_{\perp b}) \times \\
& \left\{ \hat{M}_1^0\, \hat{M}_2^0 \left[\mathcal{T}_1^g\; \cos(\varphi_1 - \varphi_2 + 2\phi_C^H) + \mathcal{T}_2^g\; \sin(\varphi_1 - \varphi_2 + 2\phi_C^H) \right] \right\}
\end{aligned}
$$

Unpolarized cross section and SSA for pp → π + X

$$\frac{E_C\, d\sigma^{(A,S_A)+(B,S_B)\to C+X}}{d^3\boldsymbol{p}_C} = \sum_{a,b,c,d} \int \frac{dx_a\, dx_b\, dz}{16\pi^2 x_a x_b z^2 s}\, d^2\boldsymbol{k}_{\perp a}\, d^2\boldsymbol{k}_{\perp b}\, d^3\boldsymbol{k}_{\perp C}\, \delta(\boldsymbol{k}_{\perp C}\cdot \hat{\boldsymbol{p}}_c)\, J(\boldsymbol{k}_{\perp C})$$
$$\times\ \Sigma(S_A, S_B)^{ab\to cd}(x_a, x_b, z, \boldsymbol{k}_{\perp a}, \boldsymbol{k}_{\perp b}, \boldsymbol{k}_{\perp C})\, \delta(\hat{s}+\hat{t}+\hat{u})$$

$$A_N(pp\to\pi+X) = \frac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^\uparrow + d\sigma^\downarrow}$$

The calculation requires the difference and sum of single-spin kernels for opposite transverse polarizations [only $q_a q_b \to q_c q_c$ and $gq \to gq$ contributions shown]

$$[\Sigma(\uparrow,0) - \Sigma(\downarrow,0)]^{q_a q_b \to q_c q_d} =$$
$$\frac{1}{2}\,\Delta\hat{f}_{a/A^\uparrow}(x_a,\boldsymbol{k}_{\perp a})\,\hat{f}_{b/B}(x_b,k_{\perp b})\left[|\hat{M}_1^0|^2 + |\hat{M}_2^0|^2 + |\hat{M}_3^0|^2\right]\hat{D}_{C/c}(z,k_{\perp C})$$
$$+\ 2\left[\Delta^-\hat{f}^a_{s_y/\uparrow}(x_a,\boldsymbol{k}_{\perp a})\,\cos(\varphi_3-\varphi_2) - \Delta\hat{f}^a_{s_x/\uparrow}(x_a,\boldsymbol{k}_{\perp a})\,\sin(\varphi_3-\varphi_2)\right]$$
$$\times\,\Delta\hat{f}^b_{s_y/B}(x_b,\boldsymbol{k}_{\perp b})\,\hat{M}_2^0\,\hat{M}_3^0\,\hat{D}_{C/c}(z,k_{\perp C})$$
$$-\ \left[\Delta\hat{f}^a_{s_x/\uparrow}(x_a,\boldsymbol{k}_{\perp a})\,\sin(\varphi_1-\varphi_2+\phi_C^H) - \Delta^-\hat{f}^a_{s_y/\uparrow}(x_a,\boldsymbol{k}_{\perp a})\,\cos(\varphi_1-\varphi_2+\phi_C^H)\right]$$
$$\times\,\hat{f}_{b/B}(x_b,k_{\perp b})\,\hat{M}_1^0\,\hat{M}_2^0\,\Delta^N\hat{D}_{C/c^\uparrow}(z,k_{\perp C})$$
$$+\ \frac{1}{2}\,\Delta\hat{f}_{a/A^\uparrow}(x_a,\boldsymbol{k}_{\perp a})\,\Delta\hat{f}^b_{s_y/B}(x_b,\boldsymbol{k}_{\perp b})\,\cos(\varphi_1-\varphi_3+\phi_C^H)\,\hat{M}_1^0\,\hat{M}_3^0\,\Delta^N\hat{D}_{C/c^\uparrow}(z,k_{\perp C})$$

$$[\Sigma(\uparrow,0)-\Sigma(\downarrow,0)]^{gq\to gq} =$$

$$\frac{1}{2}\,\Delta\hat{f}_{g/A^\uparrow}(x_g,\boldsymbol{k}_{\perp g})\,\hat{f}_{q/B}(x_q,k_{\perp q})\,\left[|\hat{M}_1^0|^2+|\hat{M}_2^0|^2\right]\,\hat{D}_{C/g}(z,k_{\perp C})$$

$$+\left[\Delta^-\hat{f}^g_{\mathcal{T}_1/\uparrow}(x_g,\boldsymbol{k}_{\perp g})\,\cos(\varphi_1-\varphi_2+2\phi_C^H)+\Delta\hat{f}^g_{\mathcal{T}_2/\uparrow}(x_g,\boldsymbol{k}_{\perp g})\,\sin(\varphi_1-\varphi_2+2\phi_C^H)\right]$$

$$\times\,\hat{f}_{q/B}(x_q,k_{\perp q})\,\hat{M}_1^0\,\hat{M}_2^0\,\Delta^N\hat{D}_{C/\mathcal{T}_1^g}(z,k_{\perp C})$$

$$[\Sigma(\uparrow,0)+\Sigma(\downarrow,0)]^{q_aq_b\to q_cq_d} =$$

$$\hat{f}_{a/A}(x_a,k_{\perp a})\,\hat{f}_{b/B}(x_b,k_{\perp b})\,\left[|\hat{M}_1^0|^2+|\hat{M}_2^0|^2+|\hat{M}_3^0|^2\right]\,\hat{D}_{C/c}(z,k_{\perp C})$$

$$+\quad 2\,\Delta\hat{f}^a_{s_y/A}(x_a,\boldsymbol{k}_{\perp a})\,\Delta\hat{f}^b_{s_y/B}(x_b,\boldsymbol{k}_{\perp b})\,\cos(\varphi_3-\varphi_2)\,\hat{M}_2^0\,\hat{M}_3^0\,\hat{D}_{C/c}(z,k_{\perp C})$$

$$+\quad\left[\hat{f}_{a/A}(x_a,k_{\perp a})\,\Delta\hat{f}^b_{s_y/B}(x_b,\boldsymbol{k}_{\perp b})\,\cos(\varphi_1-\varphi_3+\phi_C^H)\,\hat{M}_1^0\,\hat{M}_3^0\right.$$

$$\left.+\;\Delta\hat{f}^a_{s_y/A}(x_a,\boldsymbol{k}_{\perp a})\,\hat{f}_{b/B}(x_b,k_{\perp b})\,\cos(\varphi_1-\varphi_2+\phi_C^H)\,\hat{M}_1^0\hat{M}_2^0\right]\,\Delta^N\hat{D}_{C/c^\uparrow}(z,k_{\perp C})$$

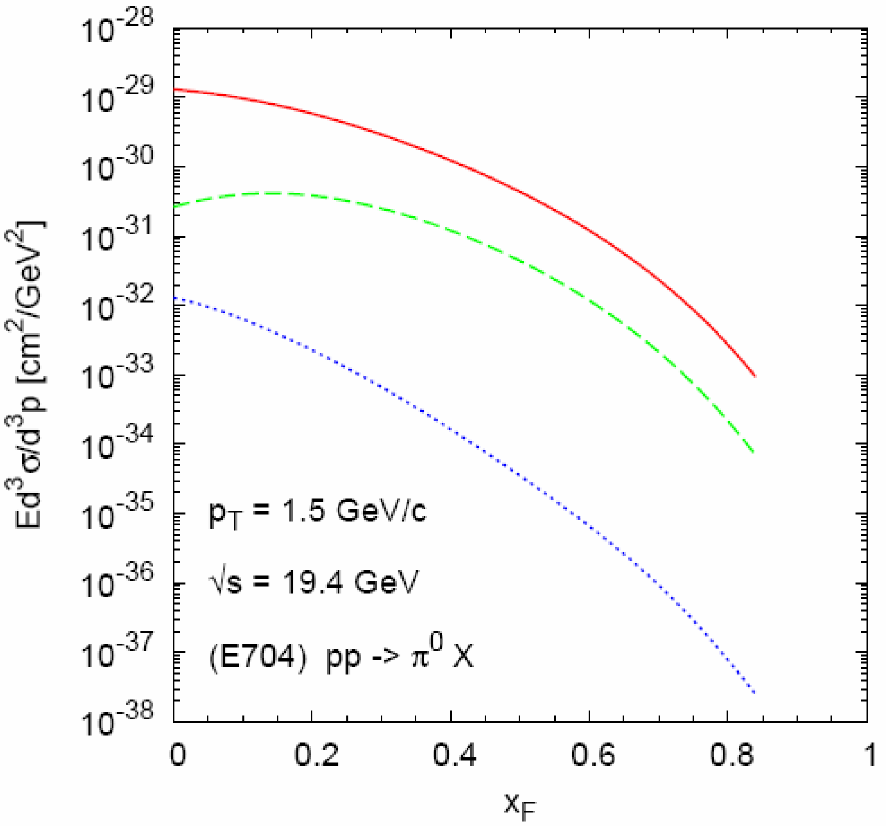
$$[\Sigma(\uparrow,0)+\Sigma(\downarrow,0)]^{gq\to gq} =$$

$$\hat{f}_{g/A}(x_g,k_{\perp g})\,\hat{f}_{q/B}(x_q,k_{\perp q})\,\left[|\hat{M}_1^0|^2+|\hat{M}_2^0|^2\right]\,\hat{D}_{C/g}(z,k_{\perp C})$$

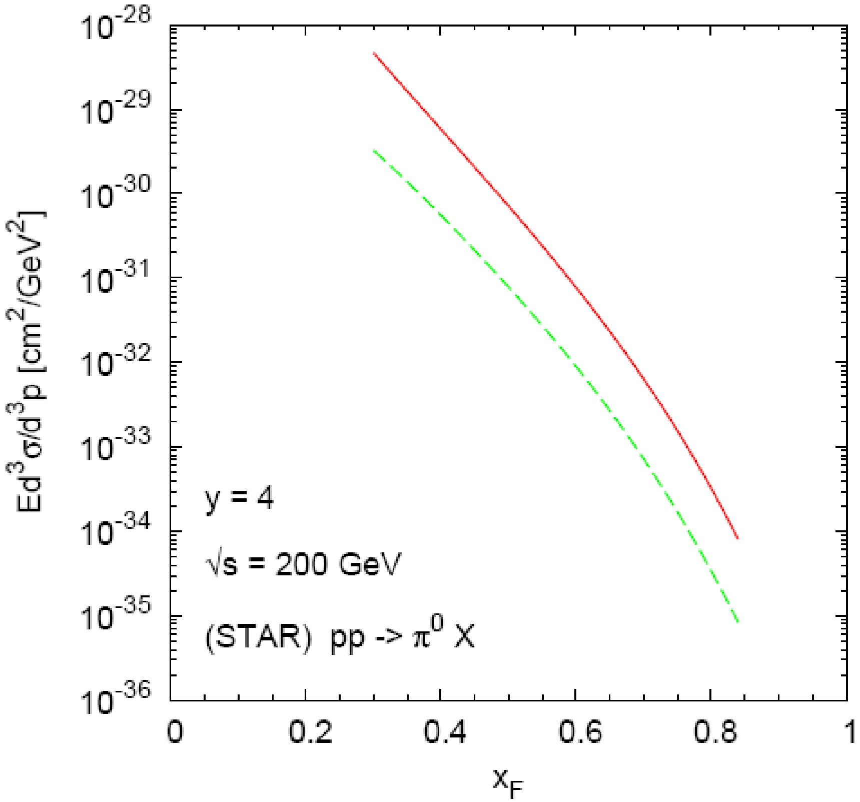
$$+\quad\Delta\hat{f}^g_{\mathcal{T}_1/A}(x_g,\boldsymbol{k}_{\perp g})\,\hat{f}_{q/B}(x_q,k_{\perp q})\,\cos(\varphi_1-\varphi_2+2\phi_C^H)\,\hat{M}_1^0\,\hat{M}_2^0\,\Delta^N\hat{D}_{C/\mathcal{T}_1^g}(z,k_{\perp C})$$

Numerical estimates: unpolarized cross section and SSA for $pp \to \pi + X$ process (E704, STAR and PAX kinematics)

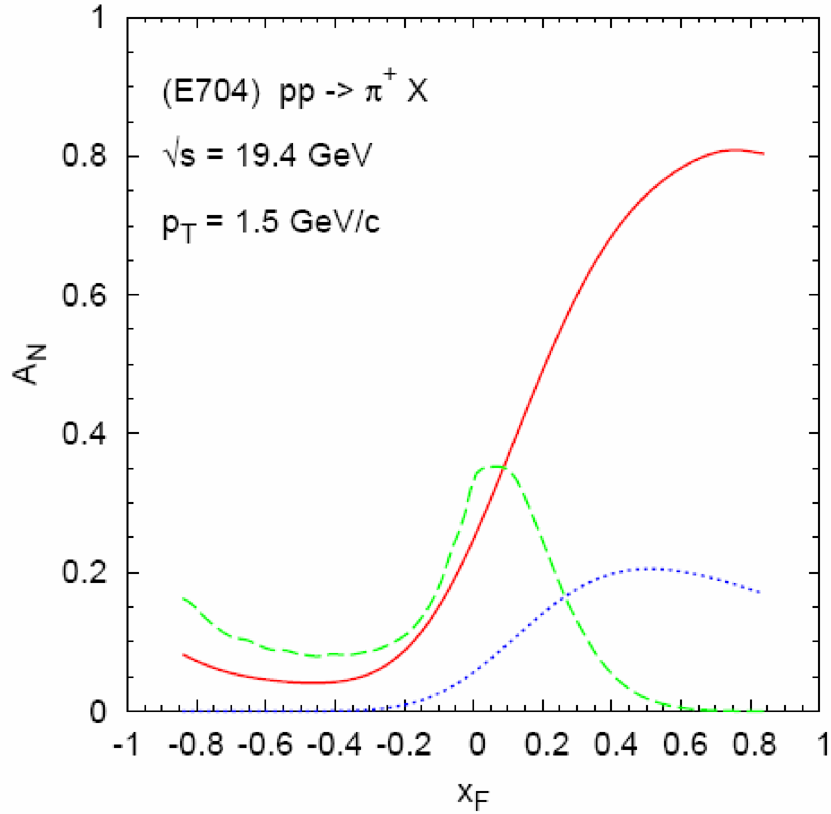
- Sivers and Collins functions: positivity bounds saturated
- All other unknown polarized PDF's and FF's replaced with the corresponding unpolarized ones [overestimated, implies some bound violation]
- Whenever different contributions could combine with different signs, we have summed them assuming the same sign [avoid (possible) cancellations not resulting from phase-space integration of the corresponding azimuthal phases]
- Flavour-independent, gaussian $k_\perp$ dependence for all distributions: $\langle k_\perp \rangle = 0.8$ GeV/*c* for PDF; $\langle k_{\perp \mathbf{C}} \rangle$ as in PRD70 (2004) 074009 for FF.
- Unpolarized, $k_\perp$–integrated functions: MRST02 PDF, KKP FF



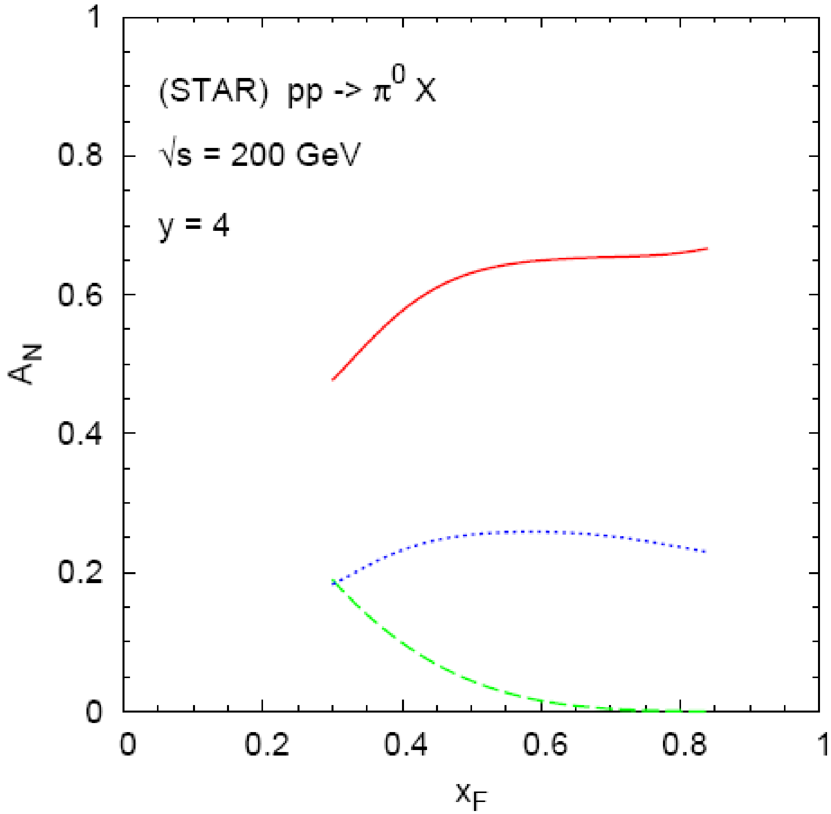
Contributions to the unpolarized cross section, plotted as a function of x_F for $pp \to \pi^0 + X$ processes and E704 kinematics, as indicated in the plot. The three curves correspond to: solid line = usual unpolarized contribution; dashed line = Boer-Mulders $\otimes$ Collins contribution (maximized); dotted line = Boer-Mulders $\otimes$ Boer-Mulders contribution (maximized).



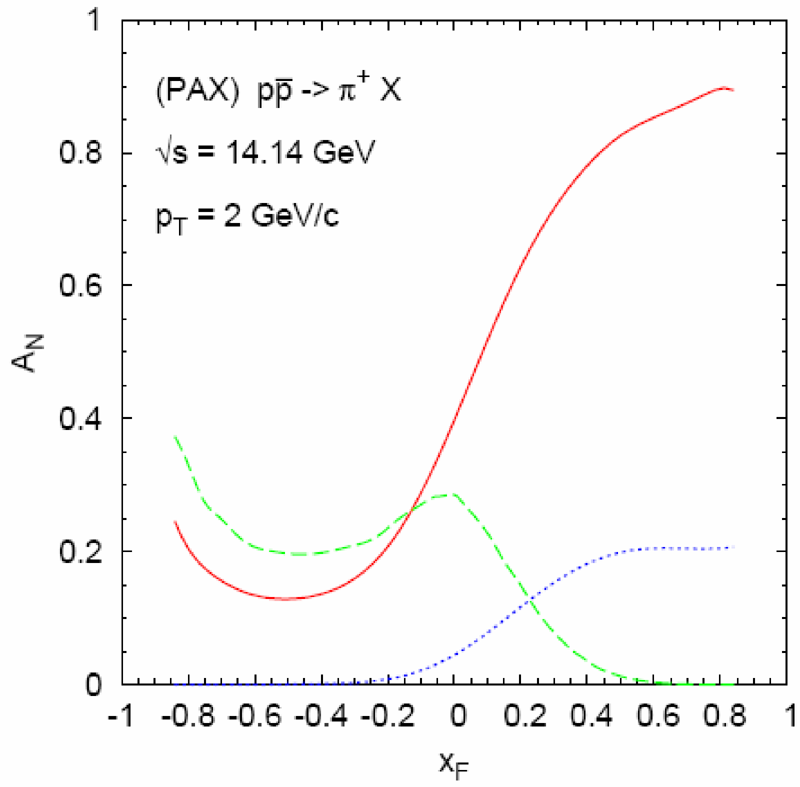
Contributions to the unpolarized cross section, plotted as a function of x_F, for pp $\rightarrow \pi^0$ + X processes and STAR kinematics, as indicated in the plot. The two curves correspond to: solid line = usual unpolarized contribution; dashed line = Boer-Mulders $\otimes$ Collins contribution (maximized); the (maximized) Boer-Mulders $\otimes$ Boer-Mulders contribution is negligible and out of scale.



Maximized contributions to the SSA A_N, plotted as a function of x_F, for pp $\rightarrow \pi^0$ + X processes and E704 kinematics, as indicated in the plot. The three curves correspond to: solid line = quark Sivers mechanism; dashed line = gluon Sivers mechanism; dotted line = transversity $\otimes$ Collins contribution; all other contributions are much smaller.



Maximized contributions to the SSA A_N, plotted as a function of x_F, for pp $\to \pi^0$ + X processes and STAR kinematics, as indicated in the plot. The three curves correspond to: solid line = quark Sivers mechanism; dashed line = gluon Sivers mechanism; dotted line = transversity $\otimes$ Collins contribution; all other contributions are much smaller. At negative x_F all contributions are vanishingly small.



Maximized contributions to the SSA A_N, plotted as a function of x_F, for $p^\uparrow \bar{p} \to \pi^+ X$ processes and PAX kinematics, as indicated in the plot. The three curves correspond to: solid line = quark Sivers mechanism; dashed line = gluon Sivers mechanism; dotted line = transversity $\otimes$ Collins contribution; all other contributions are much smaller.

Conclusions

- LO QCD hard scattering formalism to compute (un)polarized cross sections for the inclusive process $A(S_A)+B(S_B) \to C + X$ at moderately large p_T values presented
- Intrinsic parton motion fully taken into account in soft physics and in the elementary interactions; complete helicity formalism developed
- Leading-Twist, polarized and TMD PDF's and FF's with simple physical (partonic) interpretation discussed
- Formal expressions of all contributions to single and double spin asymmetries discussed, some examples given
- Complete evaluation of transverse SSA for the $pp \to \pi + X$ process and explicit expressions of some contributions given
- Numerical calculations of all maximized contributions to unpol. cross section and SSA for the $pp \to \pi + X$ process and for E704, STAR and PAX kinematics presented

Conclusions (2)

- Azimuthal phases strongly suppress many contributions after phase-space integrations, leaving at work predominantly the Sivers effect and, to a lesser extent, the Collins mechanism
- More applications are under progress/planned in the near future:
  - Sivers asymmetry in SIDIS processes already studied, extension to the Collins asymmetry in SIDIS and e^+e^- processes in progress
  - Single and double spin asymmetries in inclusive production and in Drell-Yan processes
  - Extension to the inclusive productions of spin ½ (Λ particles) and spin 1 mesons under progress; π-p collisions under consideration

Polarized PDF - details

Sivers distribution (quarks & gluons)

$$\Delta\hat f_{a/S_A}(x_a,\boldsymbol{k}_{\perp a}) \equiv \hat f_{a/S_A}(x_a,\boldsymbol{k}_{\perp a}) - \hat f_{a/-S_A}(x_a,\boldsymbol{k}_{\perp a})$$
$$= \Delta^N \hat f_{a/A^\uparrow}(x_a,k_{\perp a})\,(\hat{\boldsymbol{p}}_A \times \hat{\boldsymbol{k}}_{\perp a})\cdot \hat{\boldsymbol{P}}^A$$
$$= \Delta^N \hat f_{a/A^\uparrow}(x_a,k_{\perp a})\,\sin(\phi_{S_A}-\phi_a)$$

Boer-Mulders distribution (quarks)

$$P_j^q\,\hat f_{q/A} = \hat f^q_{s_j/A}(x,\boldsymbol{k}_\perp) - \hat f^q_{-s_j/A}(x,\boldsymbol{k}_\perp) \equiv \Delta\hat f^q_{s_j/A}(x,\boldsymbol{k}_\perp)$$
$$= \Delta^N \hat f_{q^\uparrow/A}(x,k_\perp)\,(\hat{\boldsymbol{p}}_A \times \hat{\boldsymbol{k}}_\perp)_j$$

Boer-Mulders distribution (gluons)

$$T_{ij}\,\hat f_{g/A}(x,k_\perp) = \sqrt{\frac{3}{2}}\Big[\frac{1}{2}\Delta^N \hat f^g_{\mathcal{T}_1/A}(x,k_\perp)\,(\hat u_i\hat u_j - \hat v_i\hat v_j)$$
$$- \frac{1}{6}\hat f_{g/A}(x,k_\perp)\,(\hat u_i\hat u_j + \hat v_i\hat v_j - 2\,\hat p_i\hat p_j)\Big]$$

To reach the simple configuration of the canonical amplitudes: start from the hadronic c.m. frame; perform a boost in the direction determined by $\mathbf{q} = \mathbf{p}_a + \mathbf{p}_b$ [so that the boosted three-vector $\mathbf{p}'_a + \mathbf{p}'_b$ is equal to zero]. This will provide us with a c.m.-like ref. frame S′ where partons a and b collide head-on. Here the parton a and the parton c [resulting from the hard interaction between a and b] will have directions identified by (θ'_a , ϕ'_a) and (θ'_c , ϕ'_c) respectively. In general, the parton momenta in S′ are related to the initial ones (before the boost) by [i=a,b,c,d]:

$$\boldsymbol{p}'_i = \boldsymbol{p}_i - \frac{\boldsymbol{q}}{q^0 + \sqrt{q^2}} \left(\frac{p_i \cdot q}{\sqrt{q^2}} + p_i^0 \right)$$

Perform now two subsequent rotations, one around the Z axis by an angle ϕ'_a, and one around the Y axis, by an angle θ'_a, such that the collision axis of the two colliding initial partons turns out to be aligned with the Z axis. We call this frame S″. Under these boost and rotations the helicity states and consequently the scattering amplitudes acquire phases, $\xi_{a,b,c,d}$ and $\tilde{\xi}_{a,b,c,d}$:

$$\begin{aligned} \cos\xi_j &= \frac{\cos\theta_q \sin\theta_j - \sin\theta_q \cos\theta_j \cos(\phi_q - \phi_j)}{\sin\theta_{qp_j}} \\ \sin\xi_j &= \frac{\sin\theta_q \sin(\phi_q - \phi_j)}{\sin\theta_{qp_j}} \end{aligned}$$

$$\tilde{\xi}_j = \eta'_j + \xi'_j$$

$$\cos\eta'_j = \frac{\cos\theta'_a - \cos\theta'_j \cos\theta_{p'_a p'_j}}{\sin\theta'_j \sin\theta_{p'_a p'_j}}$$

$$\sin\eta'_j = \frac{\sin\theta'_a \sin(\phi'_a - \phi'_j)}{\sin\theta_{p'_a p'_j}}$$

$$\cos\xi'_j = \frac{\cos\theta_q \sin\theta'_j - \sin\theta_q \cos\theta'_j \cos(\phi_q - \phi'_j)}{\sin\theta_{q p'_j}}$$

$$\sin\xi'_j = \frac{-\sin\theta_q \sin(\phi_q - \phi'_j)}{\sin\theta_{q p'_j}}$$

In the S″ frame the direction of the parton c is characterised by an azimuthal angle ϕ''_c given by

$$\tan\phi''_c = \frac{\sin\theta'_c \sin(\phi'_c - \phi'_a)}{\sin\theta'_c \cos(\phi'_c - \phi'_a)\cos\theta'_a - \cos\theta'_c \sin\theta'_a}$$

LO helicity amplitudes for the elementary process ab $\to$ cd

$q_a q_b \to q_c q_d$ $\qquad$ $\bar{q}_a \bar{q}_b \to \bar{q}_c \bar{q}_d$

$$|\hat{M}_1^0|^2 = \frac{8}{9}\, g_s^4 \left[\frac{\hat{s}^2}{\hat{t}^2} + \delta_{ab}\left(\frac{\hat{s}^2}{\hat{u}^2} - \frac{2}{3}\,\frac{\hat{s}^2}{\hat{t}\hat{u}}\right)\right] \qquad |\hat{M}_2^0|^2 = \frac{8}{9}\, g_s^4\, \frac{\hat{u}^2}{\hat{t}^2}$$

$$|\hat{M}_3^0|^2 = \delta_{ab}\,\frac{8}{9}\, g_s^4\, \frac{\hat{t}^2}{\hat{u}^2} \qquad \hat{M}_1^0\,\hat{M}_2^0 = \frac{8}{9}\, g_s^4 \left(-\frac{\hat{s}\hat{u}}{\hat{t}^2} + \delta_{ab}\,\frac{1}{3}\,\frac{\hat{s}}{\hat{t}}\right)$$

$$\hat{M}_1^0\,\hat{M}_3^0 = \delta_{ab}\,\frac{8}{9}\, g_s^4 \left(\frac{\hat{s}\hat{t}}{\hat{u}^2} - \frac{1}{3}\,\frac{\hat{s}}{\hat{u}}\right) \qquad \hat{M}_2^0\,\hat{M}_3^0 = \delta_{ab}\,\frac{8}{27}\, g_s^4$$

$q_a \bar{q}_b \to q_c \bar{q}_d$

$$|\hat{M}_1^0|^2 = \delta_{ac}\,\frac{8}{9}\, g_s^4\, \frac{\hat{s}^2}{\hat{t}^2} \qquad |\hat{M}_2^0|^2 = \frac{8}{9}\, g_s^4 \left(\delta_{ab}\,\frac{\hat{u}^2}{\hat{s}^2} + \delta_{ac}\,\frac{\hat{u}^2}{\hat{t}^2} - \delta_{ab}\,\delta_{ac}\,\frac{2}{3}\,\frac{\hat{u}^2}{\hat{s}\hat{t}}\right)$$

$$|\hat{M}_3^0|^2 = \delta_{ab}\,\frac{8}{9}\, g_s^4\, \frac{\hat{t}^2}{\hat{s}^2} \qquad \hat{M}_1^0\,\hat{M}_2^0 = \frac{8}{9}\, g_s^4\, \delta_{ac} \left(-\frac{\hat{s}\hat{u}}{\hat{t}^2} + \delta_{ab}\,\frac{1}{3}\,\frac{\hat{u}}{\hat{t}}\right)$$

$$\hat{M}_1^0\,\hat{M}_3^0 = \delta_{ab}\,\delta_{ac}\,\frac{8}{27}\, g_s^4 \qquad \hat{M}_2^0\,\hat{M}_3^0 = \frac{8}{9}\, g_s^4\, \delta_{ab} \left(\frac{\hat{u}\hat{t}}{\hat{s}^2} - \delta_{ac}\,\frac{1}{3}\,\frac{\hat{u}}{\hat{s}}\right)$$

LO helicity amplitudes for the elementary process ab → cd (2)

$$qg \to qg$$

$$|\hat{M}_1^0|^2 = \frac{8}{9}\, g_s^4 \left(-\frac{\hat{s}}{\hat{u}} + \frac{9}{4}\frac{\hat{s}^2}{\hat{t}^2}\right) \qquad |\hat{M}_2^0|^2 = \frac{8}{9}\, g_s^4 \left(-\frac{\hat{u}}{\hat{s}} + \frac{9}{4}\frac{\hat{u}^2}{\hat{t}^2}\right)$$

$$\hat{M}_1^0 \hat{M}_2^0 = \frac{8}{9}\, g_s^4 \left(-1 + \frac{9}{4}\frac{\hat{u}\hat{s}}{\hat{t}^2}\right).$$

$$q\bar{q} \to gg$$

$$|\hat{M}_2^0|^2 = \frac{64}{27}\, g_s^4 \left(\frac{\hat{u}}{\hat{t}} - \frac{9}{4}\frac{\hat{u}^2}{\hat{s}^2}\right) \qquad |\hat{M}_3^0|^2 = \frac{64}{27}\, g_s^4 \left(\frac{\hat{t}}{\hat{u}} - \frac{9}{4}\frac{\hat{t}^2}{\hat{s}^2}\right)$$

$$\hat{M}_2^0 \hat{M}_3^0 = \frac{64}{27}\, g_s^4 \left(1 - \frac{\hat{t}\hat{u}}{\hat{s}^2}\right)$$

$$gg \to gg$$

$$|\hat{M}_1^0|^2 = \frac{9}{2}\, g_s^4\, \hat{s}^2 \left(\frac{1}{\hat{t}^2} + \frac{1}{\hat{u}^2} + \frac{1}{\hat{t}\hat{u}}\right) \qquad |\hat{M}_2^0|^2 = \frac{9}{2}\, g_s^4\, \frac{\hat{u}^2}{\hat{s}^2} \left(1 + \frac{\hat{u}}{\hat{t}} + \frac{\hat{u}^2}{\hat{t}^2}\right)$$

$$|\hat{M}_3^0|^2 = \frac{9}{2}\, g_s^4\, \frac{\hat{t}^2}{\hat{s}^2} \left(1 + \frac{\hat{t}}{\hat{u}} + \frac{\hat{t}^2}{\hat{u}^2}\right) \qquad \hat{M}_1^0 \hat{M}_2^0 = \frac{9}{2}\, g_s^4 \left(1 + \frac{\hat{u}}{\hat{t}} + \frac{\hat{u}^2}{\hat{t}^2}\right)$$

$$\hat{M}_1^0 \hat{M}_3^0 = \frac{9}{2}\, g_s^4 \left(1 + \frac{\hat{t}}{\hat{u}} + \frac{\hat{t}^2}{\hat{u}^2}\right) \qquad \hat{M}_2^0 \hat{M}_3^0 = \frac{9}{2}\, g_s^4\, \frac{1}{\hat{s}^2}\left(\hat{u}^2 + \hat{t}^2 + \hat{u}\hat{t}\right)$$