

SSA in SIDIS & DY

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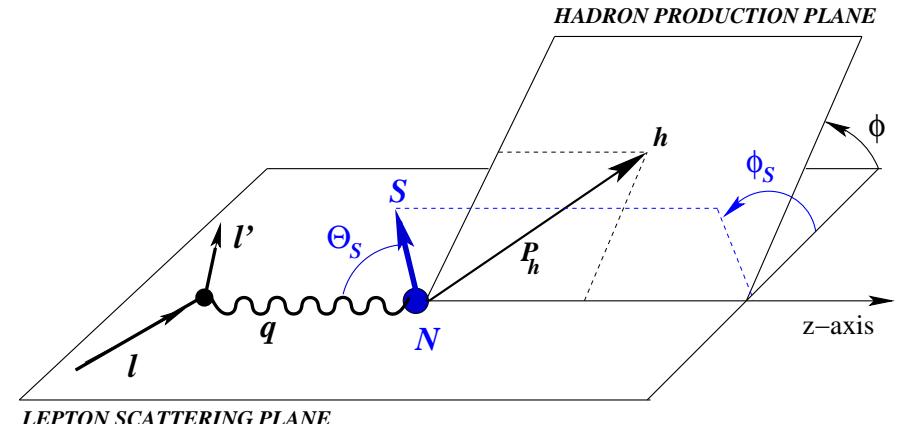
in collaboration with *J. C. Collins, A. V. Efremov, K. Goeke, M. Grosse Perdekamp, S. Menzel, A. Metz*

Overview:

- Sivers effect in SIDIS
- Preliminary HERMES data on $A_{UT}^{\sin(\phi-\phi_S)P_{h\perp}/M_N}$ & final HERMES data on $A_{UT}^{\sin(\phi-\phi_S)}$
 - Use & test(!) large N_c (= number of colours) QCD limit for Sivers function
 - Gauss model for distribution of transverse parton momenta
 - COMPASS (deuteron) vs. HERMES (proton) data
- Drell-Yan process
- Conclusions

HERMES & COMPASS data

$$\frac{N^\uparrow - N^\downarrow}{N^\uparrow + N^\downarrow} \propto \underbrace{\dots \sin(\phi - \phi_S)}_{\text{Sivers}} + \underbrace{\dots \sin(\phi + \phi_S)}_{\text{Collins effect}}$$



- **Sivers effect:** first *unambiguous* evidence for “naively/artificially T-odd” distribution in SIDIS
- Sivers SSA $\propto \sum_a e_a^2 f_{1T}^{\perp a} D_1^a$ and $D_1^a(z)$ known, e.g. Kretzer, Leader, Christova, EPJC 22 (2001) 269
- $f_{1T}^{\perp a}$ only unknown, i.e. possible to extract direct information on Sivers function
- subject to considerable interest in literature:
[Efremov, Goeke, Menzel, Metz and PS, PLB 612 \(2005\) 233, hep-ph/0412353](#)
[Anselmino, Boglione, D'Alesio, Kotzinian, Murgia and Prokudin, PRD 71 \(2005\) 074006, hep-ph/0501196](#)
[Anselmino, Boglione, D'Alesio, Kotzinian, Murgia and Prokudin, hep-ph/0507181](#)
[Vogelsang and Yuan, hep-ph/0507266](#)
[Collins, Efremov, Goeke, Menzel, Metz and PS, hep-ph/0509076](#)

- **Collins effect** $\propto \sum_a e_a^2 h_1^a H_1^{\perp a}$ equally interesting, first *unambiguous* evidence in SIDIS.
 Data indicate fascinating properties → favoured vs. unfavoured H_1^\perp .
But: Two unknowns, more difficult. See [Vogelsang and Yuan, *op. cit.*](#); Efremov et al., talk at SIR'05

- FINAL HERMES & COMPASS data PRL 94 (2005) 012002 & PRL 94 (2005) 202002

$$A_{UT}^{\sin(\phi-\phi_S)} \equiv \frac{\int d\phi \sin(\phi - \phi_S)(N^\uparrow - N^\downarrow)}{\frac{1}{2} \int d\phi (N^\uparrow + N^\downarrow)}$$

- “preferable” PRELIMINARY HERMES data Makins, “*Transversity Workshop*”, Oct. 2003, Athens. Seidl, *DIS’2004*, April 2004, Štrbské Pleso. Gregor, Acta Phys. Pol. B 36 (2005) 209

$$A_{UT}^{\sin(\phi-\phi_S)P_{h\perp}/M_N} \equiv \frac{\int d\phi \sin(\phi - \phi_S)P_{h\perp}/M_N(N^\uparrow - N^\downarrow)}{\frac{1}{2} \int d\phi (N^\uparrow + N^\downarrow)} = (-2) \frac{\sum_a e_a^2 x f_{1T}^{\perp(1)a}(x) z D_1^{a/\pi}(z)}{\sum_a e_a^2 x f_1^a(x) D_1^{a/\pi}(z)}$$

with $f_{1T}^{\perp(1)a}(x) \equiv \int d^2 \mathbf{p}_T \frac{\mathbf{p}_T^2}{2M_N^2} f_{1T}^{\perp a}(x, \mathbf{p}_T^2)$ Boer and Mulders, PRD 57 (1998) 5780

- In which sense is $A_{UT}^{\sin(\phi-\phi_S)}$ “not preferable”? See later in detail.
- **WARNING!** Keep in mind: In any case **approximation** – neglect soft factors (!)
Ji, Ma and Yuan, PRD 71 (2005) 034005, PLB 597 (2004) 299. Collins and Metz, PRL 93 (2004) 252001.

$$A_{UT}^{\sin(\phi - \phi_S) P_{h\perp}/M_N} \xleftarrow{\text{directly}} f_{1T}^{\perp(1)a}(x) \quad \text{explored by Efremov et al., PLB 612 (2005) 233, hep-ph/0412353}$$

- error bars sizeable → minimize # of free parameters in fit Ansatz → **theoretical constraints**

I use large- N_c limit $f_{1T}^{\perp u} = -f_{1T}^{\perp d}$ modulo $1/N_c$ corrections Pobylitsa hep-ph/0301236 (★)

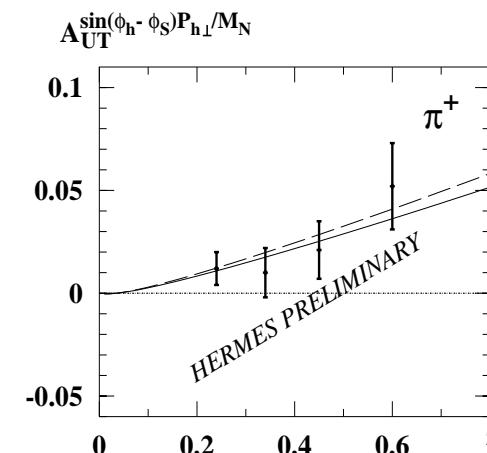
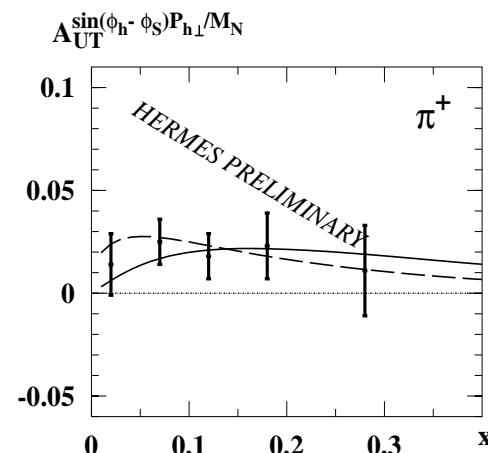
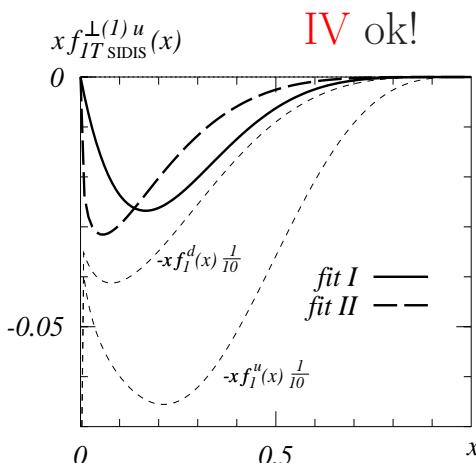
II satisfy sum rule $\sum_{a=g,u,d,\dots} \int dx f_{1T}^{\perp(1)a}(x) = 0$ Burkardt, PRD 69 (2004) 057501 and 091501

III explore “connection” of $f_{1T}^{\perp a}$ to GPD, $E^q(x, 0, 0) \propto (1-x)^4$ or $(1-x)^5$ Brodsky et al, Burkardt

IV respect positivity $\frac{|\mathbf{p}_T|}{M_N} |f_{1T}^{\perp a}(x, \mathbf{p}_T^2)| \leq f_1^a(x, \mathbf{p}_T^2)$ Bacchetta, et al., PRL 85 (2000) 712

- fit Ansatz: $x f_{1T}^{\perp(1)u} = -x f_{1T}^{\perp(1)d} = A x^b (1-x)^5$ neglect \bar{q}, s, \dots Respects I, II, III!

- fit result: $x f_{1T}^{\perp(1)u} = \begin{cases} -0.4x (1-x)^5 & \text{for } b=1 \text{ fixed} \\ -0.1x^{0.3}(1-x)^5 & \text{both } A, b \text{ free} \end{cases}$ Cross check!



★ Footnote for historical correctness:

- M. Anselmino, V. Barone, A. Drago and F. Murgia,
“Non-standard time reversal for particle multiplets and the spin-flavor structure of hadrons,”
Nucl. Phys. Proc. Suppl. **105** (2002) 132, hep-ph/0111044
- M. Anselmino, V. Barone, A. Drago and F. Murgia,
“Non-standard time reversal and transverse single-spin asymmetries”,
hep-ph/0209073
- P. V. Pobylitsa,
“T-odd quark distributions: QCD versus chiral models”,
hep-ph/0212027
- P. V. Pobylitsa,
“Transverse-momentum dependent parton distributions in large- N_c QCD”,
hep-ph/0301236.
- A. Drago,
“Time-reversal odd distribution functions in chiral models with vector mesons”,
Phys. Rev. D **71** (2005) 057501, hep-ph/0501282

Comments

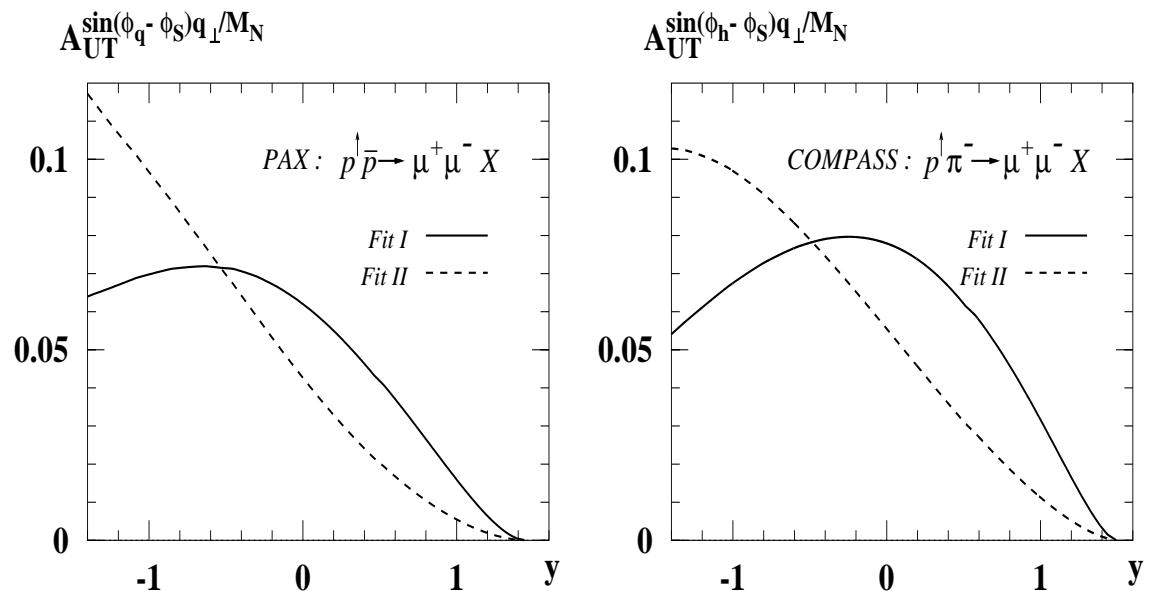
- supports picture by Burkardt, *op. cit.* $\int dx f_{1T}^{\perp(1)u}_{\text{SIDIS}}(x) \propto -\kappa^u < 0, \quad \int dx f_{1T}^{\perp(1)d}_{\text{SIDIS}}(x) \propto -\kappa^d > 0$
- Where are $1/N_c$ corrections? Good place to see: $f_{1T}^{\perp u/\text{deuteron}} \approx f_{1T}^{\perp u/p} + f_{1T}^{\perp u/n} \approx f_{1T}^{\perp u} + f_{1T}^{\perp d}$
- COMPASS $A_{UT, \text{deuteron}}^{\sin(\phi - \phi_s)} \sim 1/N_c \sim 0$ within error bars
- Thus HERMES & COMPASS compatible, $f_{1T}^{\perp a}|_{\text{SIDIS}}$ obeys large N_c predictions

Application

- important prediction
Collins, PLB 536 (2002) 43

$$f_{1T}^{\perp a}|_{\text{SIDIS}} = - f_{1T}^{\perp a}|_{\text{DY}}$$

- check in experiment!
- PAX at GSI $p^\uparrow \bar{p} \rightarrow \mu^+ \mu^- X$
- COMPASS $p^\uparrow \pi^- \rightarrow \mu^+ \mu^- X$
- possible to confirm (or ...)



Complication at the present stage: Status of used HERMES data PRELIMINARY.

Not recommended to use these data, because analysis of possible error correlations not finished
HERMES Collaboration, private communication

Necessary to solidify our conclusions → study the *FINAL* but “less preferably” weighted data.

$$A_{UT}^{\sin(\phi_h - \phi_S)} = (-2) \frac{\sum_a e_a^2 \int d^2 \mathbf{P}_{h\perp} \int d^2 \mathbf{p}_T \int d^2 \mathbf{K}_T \sin(\phi_h - \phi_S) \sin(\phi_{\mathbf{p}_T} - \phi_S) \frac{|\mathbf{p}_T|}{M_N} \delta^{(2)}(\mathbf{p}_T - \mathbf{K}_T - \mathbf{P}_{h\perp}/z) x f_{1T}^{\perp a}(x, \mathbf{p}_T^2) D_1^a(z, \mathbf{K}_T^2)}{\sum_b e_b^2 x f_1^b(x) D_1^b(z)}$$

→ Have to assume some **model** for transverse parton momenta to solve convolution integrals.

Already committed a crime: soft factors → $\delta^{(2)}(\dots)$. Why not another?

Let us use **Gaussian model**

$$\begin{aligned} f_1^a(x, \mathbf{p}_T^2) &\equiv f_1^a(x) \frac{\exp(-\mathbf{p}_T^2/p_{\text{unp}}^2)}{\pi p_{\text{unp}}^2}, \\ f_{1T}^{\perp a}(x, \mathbf{p}_T^2) &\equiv f_{1T}^{\perp a}(x) \frac{\exp(-\mathbf{p}_T^2/p_{\text{Siv}}^2)}{\pi p_{\text{Siv}}^2} \quad \xrightarrow{\textcolor{red}{\text{red}}} \quad A_{UT}^{\sin(\phi - \phi_S)} = (-2) \frac{a_{\text{Gauss}} \sum_a e_a^2 x f_{1T}^{\perp(1)a}(x) D_1^a(z)}{\sum_b e_b^2 x f_1^b(x) D_1^b(z)}, \\ D_1^a(z, \mathbf{K}_T^2) &\equiv D_1^a(z) \frac{\exp(-\mathbf{K}_T^2/K_{D_1}^2)}{\pi K_{D_1}^2} \quad \text{with} \quad a_{\text{Gauss}} = \frac{\sqrt{\pi}}{2} \frac{M_N}{\sqrt{p_{\text{Siv}}^2 + K_{D_1}^2/z^2}}. \end{aligned}$$

Mulders and Tangerman, NPB 461 (1996) 197. Efremov, Goeke and PS, PLB 568 (2003) 63.

Price to pay: 2 free parameters p_{unp}^2 , $\underbrace{p_{\text{Siv}}^2}_{\text{How to fix?}}$, $\underbrace{K_{D_1}^2}$ if we assume them flavour- and x - or z -independent.

Note: In numerous processes reasonable **model**, if: transv. momenta \ll hard scale
 D'Alesio and Murgia, PRD 70 (2004) 074009.

- In HERMES experiment $\langle P_{h\perp} \rangle \sim 0.4 \text{ GeV} \ll \sqrt{\langle Q^2 \rangle} \sim 1.5 \text{ GeV}$.

- more precisely: $\langle P_{h\perp}(z) \rangle$ of hadrons produced from a deuterium target [HERMES, PLB 562 \(2003\) 182](#)

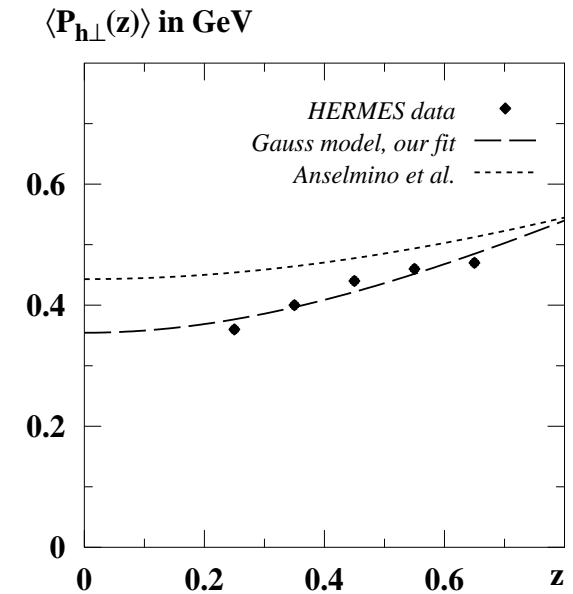
- $\langle P_{h\perp}(z) \rangle \stackrel{\text{Gauss}}{=} \frac{\sqrt{\pi}}{2} \sqrt{z^2 p_{\text{unp}}^2 + K_{D_1}^2}$

- $p_{\text{unp}}^2 = 0.33 \text{ GeV}^2$ and $K_{D_1}^2 = 0.16 \text{ GeV}^2 \rightarrow \text{Ok!}$

- compare to study of Cahn effect by [Anselmino et al.](#) where

$p_{\text{unp}}^2 = 0.2 \text{ GeV}^2$ and $K_{D_1}^2 = 0.25 \text{ GeV}^2$ obtained.

\Rightarrow Good qualitative agreement!



Ok, p_{unp}^2 , $K_{D_1}^2$ fixed. But what is p_{Siv}^2 ?

- Of course, cannot be zero! Otherwise $f_{1T}^\perp(x, \mathbf{p}_T^2) \rightarrow f_{1T}^\perp(x) \delta^{(2)}(\mathbf{p}_T) \Rightarrow A_{UT}^{\sin(\phi - \phi_S)} \rightarrow 0$
- Also, cannot be arbitrarily large. To satisfy positivity in Gauss Ansatz

$$p_{\text{Siv}}^2 \leq \frac{p_{\text{unp}}^2}{1 + \frac{2M_N^2 p_{\text{unp}}^2}{e p_{\text{Siv}}^4} \left(\frac{f_{1T}^{\perp(1)a}(x)}{f_1^a(x)} \right)^2}.$$

Parameter restricted $0 < p_{\text{Siv}}^2 < p_{\text{unp}}^2$. Admittedly vague. But, which is astonishing, . . .

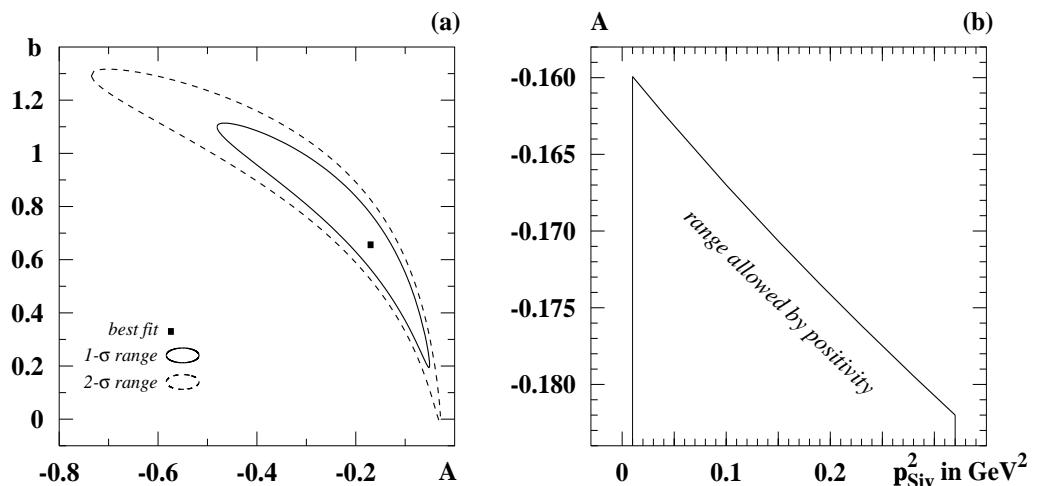
- $0 < p_{\text{Siv}}^2 < 0.33 \text{ GeV}^2$ **sufficient**, if we extract $f_{1T}^{\perp(1)a}(x) = \frac{p_{\text{Siv}}^2}{2M_N^2} f_{1T}^{\perp a}(x)$ and not $f_{1T}^{\perp a}(x)$ itself.
- $$\Rightarrow \text{needed only in } a_{\text{Gauss}} = \frac{\sqrt{\pi}}{2} \frac{M_N}{\sqrt{p_{\text{Siv}}^2 + K_{D_1}^2/z^2}} \Rightarrow 0.72 < a_{\text{Gauss}} < 0.83 \Rightarrow 10\% \text{ uncertainty!}$$

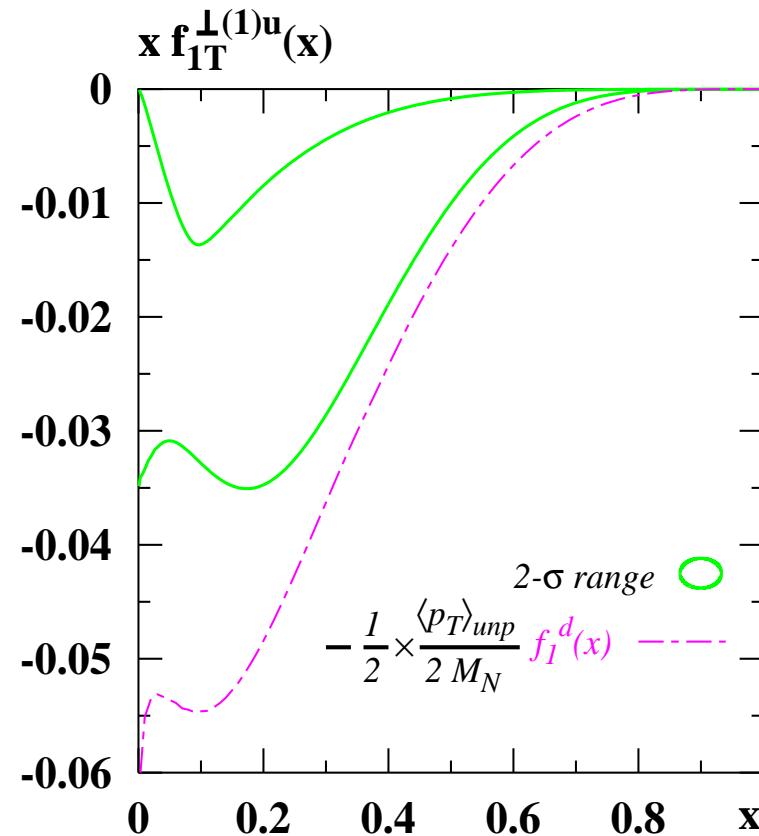
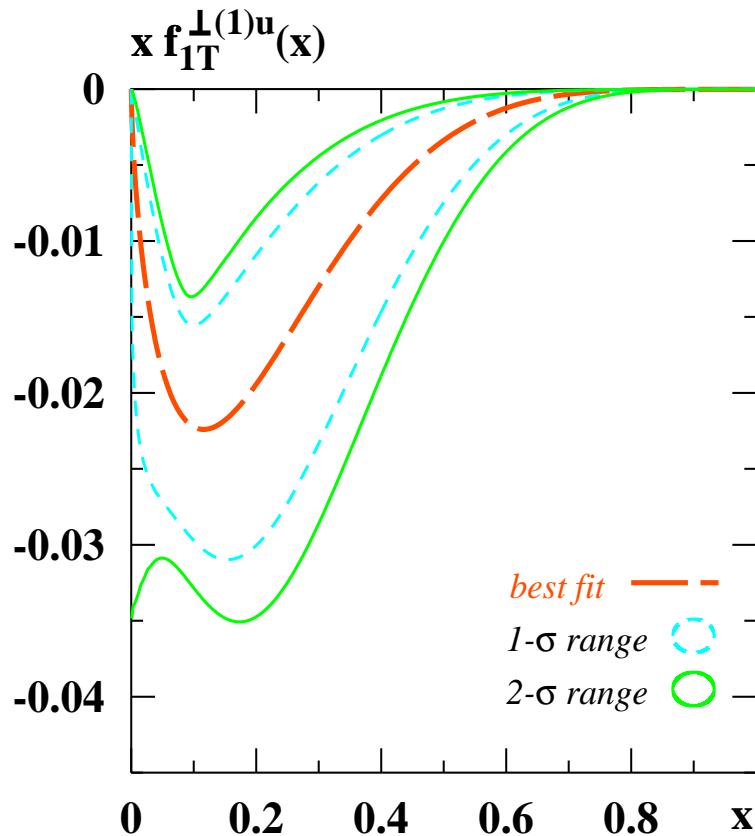
Procedure

- Ansatz $x f_{1T}^{\perp(1)u}(x) = -x f_{1T}^{\perp(1)d}(x) = A x^b (1-x)^5$, neglect anti- and heavier quarks.
- Choose some $p_{\text{Siv}}^2 \in [0, 0.33 \text{ GeV}^2]$
- Fit to *FINAL* HERMES data
- Check (within 2σ) positivity $p_{\text{Siv}}^2 \leq \frac{p_{\text{unp}}^2}{1 + \frac{2M_N^2 p_{\text{unp}}^2}{e p_{\text{Siv}}^4} \left(\frac{f_{1T}^{\perp(1)a}(x)}{f_1^a(x)} \right)^2}$

Result

- Best fit ($\chi^2 \sim 2.2$)
- $$x f_{1T}^{\perp(1)u}(x) = -(0.16 \dots 0.18) x^{0.66} (1-x)^5$$
- for $p_{\text{Siv}}^2 = (0.01 \dots 0.32) \text{ GeV}^2$
- refers to scale $\sim 2.5 \text{ GeV}^2$





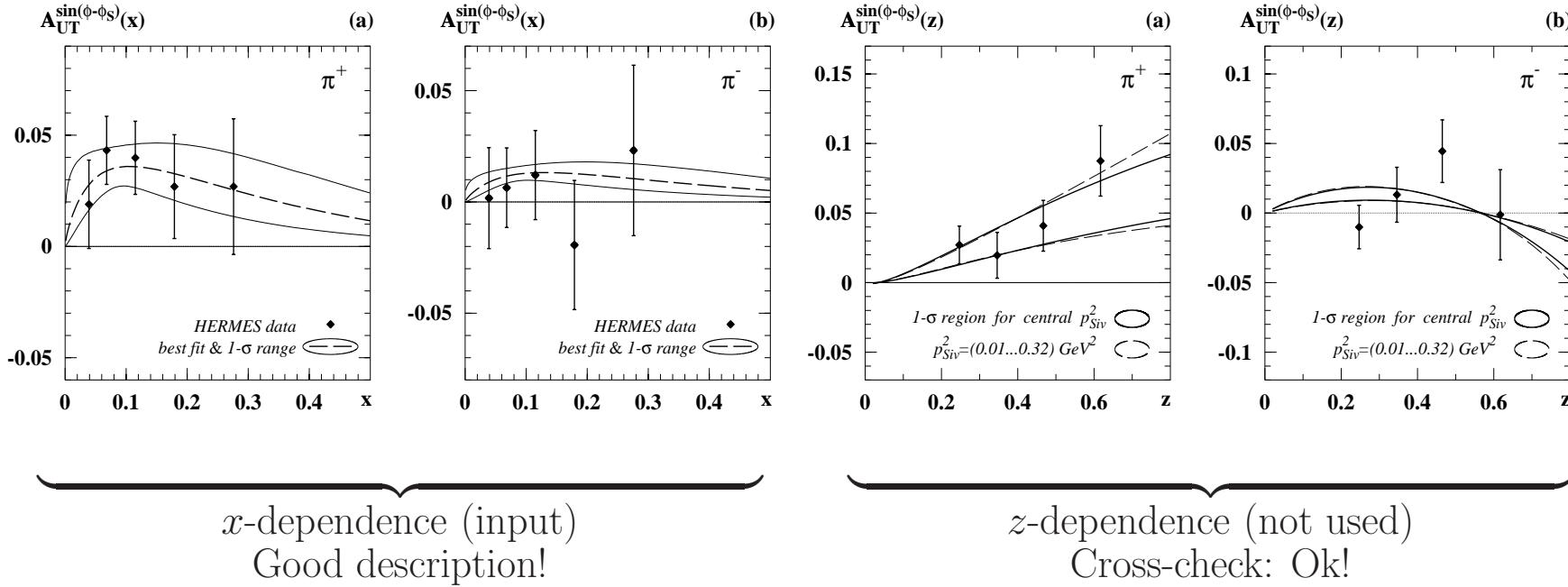
Positivity ok! & Not small!!!

- In large- N_c : $\frac{|\mathbf{p}_T|}{M_N} |f_{1T}^{\perp u}(x, \mathbf{p}_T^2)| = \frac{|\mathbf{p}_T|}{M_N} |f_{1T}^{\perp d}(x, \mathbf{p}_T^2)| \leq f_1^u(x, \mathbf{p}_T^2) = f_1^d(x, \mathbf{p}_T^2)$
- In nature $f_1^u(x, \mathbf{p}_T^2) > f_1^d(x, \mathbf{p}_T^2) \Rightarrow$ for d -quark stronger. To check multiply by $|\mathbf{p}_T|$ & integrate

$$|f_{1T}^{\perp(1)u,d}(x)| \leq \frac{\langle p_T \rangle_{\text{unp}}}{2M_N} f_1^d(x)$$

$$\langle p_T \rangle_{\text{unp}} = \frac{\int d^2 \mathbf{p}_T |\mathbf{p}_T| f_1^q(x, \mathbf{p}_T^2)}{\int d^2 \mathbf{p}_T f_1^q(x, \mathbf{p}_T^2)} \stackrel{\text{Gauss}}{=} \frac{\sqrt{\pi}}{2} p_{\text{unp}} = 0.51 \text{ GeV}$$

How does it describe the HERMES data? Well!



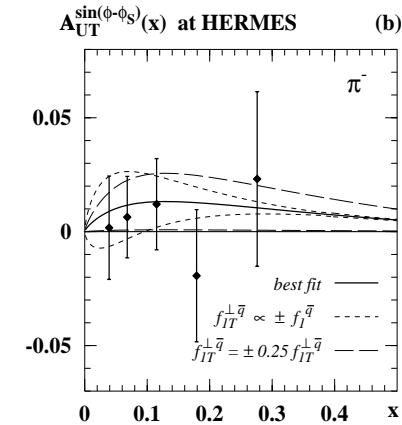
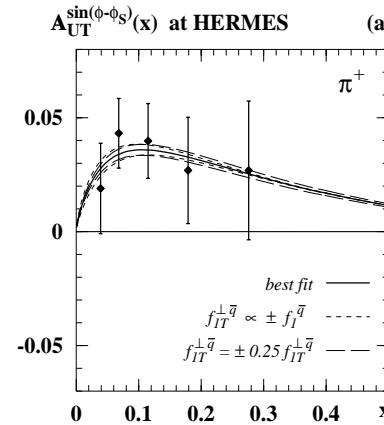
Good description. However, two questions arise:

- $f_1^{\bar{d}}(x)/f_1^d(x) \sim 25\%$ at small HERMES- x . Justified to neglect Sivers- \bar{q} ?
 - Where are $1/N_c \sim 30\%$ corrections?

Sivers- \bar{q}

- Let us simulate

$$f_{1T}^{\perp(1)\bar{q}}(x) = \underbrace{f_{1T}^{\perp(1)q}(x)}_{\text{best fit}} \times \begin{cases} \pm 25\% \\ \pm \frac{(f_1^{\bar{u}} + f_1^{\bar{d}})(x)}{(f_1^u + f_1^d)(x)} \end{cases}$$



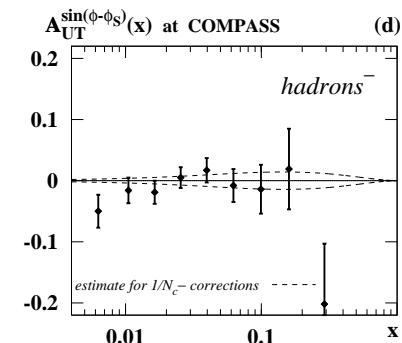
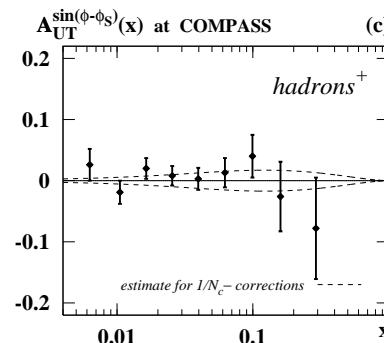
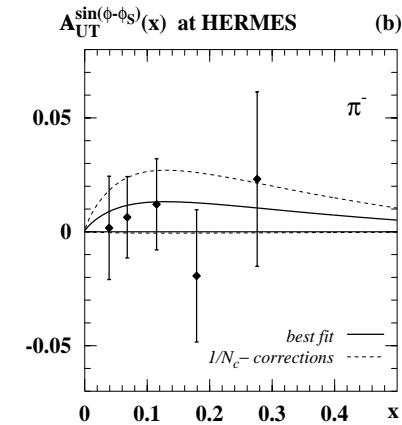
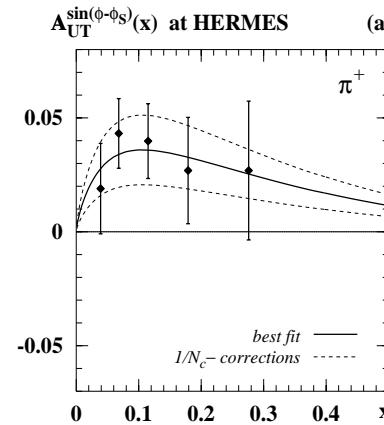
$1/N_c$ -corrections

- $\underbrace{|(f_{1T}^{\perp u} + f_{1T}^{\perp d})(x)|}_{\mathcal{O}(N_c^2)} \ll \underbrace{|(f_{1T}^{\perp u} - f_{1T}^{\perp d})(x)|}_{\mathcal{O}(N_c^3)}$

$$(f_{1T}^{\perp u} + f_{1T}^{\perp d})(x) = \pm \frac{1}{N_c} \underbrace{(f_{1T}^{\perp u} - f_{1T}^{\perp d})(x)}_{\text{best fit}} \neq 0$$

- $f_{1T}^{\perp u/\text{deuteron}} = (f_{1T}^{\perp u} + f_{1T}^{\perp d})$, etc.

Sivers SSA on deuterium $\stackrel{!}{=} "1/N_c\text{-correction}"$
(Beware: Small x !)



Conclusion: Neglect of \bar{q} -effects and $1/N_c$ -corrections in fit Ansatz **not unjustified** for present data

Maybe, suspicion(!), large- N_c works even *particularly well* for Sivers function because it *happens to work* particularly well for the anomalous magnetic moments “related” by $\int dx f_{1T}^{\perp(1)q} \text{SIDIS}(x) \propto -\kappa^q$, Burkardt, *op. cit.*

Recall: $\kappa^u = 1.673$ and $\kappa^d = -2.033$

In nature: $\underbrace{|\kappa^u - \kappa^d|}_{\mathcal{O}(N_c^2)} \sim 3.706 \gg \underbrace{|\kappa^u + \kappa^d|}_{\mathcal{O}(N_c)} \sim 0.360$

Is this an additional reason why large- N_c useful for Sivers effect? The future will show.

There are large- N_c predictions also for all the other novel twist-2 functions. E.g. Boer-Mulders function

$$h_1^{\perp u} = h_1^{\perp d} \quad \text{modulo } 1/N_c \text{ corrections, ...}$$

[Pobylitsa hep-ph/0301236](#)
Worthile to be kept in mind.

Where else is large- N_c useful?

Worthwhile recalling $\frac{\Delta G(x)}{G(x)} \sim \frac{1}{N_c}$ Efremov, Goeke and Pobylitsa, PLB 488 (2000) 182, hep-ph/0004196

$(\Delta \bar{u} - \Delta \bar{d})(x) > |(\bar{u} - \bar{d})(x)| \neq 0$ Diakonov, NPB 480 (1996) 341, hep-ph/9606314 Being tested ...

$f_{1T}^{\perp(1)q}$ from FINAL
HERMES data analysed
without $P_{h\perp}$ -weight
hep-ph/0509076

$f_{1T}^{\perp(1)q}$ from PRELIMINARY
but “more preferably”
 $P_{h\perp}$ -weighted data
PLB 612 (2005) 233, hep-ph/0412353

Here FINAL data on $A_{UT}^{\sin(\phi-\phi_S)}$

Gauss \longrightarrow ... constrain parameters ... **etc.** $\longrightarrow f_{1T}^{\perp(1)q}(x)|_{\text{model-dependent}}$
 $\updownarrow ?$
 directly $\longrightarrow f_{1T}^{\perp(1)q}(x)|_{\text{“model-independent”}}$

There PRELIMINARY data on $A_{UT}^{\sin(\phi-\phi_S)P_{h\perp}/M_N}$

Does the Gauss Ansatz work?

- The same events with/without $P_{h\perp}$ -weight!

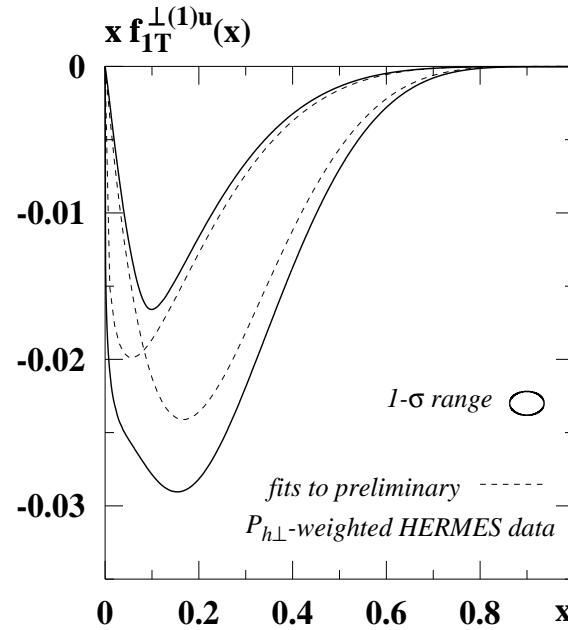
If Gauss ok \implies the same $f_{1T}^{\perp(1)q}(x)$

\implies **Yes, it works!**

Grain of salt:

- Be careful: Preliminary data.

“Indication”: Effects of error correlations
less dominant than statistical error of data



Intermediate conclusion:

- Gauss Ansatz & large- N_c presently useful tools
- Extraction of $f_{1T}^{\perp(1)q}(x)$ from preliminary data “confirmed”
- Previous conclusions solidified (DY, change of sign, PAX & COMPASS, etc.)

see also Anselmino *et al.*, and Vogelsang and Yuan, *op. cit.*

How to test further Gauss model?

- Range of reliability? Limitations?
- e.g. for π^0 (neglecting s , ...)

$$A_{UT}^{\sin(\phi-\phi_S)}(\pi^0)(z) = a_{\text{Gauss}}(z) \frac{\sum_a e_a^2 \langle x f_{1T}^{\perp(1)a} \rangle}{\sum_b e_b^2 \langle x f_1^b \rangle}$$

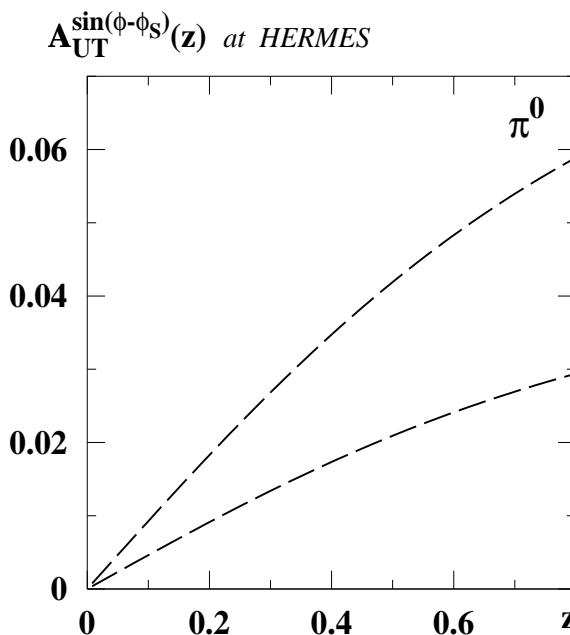
$$\propto \frac{z}{\sqrt{\underbrace{z^2 p_{\text{Siv}}^2}_{\text{smaller}} + \underbrace{K_{D_1}^2}_{\text{larger}}}}$$

How to see $1/N_c$ corrections?

- More precise deuteron & proton data
most recent PRELIMINARY HERMES data Diefenthaler, DIS'05, hep-ex/0507013 still compatible

How to see Sivers- \bar{q} ?

- Drell-Yan



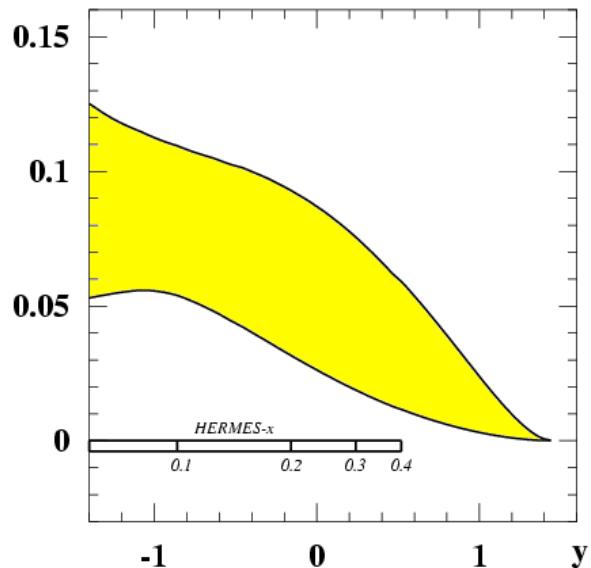
Sivers effect in DY

PAX & COMPASS

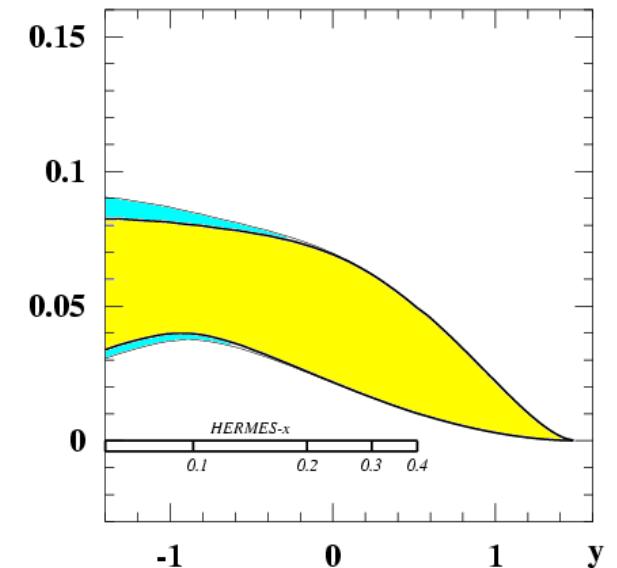
- $p^\uparrow \bar{p}$
- $p^\uparrow \pi^- \} \rightarrow l^+ l^- X$

dominated by annihilations of valence q and valence \bar{q}
 \Rightarrow not sensitive to Sivers- \bar{q} , good!

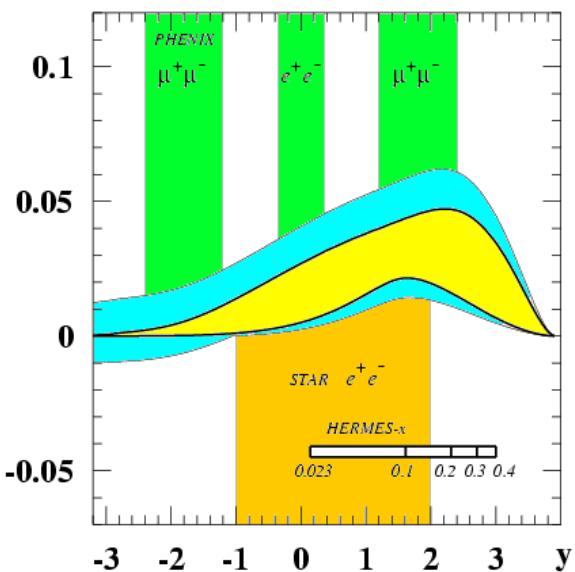
$A_{UT}^{\sin(\phi - \phi_S)}$ in $p^\uparrow \bar{p} \rightarrow l^+ l^- X$ at PAX



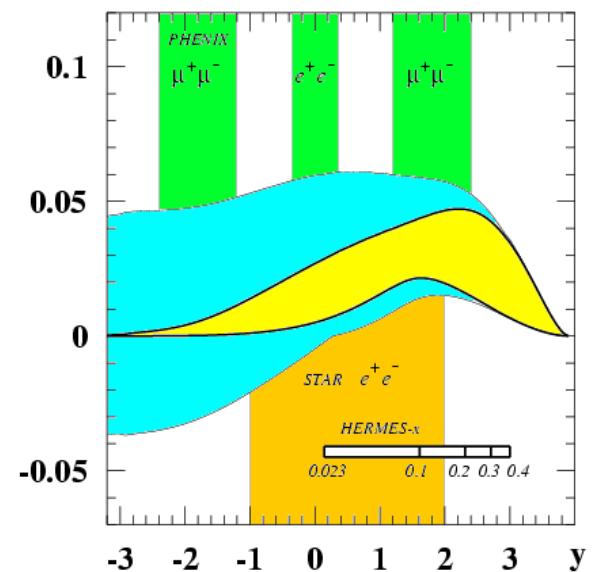
$A_{UT}^{\sin(\phi - \phi_S)}$ in $p^\uparrow \pi^- \rightarrow l^+ l^- X$ at COMPASS



$A_{UT}^{\sin(\phi - \phi_S)}$ in $p^\uparrow p \rightarrow l^+ l^- X$ at RHIC $Q=4\text{GeV}$



$A_{UT}^{\sin(\phi - \phi_S)}$ in $p^\uparrow p \rightarrow l^+ l^- X$ at RHIC $Q=4\text{GeV}$



yellow = $1-\sigma$ region

blue = effects due to Sivers- \bar{q}

RHIC

- $p^\uparrow p \rightarrow l^+ l^- X$

valence q and sea \bar{q} on equal footing
sensitive to Sivers- \bar{q} in certain y -region

- RHIC can test “change of sign”
& provide information on Sivers- \bar{q} !

Conclusions

- Previous study of preliminary HERMES data on $A_{UT}^{\sin(\phi-\phi_S)P_{h\perp}/M_N}$
solidified by study of final HERMES data on $A_{UT}^{\sin(\phi-\phi_S)}$
- Gaussian model \Rightarrow parameters constrained in terms of HERMES data \Rightarrow consistent with data
- Large- N_c predictions used (and: tested!). Useful guideline!
 \Rightarrow naturally explains compatibility of HERMES (proton) and COMPASS (deuteron) data
- Obtained fit satisfies all presently known theoretical constraints
(large- N_c , Burkardt sum-rule, “connection” to GPD, positivity)
- Consistent (of course) with fits by Anselmino *et al.*, Vogelsang and Yuan (but different assumptions!)
- Change of sign $f_{1T}^{\perp a}|_{\text{SIDIS}} = - f_{1T}^{\perp a}|_{\text{DY}}$ can be tested at COMPASS & PAX (byproduct) for Sivers- q . . .
- . . . also at RHIC, where first glimpse on Sivers- \bar{q} possible
- Most recent & more precise preliminary HERMES data somehow larger Sivers effect.
Still compatible with large- N_c . \Rightarrow even more optimistic estimates for Drell-Yan

Thank you!