Quantum Structures

in

Chromodynamics

Como
Sep. 2005

OUTLINE

I. The Complexity of the Proton

a) the message of sum rules

b) confinement & chiral symmetry

II. Ratcliffe Resolution Structures

- massless chiral constituents
- example - constituent quark

III. Orbital Structures

- single spin asymmetries
- normalization/factorization
- connection to spectator models

I. COMPLEXITY

understanding of the standard model is complicated by the complexity of the structure of the proton (the only stable hadron) & by inference, othe hadrons. (excepting only those with only <u>massive</u> quarks (c, b, t)

we have to resort to lengthy computer simulations using lattice regularization to quantify simplest system.

The proton is a reflection of the interaction of its constituents. Many of its properties are dynamic rather than an apportionment of the properties of the underlying quanta

- < 5% proton mass associated with quark masses
- < 45% of momentum associated with quarks
- < 30% of J_z carried by J_z of quarks

...

The interactions lead to Stable Quantum Structures

RATCLIFFE RESOLUTION STRUCTURES

In 1978 paper Babcock, Monsay, Sivers
[$J_z = 1/2$ sum rule]

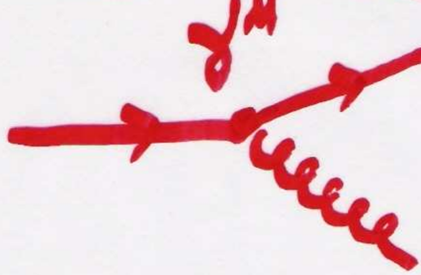
$$J_z = 1/2 = \sum_i \Delta q^i + \sum_i \Delta \bar{q}_i + \Delta G + \sum_k \langle L_z \rangle_k$$

$\lim Q^2 \to \infty$ Δq_i, $\Delta \bar{q}_i$ const

$\Delta G \to +\infty$, $\sum_k \langle L_z \rangle_k \to -\infty$

Phil Ratcliffe examined the Spin-Dependent DGLAP 'eq'n's & showed

DGLAP Kernels



Resolve one object into 2

$q \to q' G$

$$\left\{ \begin{array}{l} J_z^q = J_z^{q'} \\ J_z^G = -L_z \end{array} \right\}$$ quark helicity conservation

This gave (order-by-order) a way to understand the content of the sum rule

For light quarks this should have a "non-perturbative" extension ....

you can always "resolve" a massive object into chiral objects

$$P^\mu = a\left(k^\mu + l^\mu/a\right) \qquad l\cdot l = k\cdot k = 0$$

$$W^\mu = \left(k^\mu - l^\mu/a\right) \qquad P^2 = a\, 2k\cdot l = m^2$$

$$W^2 = -\frac{2k\cdot l}{a} = \frac{m^2}{a^2}$$

$a^2 = \frac{1}{s(s+1)}$ if you identify W with Wightman-Pauli-Lubanski spin vector

Example: Constituent quark mass m with spin "quantized" in the $\hat{y}$ direction

$$ak = \frac{m}{2}(1, \sin\theta\sin\phi, \cos\theta, \sin\theta\cos\phi)$$

$$l = \frac{m}{2}(1, -\sin\theta\sin\phi, -\cos\theta, -\sin\theta\cos\phi)$$

$$p = ak + l = m(1, 0, 0, 0)$$

$$W \quad k - l/a = m\left(\tfrac{3}{4}\right)^{1/2}(0, \sin\theta\,\overset{\sin\phi}{\cancel{\cos\phi}}, \cos\theta, \sin\theta\cos\phi)$$

for $s = 1/2$

$$\left(3/4\right)^{1/2}\cos\theta = 1/2 \qquad \cos\theta = 1/\sqrt{3}$$

$$\sin\theta = \sqrt{2/3}$$

$$\tilde{W} = m\,(0, \tfrac{1}{\sqrt{2}}\sin\phi, \tfrac{1}{2}, \tfrac{1}{\sqrt{2}}\cos\phi)$$

$$k = \frac{m}{4}(3)^{1/2}(1, \sqrt{2/3}\sin\phi, \sqrt{1/3}, \sqrt{2/3}\cos\phi)$$

$$l = \frac{m}{2}(1, -\sqrt{2/3}\sin\phi, -\sqrt{1/3}, -\sqrt{2/3}\cos\phi)$$

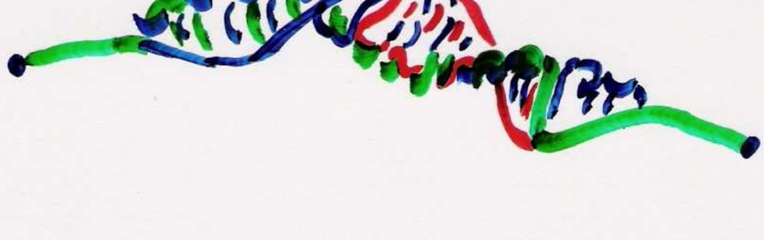
the "precession" of spin vector can be associated with the "momenta" of chiral <u>constituents</u>.

Ratcliffe Resolution Structures are ubiquitous. Resolving composite systems involving <u>spin</u> lead to "orbital" motions for chiral constituents.

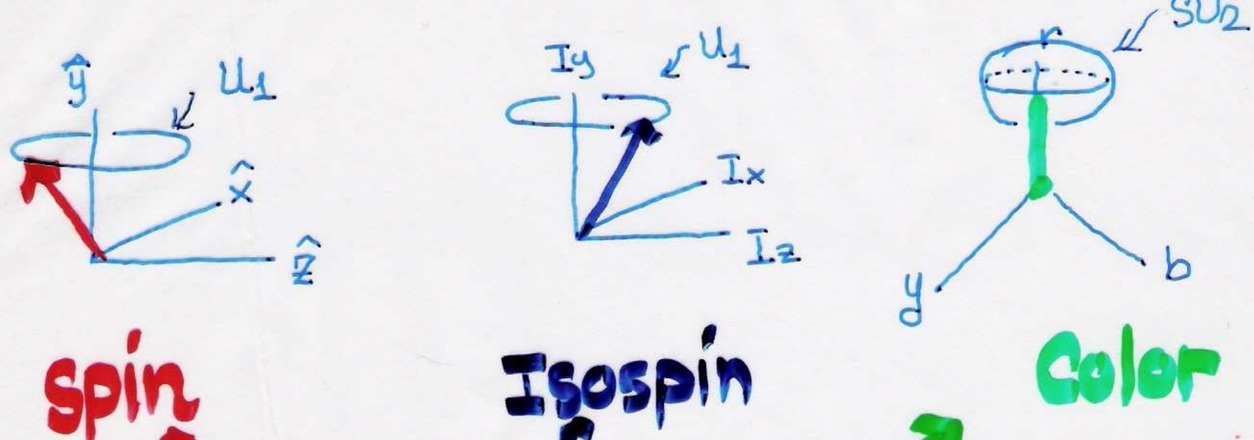
<u>and</u>

ORBITING CONSTITUENTS CAN LEAD TO HARD/SCATTERING SPIN ASYMMMETRIES !!

as we will discuss



This decomposition is part of the "null-tetrad" classification of gauge fields (Carmeli/Charak/Kaye (77))



Local direct product

$$\psi = \{ \xi \otimes \eta \otimes t \}$$

chiral objects frame the manifolds defined by the uncertainty relations of the groups

Study of classical gauge structures & Weyl-Dirac eqn.

Ralston, Sivers PR (83)
PR (84)

Sivers PR (83)
PR (86)
PR (87)
PR (89)

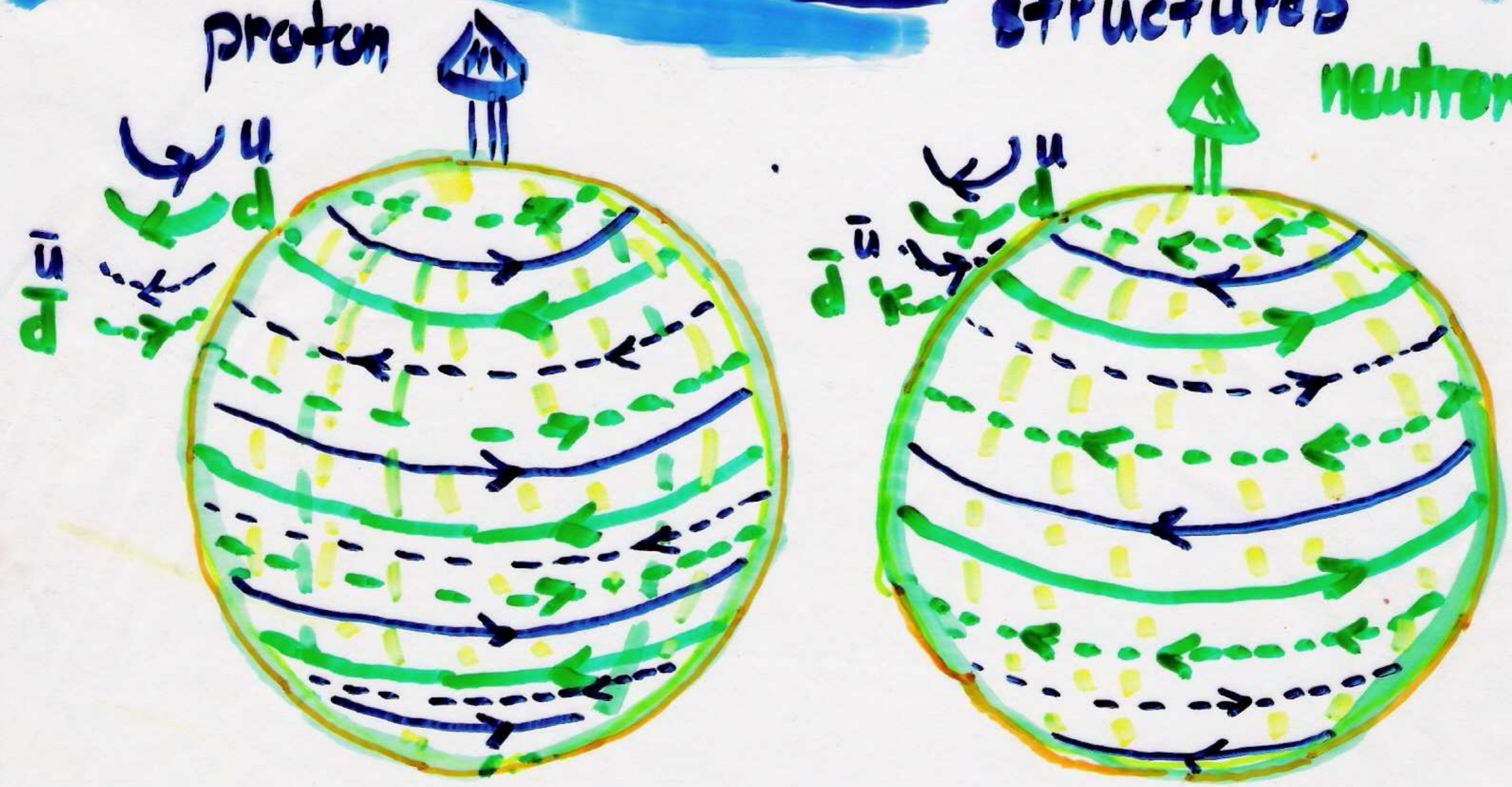
Long study of these results

There exist simple, cogent theoretical arguments to suggest:

- flavor/isospin currents exist on the surface of a confined system in chromodynamics
- In the proton, these currents are correlated with the proton's spin orientation
- the surface currents play a significant role in multi-baryon composite systems (nuclei)
- associated with "meson cloud" of classical nuclear physics -- however meson description inadequate

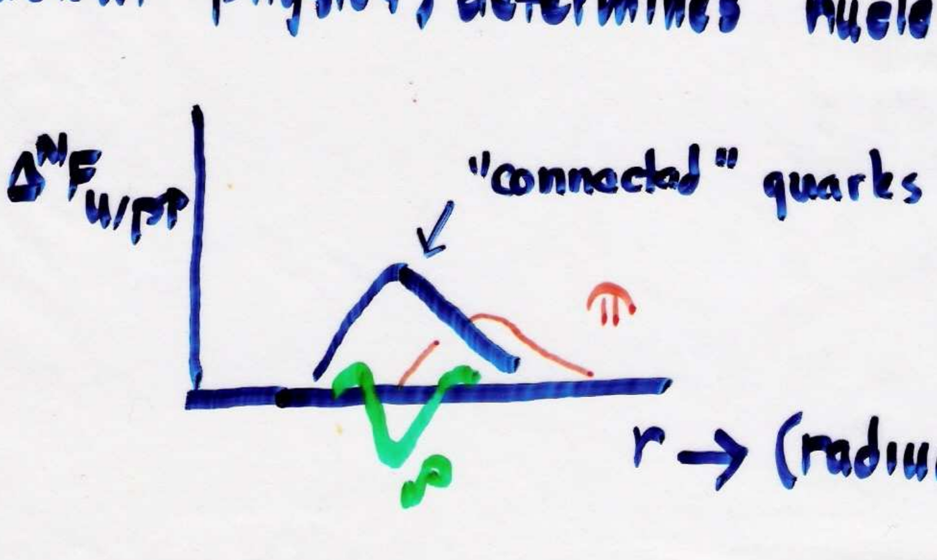
Isospin and Charge Conjugation
G - Parity

stable orbital structures



approx. cancellation $\Delta^u F$ for deuteron confirmed by [redacted]! (compass)

The "skin" of Nucleon (AKA "meson cloud" in nuclear physics) determines nuclear interaction

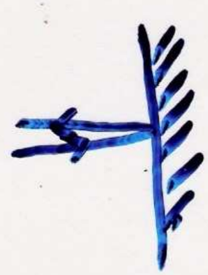


Like a Littoral zone the surface has its own rules

III ORBITAL STRUCTURES

Confinent and chiral symmetry combine to create spin-orbit correlations

chirality & confinement



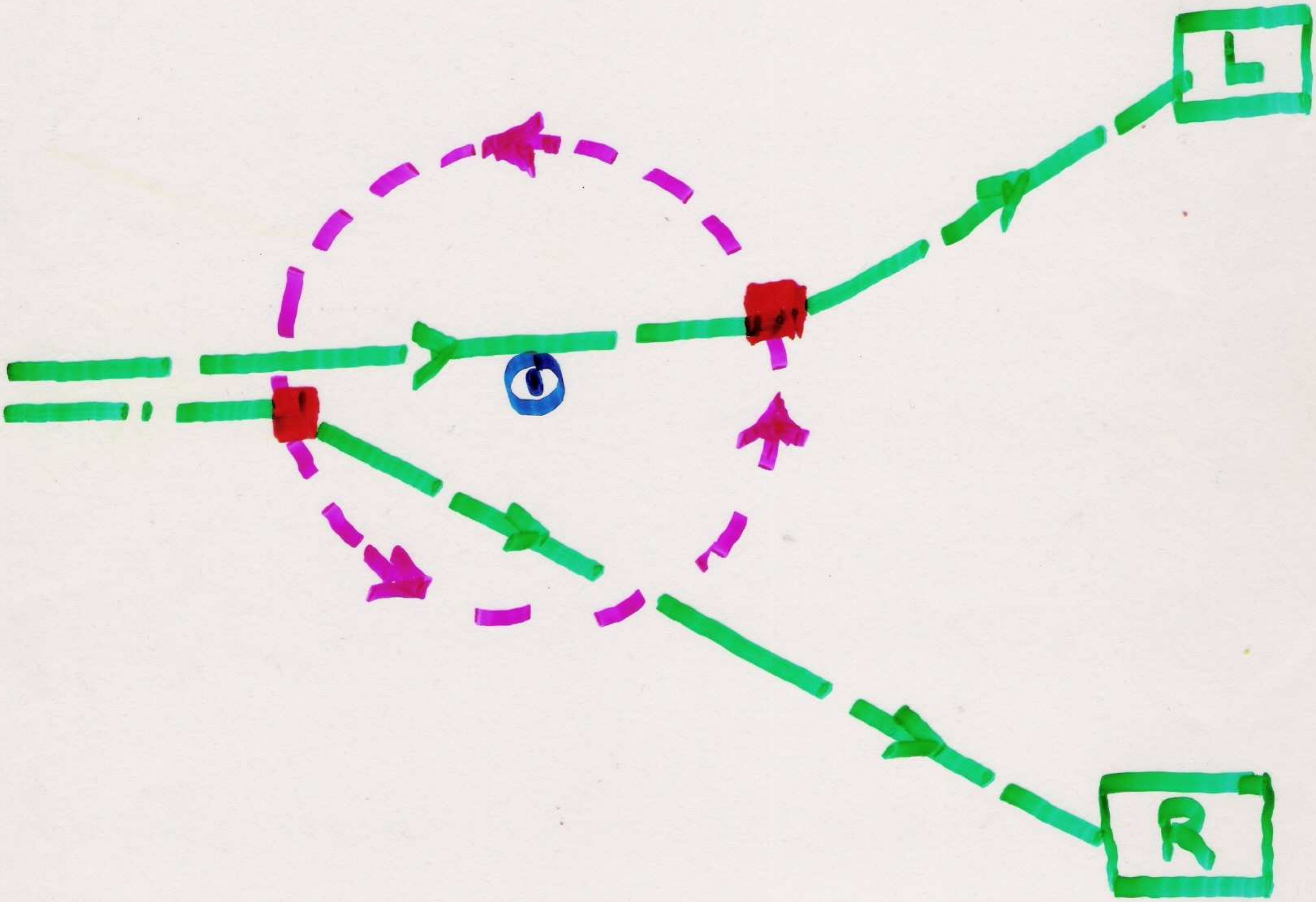
$\Delta J_z = -1$

I. Single-Spin Asymmetries & Orbital Structures in Hadronic Distributions

The quantum Description of scattering from rotating objects (such as "components" of a quantum "fan") leads directly to single-spin observables

(demonstration)

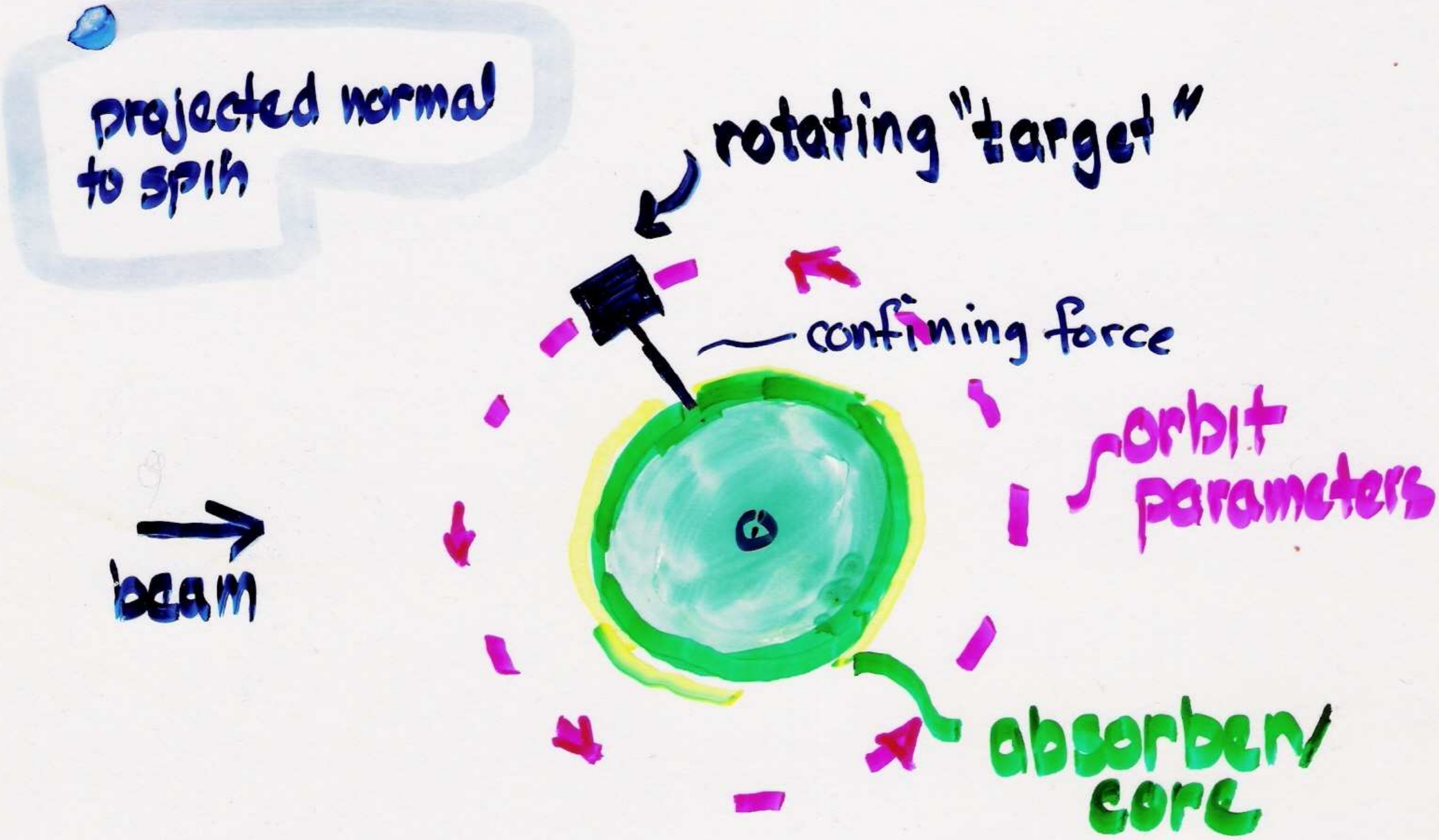
QUANTUM ASYMMETRIES from Rotating Body



apply Cauchy inequalities
necessary (but not sufficient)
for L-R asymmetry - initial and/or
final state interactions

shadowing - amplitude
parton energy loss - hard scattering
kinematics

components of a quantum fan



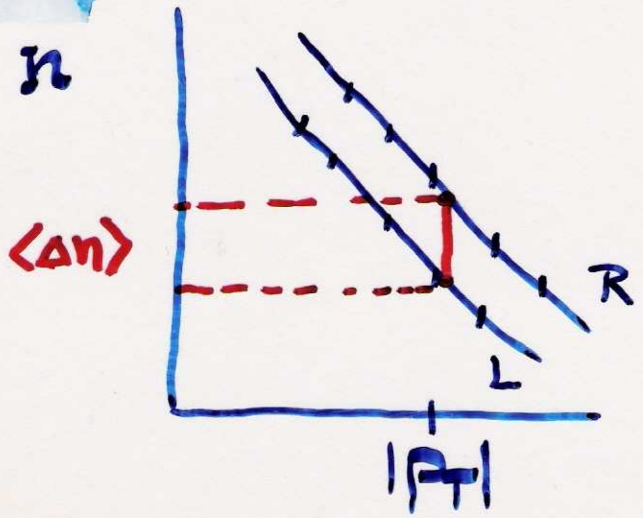
Strong initial-state interactions:
(Drell-yan) dominant scattering from front

Strong final-state interactions:
(SIDIS) dominant scattering from back

Both initial & final state interactions
(pp → jet x) dominant scattering from "top"
(large-x) with "Regge" treatment
of interactions with absorber

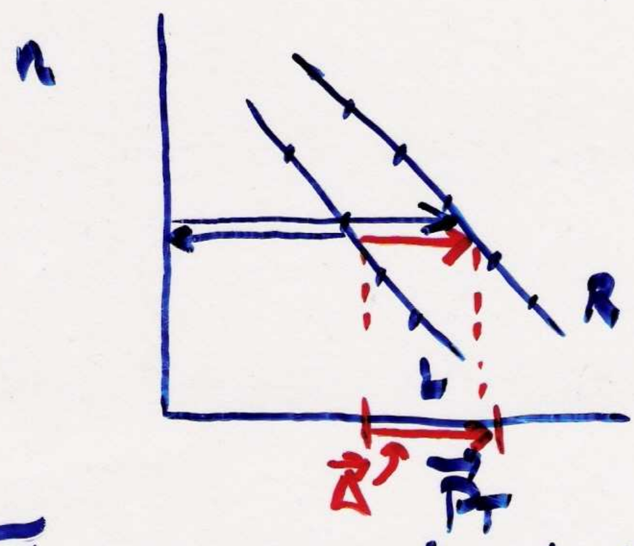
Take absorber away ... No asymmetries

A comment on Asymmetries in Gauge Theories & the Trento Conventions & Bohm-Aharanov



$\Delta^N_{R-L}(n)$

$\boxed{\text{two definitions of - sign}}$



$\vec{\Delta}^N_{R-L}(\vec{p}_T)$

$\vec{\Delta}$

$\langle \vec{p}_T \rangle_{R-L} = \bar{n}\vec{\Delta}$

missing momentum

[in an experiment these are equivalent]

However in a gauge theory, the L-R scales (can be different)

$\Delta, \vec{\Delta}$ $\quad p_T^{\mu} - ig A_T^{\mu}$ $\quad \vec{\Delta}$ gauge covariant

Distinguish by arrow (Rotate · Detector $\dot{L} \to \dot{R}$)

II. Preliminaries

Single-Spin observables are tightly constrained by finite symmetries and rotational invariance

All single-spin observables are <u>odd</u> under the combination of <u>Parity</u> (P) and another symmetry

A_τ

$$\boxed{PA_\tau = - \quad \text{for } \underline{\text{any}} \text{ single spin observable}}$$

A_τ 

A_τ projects out Transversity amplitudes (Moravcik, Goldstein)

<u>mom.)</u>

$P(\vec{p}_i, \vec{\sigma}_i) \to (-\vec{p}_i, \vec{\sigma}_i)$

$A_\tau(\vec{p}_i, \vec{\sigma}_i) \to (-\vec{p}_i, -\vec{\sigma}_i)$

$PA_\tau(\vec{p}_i, \vec{\sigma}_i) \to (\vec{p}_i, -\vec{\sigma}_i)$

A_τ is linear and, unlike T, does <u>not</u> reverse the order of Field theory operators

$$T(Q_1 Q_2 Q_3)T^{-1} = (\tilde{Q}_3 \tilde{Q}_2 \tilde{Q}_1)$$

without regard for commutation relations

$$A_\tau (Q_1 Q_2 Q_3) = Q_1' Q_2' Q_3'$$

$\hat{S}\cdot(\vec{p}_1 \times \vec{p}_2)$ <u>even</u> under time reversal and <u>odd</u> unde A_τ

I now call it Artificial time reversal

$$A_\tau \cdot A_\tau = \mathbb{1}$$

$$\left(\frac{\mathbb{1} \pm A_\tau}{2}\right) = \mathcal{P}_\pm$$ Projection Operators

amplitude $\mathcal{M} = \mathcal{M}_+ + \mathcal{M}_-$

$$|\mathcal{M}|^2 = |\mathcal{M}_+|^2 + |\mathcal{M}_-|^2$$

In terms of helicity amplitudes

$$2\,d\sigma_R = K\{|N|^2 + |F|^2 + \mathrm{Im}(F^*N)\}$$

$$2\,d\sigma_L = K\{|N|^2 + |F|^2 - \mathrm{Im}(F^*N)\}$$

where N = nonflip amplitude
F = flip amplitude

$$A_N d\sigma = (d\sigma_L - d\sigma_R) = -K\,\mathrm{Im}(F^*N)$$

$$d\sigma_0 = (d\sigma_L + d\sigma_R) = K(|N|^2 + |F|^2)$$

Phase difference signal of SSA

A^γ projects transversity amplitudes
(Moravcik & Goldstein)

$$2\,d\sigma_L = K\{|m^{(+)}|^2 + |m^{(-)}|^2\}$$

$$2\,d\sigma_R = K\{|m^{(+)}|^2 - |m^{(-)}|^2\}$$

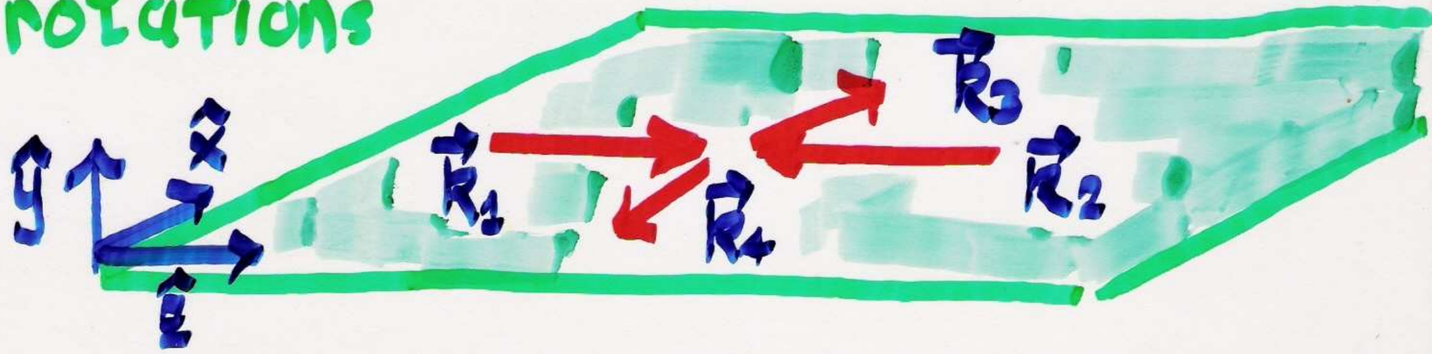
$$A_N d\sigma = K|m^{(-)}|^2$$

$$d\sigma_0 = K|m^{(+)}|^2$$

The change in basis provides simplification
→ directly to parton interpretation.

Jacob - Wick Helicity Amplitudes

& rotations



Scattering Process

$$(\vec{k}_1, \vec{\sigma}_1)(\vec{k}_2, \vec{\sigma}_2) \to (\vec{k}_3, \vec{\sigma}_3)(\vec{k}_4, \vec{\sigma}_4)$$

only one spin is measured $\vec{\Sigma}_j$

normal to the scattering plane

$$\vec{y} = \vec{k}_1 \times \vec{k}_3$$

In the time-reversed process

$$\vec{y}_T = (-\vec{k}_3) \times (-\vec{k}_1) = -\vec{y}$$

"<u>naive</u>" or "<u>artificial</u>" time reversal

$$\vec{y}_{A\tau} = (-\vec{k}_1) \times (-\vec{k}_3) = \vec{y}$$

the difference between T & Aτ <u>significant</u>

Repeat

	Σ_x	Σ_y	Σ_z
P	−	+	−
C	+	+	−
T	−	+	+
(PCT)	+	+	+
A_τ	+	−	+
(PA_τ)	−	−	−

A_τ artificial time reversal

$PA_\tau = -1$ for <u>all</u> single-spin observables.

$A_\tau = 1$ for light quark QCD pert theory

$A_\tau : \{ \vec{p}_i \to -\vec{p}_i$ (all i) $(\hat{\sigma}_i \to -\hat{\sigma}_i)$ all i $\}$

Confinement & Quantum Indeterminacy

$$[L_y, \sin\phi] = i\cos\phi$$

$$[L_y, \cos\phi] = -i\sin\phi$$

mom. space

$$[L_y, p_x/p_0] = i\, p_z/p_0$$

$$[L_y, p_z/p_0] = -i\, p_x/p_0$$

L_y plays the role of "position" operator

pos. space

$$[L_y, x/R] = -i\, z/R$$

$$[L_y, z/R] = i\, x/R$$

L_y plays the role of "momentum" operator

quantum confinement $L_y = m \rightarrow$

orbit singularity

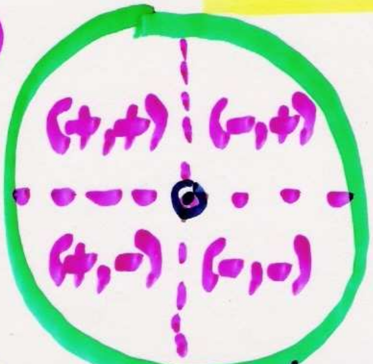
pole in complex m plane

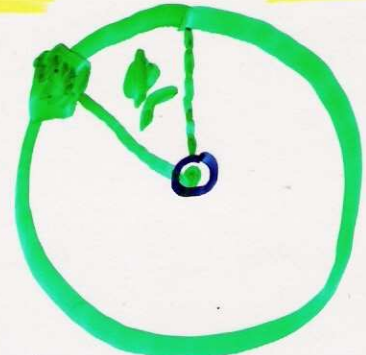
(this is where the fundamentals of the complex-J (Regge) plane come in)

Characteristics of an Orbit

$\vec{L} \cdot \hat{B} = +$

(sin, cos)





project on x-z plane - integrate over k_y

$$P_\mu = \left((m_q^2 + \langle k_y^2 \rangle + k_0^2)^{1/2},\ k_0 \sin\phi,\ 0,\ k_0 \cos\phi \right)$$

$$\mu = (m_q^2 + \langle k_y^2 \rangle + k_0^2)^{1/2}$$

energy of orbiting const. in proton rest-frame

orbit stability

$$\frac{\mu}{M_p} < 1/2$$

Project on light-cone

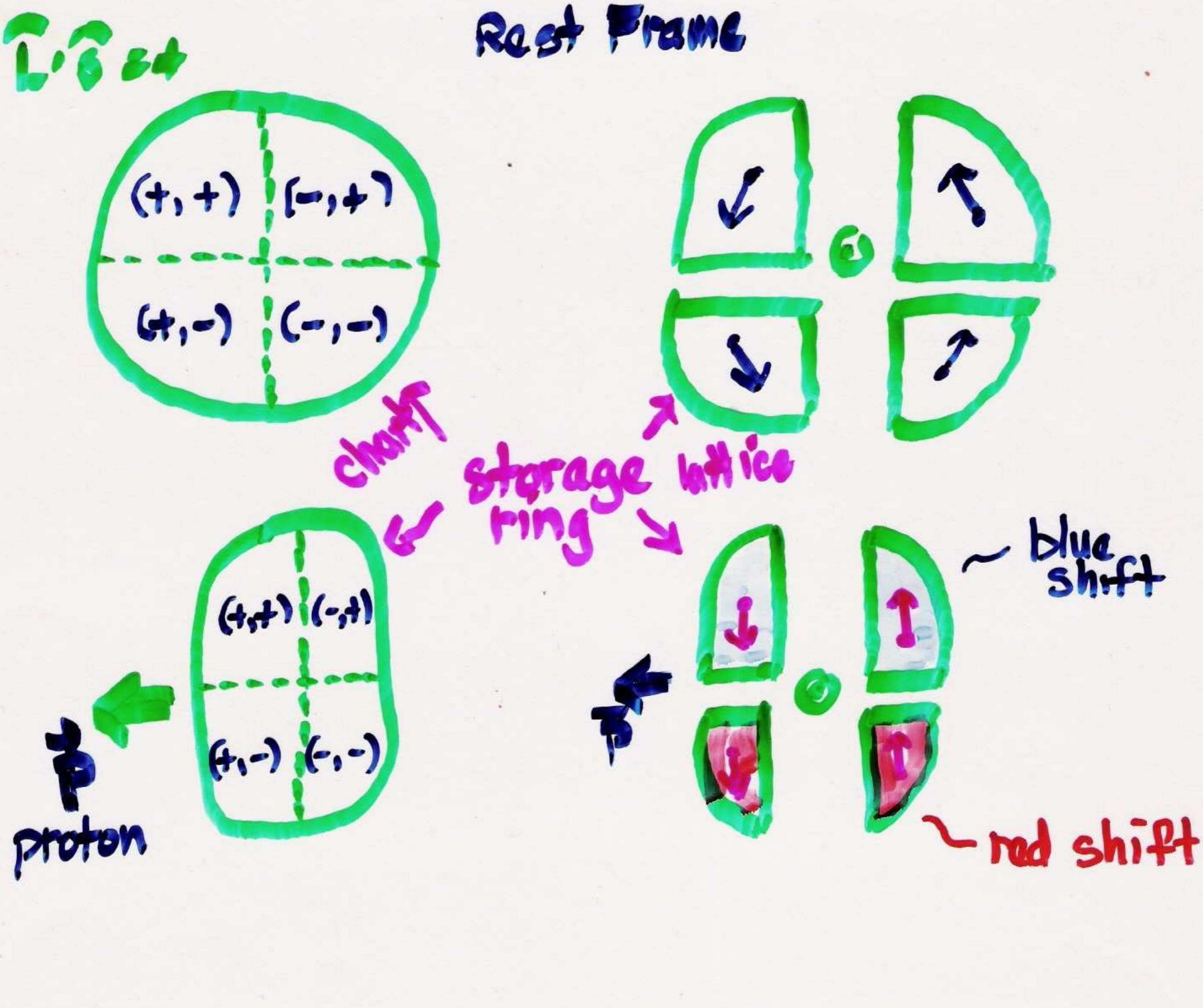
$$P_+ = \left[(m_q^2 + \langle k_y^2 \rangle + k_0^2)^{1/2} + k_0 \cos\phi \right]$$

$$P_- = \left[(m_q^2 + \langle k_y^2 \rangle + k_0^2)^{1/2} - k_0 \cos\phi \right]$$

top $\rightleftarrows$ bottom $\quad$ $P_+ \rightleftarrows P_-$ $\quad$ front $\rightleftarrows$ back $\quad$ $k_T \rightleftarrows -k_T$

to have a consistent Local quantum field theory, you need to respect the signs in the commutators and this implies

QFT Requires at <u>least</u> 4-sector representation of a internal rotator



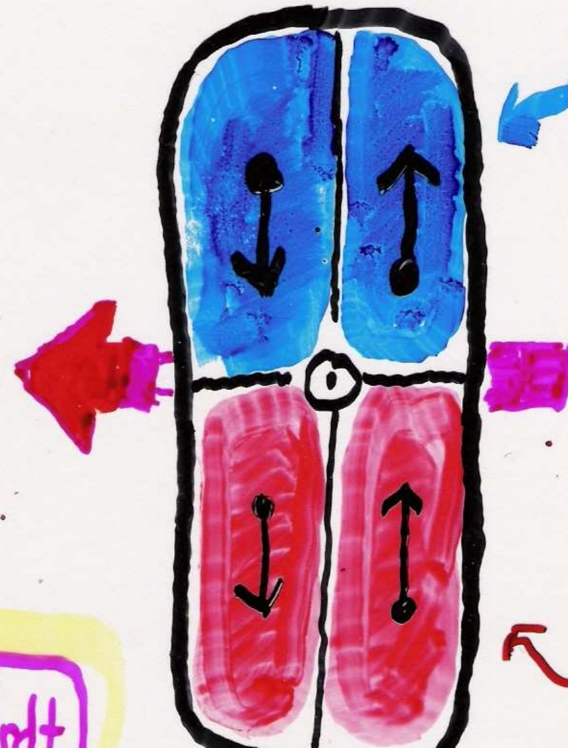
rotating

KINEMATICS in large P frame

$\Delta^N G(x, p_T)$

or

$\underline{f_{1T}^{\perp}(x, p_T)}$



blue shift

(large x)

Hadron momentum

red shift

(small x)

Burkhardt

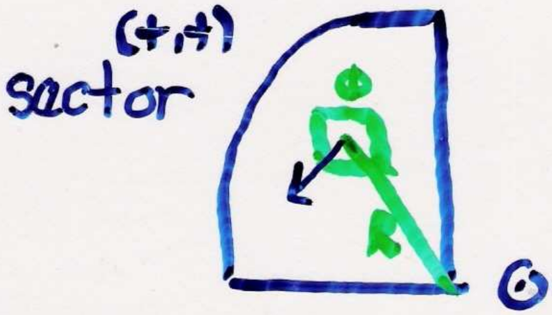
CN YANG (1978) → SINGLE-SPIN ASYMMETRY from rotating constituents (elastic - Chou Yang Model)

approx. same time as Kane et. al

III. Quantum Field Theory "Construction" of $\Delta^N G_{q/\vec{p}}(x, \vec{k}_T; Q^2)$

For a proton polarized in the $+\hat{y}$ direction project onto the (x, z) plane E_2 by integrating over dk_y or dy (respectively)

$[\vec{R}]$ Base space $\in E_2$
$[\vec{P}]$ tangent space $\in E_2$



In each of the 4 quadrants which represent the stable-orbit composition of a internal "storage ring" $\langle L_y \rangle = +$

define ψ^i_P, ψ^i_R quark fields of flavor i

A_P, A_R curvature potential, Gluon field

on $\{\phi\}$ patches

these fields are the "averages" of the $E_4 \otimes SU_3$ fields

$$\int dt\, dy\, \psi^i_R \quad \in \quad E_2 \otimes SU_3 \quad \text{etc.}$$

$$[\vec{R}\,;\, \psi^i_R, A_R] \in E_2 \otimes SU_3 \text{ vector bundle}$$
$$[\vec{P}\,;\, \psi^i_P, A_P] \in E_2 \otimes SU_3 \text{ tangent bundle}$$

this space supports all differential forms local in E_2 !!

$|\psi^i_p|^2$ = local number density in $\{\phi\}$ for quark flavor i

helicity av.-d.

Use A_γ : $\Pi_\pm$ to decompose

$$|\psi^i_p|^2 = (1-|\varepsilon_A|^2)|\psi^i_+|^2 + |\varepsilon_A|^2|\psi^i_-|^2$$

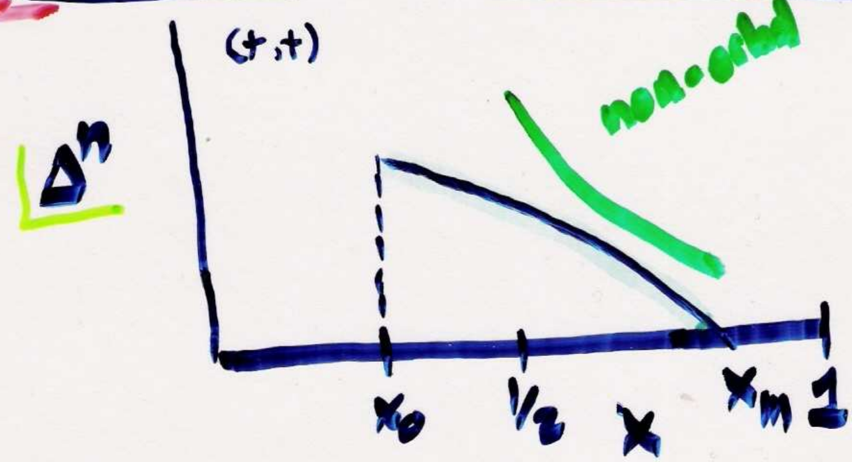
each $\{\phi\}$

[valuable simplification !]

the orbit defined in II contributes to

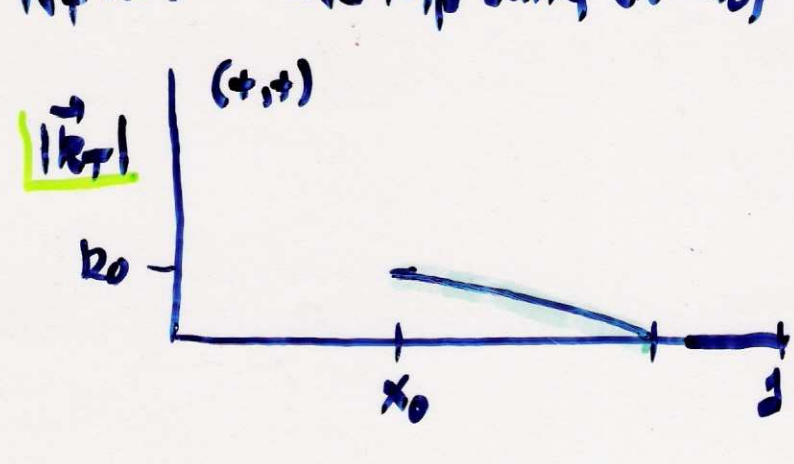
$$\Delta^N G^{(+,+)}_{q/p\uparrow}(x, \vec{k}_T; Q^2)$$

$$\boxed{x_0 = \mu/M_p < 1/2 \qquad \mu = \text{Proton Rest frame energy of orbit} = (m_q^2 + \langle k_y \rangle^2 + p_0^2)^{1/2}}$$



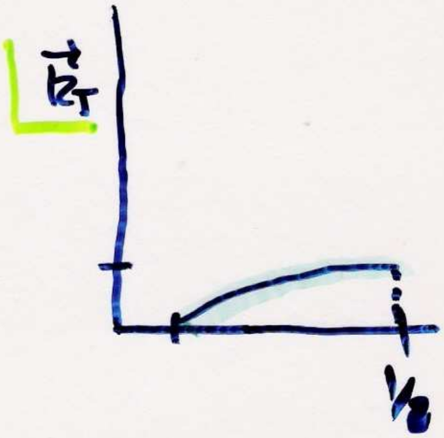
$$x = x_0 + \frac{k_0}{M}\cos\phi \qquad x_{max} = x_0 + \frac{k_0}{m}$$

$$\vec{k}_T(x) = k_0 M_p \tan\phi\,(x - x_0)$$



a non-orbit

the contribution to $(+,-)$ sector



$$x_{MIN} = x_0 - k_0/M \qquad \vec{k}_T(x) = M_p\,(x+x_0)\tan\phi$$

these two sectors can be patched together for each orbit at x_0

$$\Delta^{N\,\text{front}} = \{(+,+) \oplus (+,-)\}\ \text{each } \mu \equiv \Delta^{N(+,0)}$$

average over μ^2

Normalization

$$\int_0^1 dx\, d^2k_T\, \Delta^N G^{\text{Front}}_{q^i/p\uparrow}(x, \vec{k}_T\, ; Q^2) = \frac{1}{2}|\epsilon_A|^2 \int_0^{O(Q^2)} d\mu^2 \left(n_+(\mu^2) - n_-(\mu^2)\right)$$

Repeat Construction

$$\Delta^N G^{\text{"Back"}}_{q/p\uparrow}(x, \vec{k}_T ; Q^2) = \Delta^N G^{\text{"FRONT"}}_{q/p\uparrow}(x, -\vec{k}_T ; Q^2)$$

$$= -\Delta^N G^{\text{"FRONT"}}_{q/p\uparrow}(x, \vec{k}_T ; Q^2)$$

number valued

Evolution in Q^2

$\int d\mu^2\, n_+(\mu^2) = \langle n_+ \rangle$ does not change from perturbative effects

this means - to leading twist

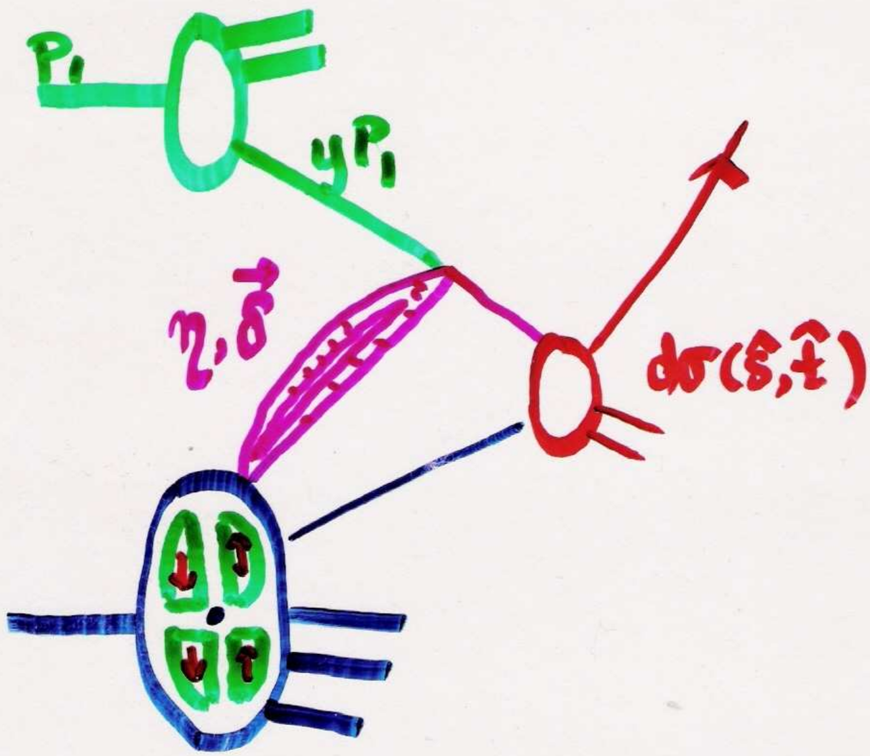
$$\frac{\partial}{\partial \ln Q^2} \Delta^N G^F(x, \vec{k}_T(x) ; Q^2) = \frac{\alpha_s}{2\pi} \int dz\, P_{q/q}(z) \Delta^N G\left(\frac{x}{z} ; \vec{k}_T\left(\frac{x}{z}\right) ; Q^2\right)$$

nonsinglet DGLAP of "k_T averaged" dstn.

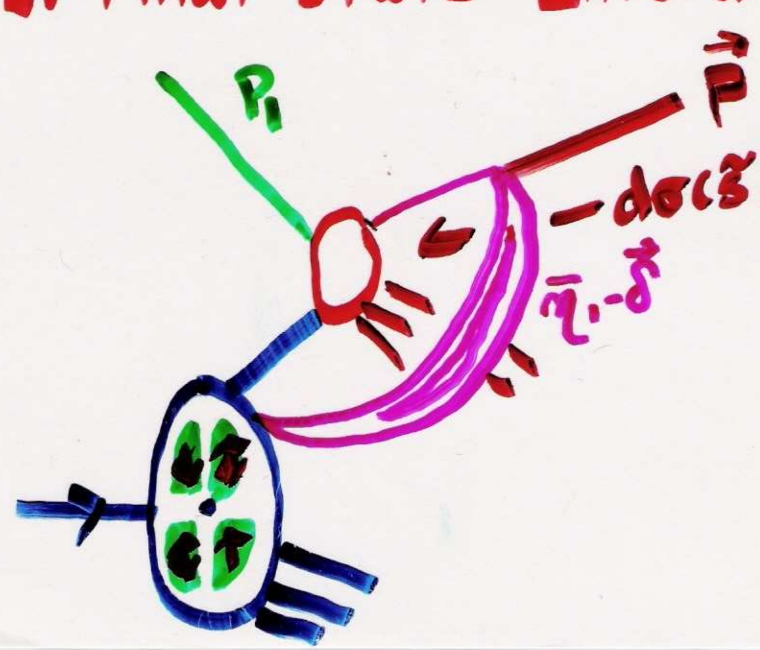
Factorization

Inclusion of $\Delta^N G_{q/p\uparrow}(x, \vec{k}_T; Q^2)$ into Inclusive Jet asymmetries

1. Initial State Interactions (DY)



2. Final state Interactions (SIDIS)

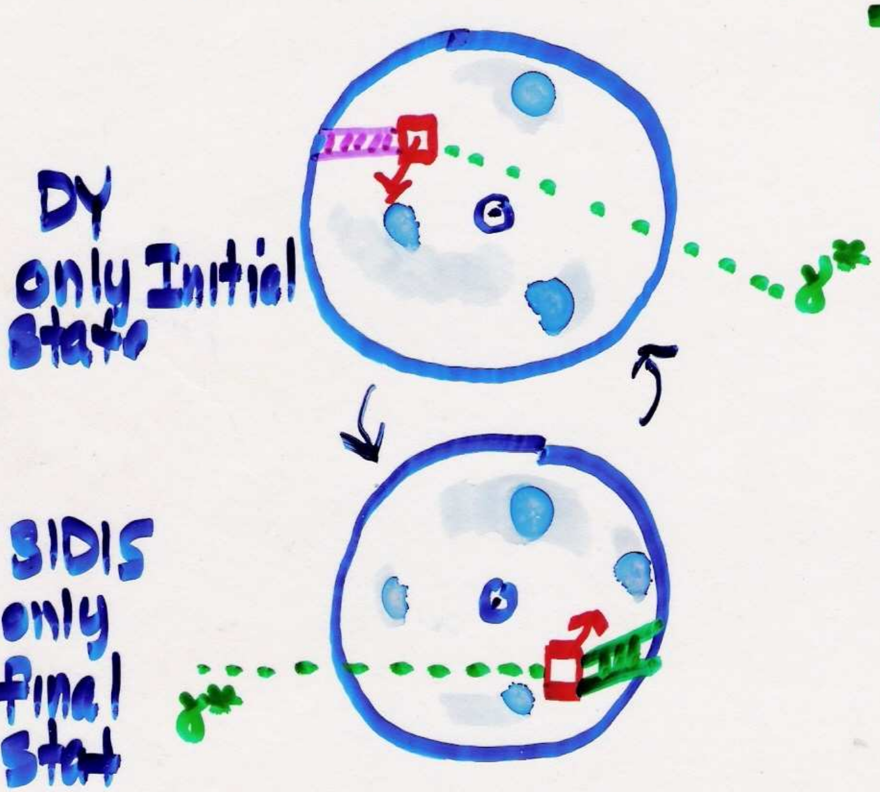


Orbital Symmetry

Compare

$\bar{p}\, p\uparrow \to \gamma^* X$ (Drell-Yan)

$\gamma^* p\uparrow \to$ jet X (SIDIS)



These two situation related by C (or T) + rotation

180° rotation produces − sign

without any restriction on form of soft interactions

$$\Delta^N G^{DY}_{q/p\uparrow} = - \Delta^N G^{SIDIS}_{q/p\uparrow}$$

1-1 map preserves kinematics

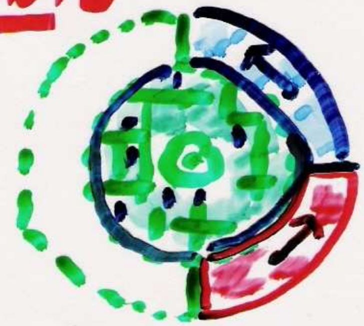
[Collins Symmetry]

(stronger than factorization)

he derived symmetry using gauge-link insertions!

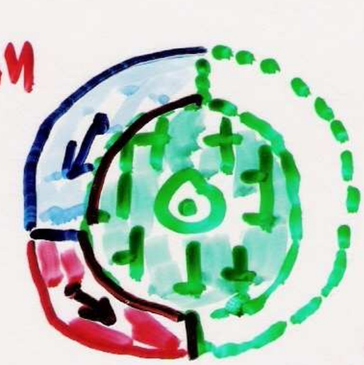
these can be combined in an eikonal approximation

SIDIS



$$\Delta^N G_{q/p\uparrow}(x, k_T; \mu^2)\Big|_{\gamma^* p \to jet} = (1-\eta)\,\Delta^N G^{Back}(x, k_T; \mu^2)$$

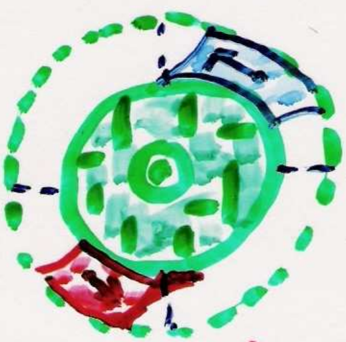
Drell-Yan



$$\Delta^N G_{q/p\uparrow}(x, k_T; \mu^2)\Big|_{pp \to \gamma^*} = (1-\eta)\,\Delta^N G^{Front}(x, k_T; \mu^2)$$

$$= -\Delta^N G(x, k_T)^{Drell-Yan}$$

PP → jet



→ SIDIS large x

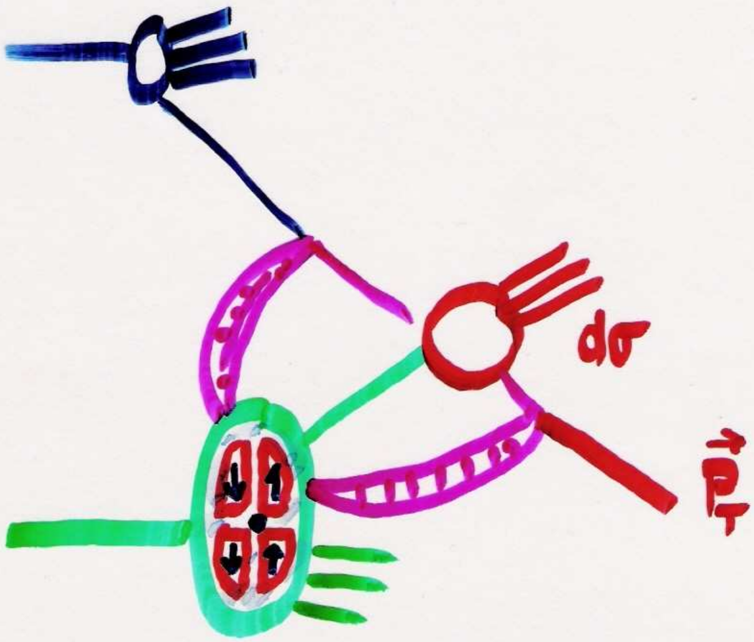
→ DY small x

in the eikonal approx

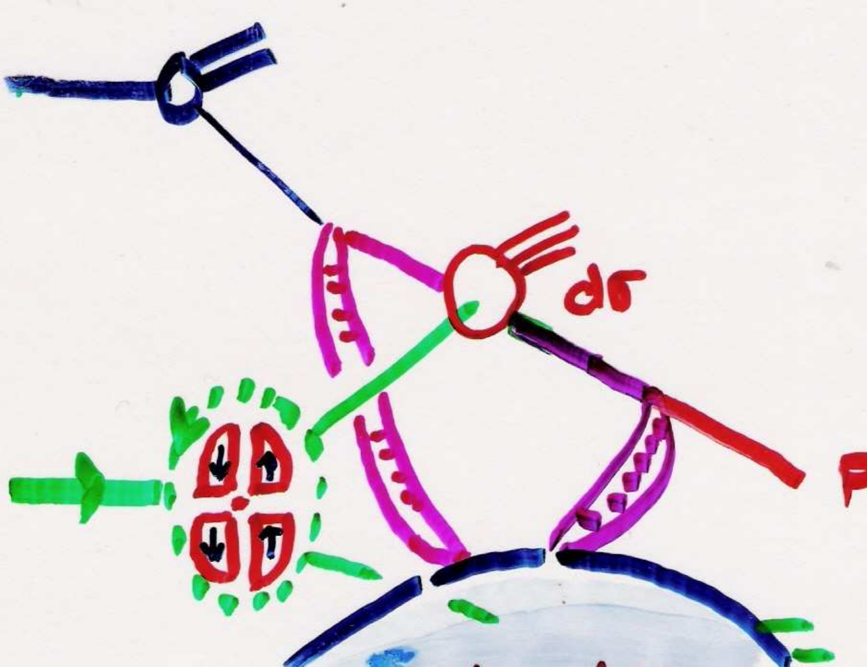
$$\Delta^N G_{q/p\uparrow}(x, k_T; \mu^2)\Big|_{pp \to jet} = (1-\eta)\,\Delta^N G^{FRONT}(x, k_T; \mu^2)$$

$$+ R^{-1}(1-\eta')\,R\,\Delta^N G^{Back}(x, k_T; \mu^2)$$

$A_N d\sigma(p\,p\uparrow \to \text{Jet}\ X)$



mixture of ISI
& final state I
varies with x

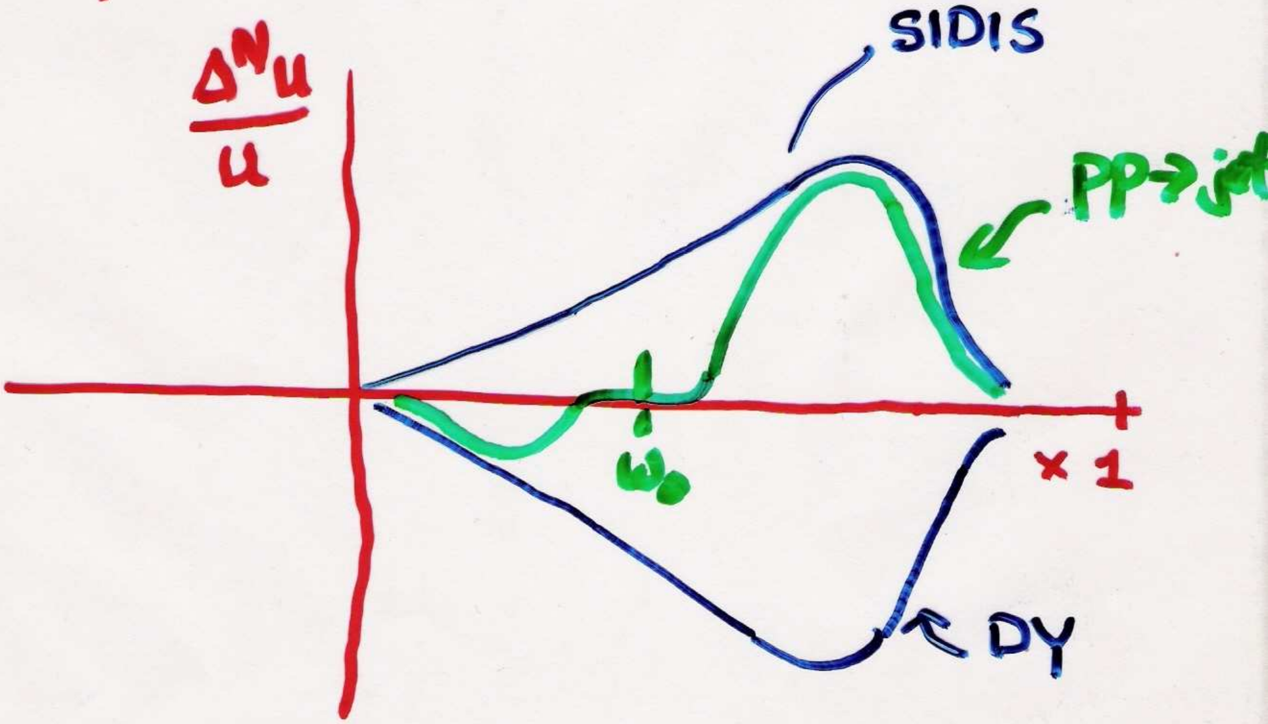


Plenty of
reasons to
consider possibility
of interesting
"coherent" effects

Regge
treatment

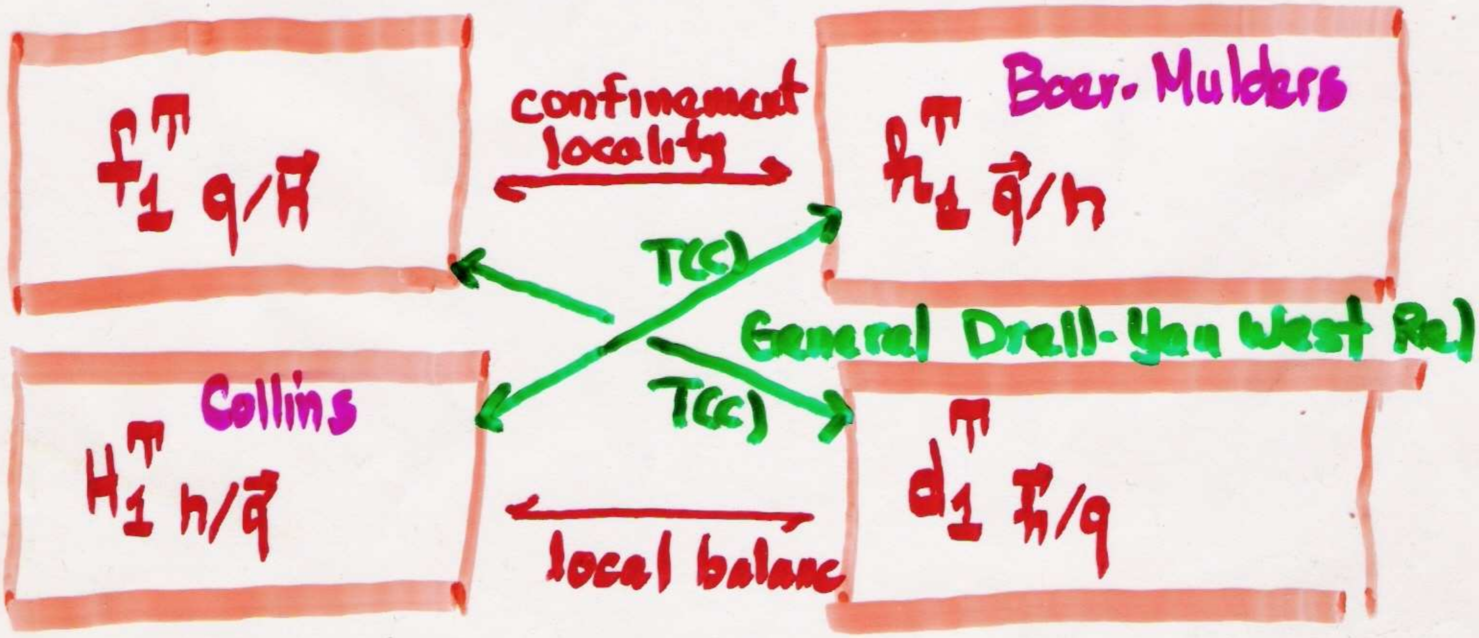
This process a real test of
"Spectator" models

For up quarks



jet production from gluons / photons
involves different flavor mix

The "Mulders/Tangerman" Barbarshop String Quartet & Yang-Pijlman Gauge Structures



There are 4-"leading-twist" manifestation of an SU_3 Yang-Pijlman extensión of a Bohm-Aharanov structure in "Missing momentum" or Pol.

They are closely related in Quantum Field theory. Collins-Fcn → Boer Mulder

$$f_{1\,q/\vec{h}}^T \rightarrow d_{1\,\vec{h}/q}^T$$

It has been amply verified that there are no "leading-twist" A_T-odd in the perturbative light-quark sector of the standard

This is the "currently-correct" interpretation of the Kane, Pumplin, Repko (1978) result

$$A_N \propto \frac{m_q}{\sqrt{s}} \alpha_s \, \mathrm{Im}\{F_{\sigma}\}$$

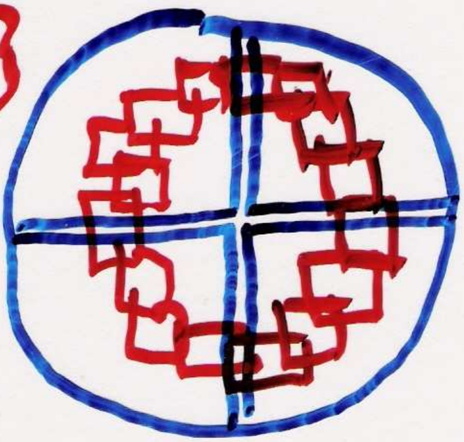
Hence, it is possible to interpret (if you choose) the Qiu-Sterman result (and variations) in terms of $f_1^T(x, \vec{k}; Q^2)$. They are complamentairy approaches

It is also possible to interpret "Spectator Models" for asymmetries as "models" for orbital distributions.

However, the parton content is somewhat hidden.

The possibility of dealing with coherence can be contemplated

$\{\psi_A^i(\bar{R},\bar{\phi})\}$



A_χ Projection works at amplitude level

$\vec{\beta} = \beta_x \hat{x} + \beta_z \hat{z}$
different from
$\vec{b} = b_x \hat{x} + b_y \hat{y}$

$$\psi_A^i = |\varepsilon_A|\langle n_+\rangle|^{1/2} \exp\{i\langle m\rangle\phi\} \tilde{f}(\bar{R},\phi)$$
$$\cong |\varepsilon_A|\langle n_+\rangle|^{1/2} \exp\{i\langle\vec{\beta}\rangle\cdot\vec{k}_T\} \tilde{f}(\beta)$$

There is the ability to compare amplitudes including a Regge treatment of both ISI & FSI (Regge calculus/ cutting rules) involving hard scattering at different locations in orbit