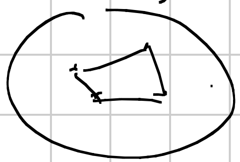
Se F figura, $F \subseteq \Pi$ piano, allora se F è limitata

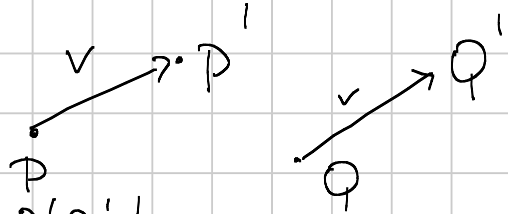
allora $\mathcal{G}(F) = \{a, \ a \text{ isometrie}, \ a(F) = F\}$



$\not\ni \tau, \gamma$.

Se F è una figura $|\mathcal{G}(F)| < \infty$

$\Longrightarrow \tau, \gamma \notin \mathcal{G}(F)$. Se $\tau = \tau_v$



$v \neq \underline{0}$ $\tau_v \neq id$ $|PQ| = |P'Q'|$

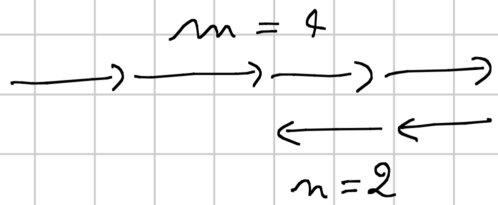
$\tau_v^n = \underbrace{\tau_v \circ \quad \circ \tau_v}_{n \text{ volte}}$ $\tau_v^2 = \tau_v \circ \tau_v$, $\tau_v^3 = \tau_v \circ \tau_v \circ \tau_v$

$\tau_v^n = \tau_{nv}$ v vettore $n \in \mathbb{N}$ $2v$

$|nv| = |n||v|$ Se $nv = \underline{0}$ $|nv| = 0 = |n||v|$

Solo se $n = 0$ $\tau_v^0 = id$

$\tau^n \in \mathcal{G}$ <u>Apparentemente</u> ho infinite traslazioni in $\mathcal{G}$. Siano $m > n$ $\tau^m = \tau^n$ $\tau^m \tau^{-n} = \tau^n \tau^{-n} = id$

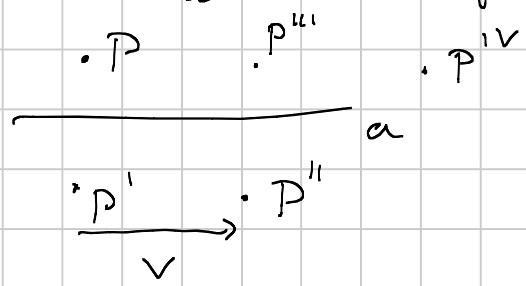


$\tau^{m-n} = id \Rightarrow m = n$

Siccome $\mathcal{G}$ è finito non esiste traslazione in $\mathcal{G}$

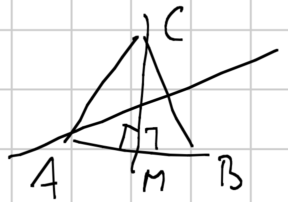
Analogamente non esiste glissoriflessione in $\mathcal{G}$

$\gamma = \tau_v \circ \sigma_a$ $\gamma^2 = \tau_{2v} \notin \mathcal{G}$



$\mathcal{G}(F')$ è composto solo da rotazioni e/o riflessioni

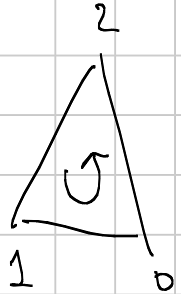
Esempio:

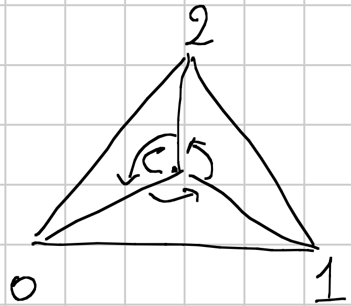


equilatero,

asse di AB = la retta perpendicolare ad AB passante per il suo punto medio

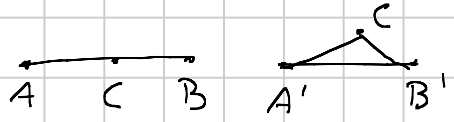
= insieme dei punti C : $|AC| = |BC|$





$$\{\underline{id}, \underset{V}{\sigma_0}, \underset{V}{\sigma_1}, \underset{V}{\sigma_2}, \underline{\rho}, \underline{\rho^2}\}$$

$$\rho^2_{C,120°} = \rho_{C,240°}$$



$$\rho\sigma_0 : \begin{aligned} 0 &\longmapsto 1 \\ 1 &\longmapsto 0 \\ 2 &\longmapsto 2 \end{aligned}$$

$\rho\sigma_0(V) = \sigma_2(V)$ $\quad V$ vertice

$\Longrightarrow \rho\sigma_0 = \sigma_2$

Osservazione: Se α isometria del triangolo

allora $\{\alpha(0), \alpha(1), \alpha(2)\} = \{0, 1, 2\}$

i) α(segmento) = segmento

ii) vertice = segmento1 $\cap$ segmento2

$\alpha(\text{vertice}) = \alpha(\quad " \quad) \cap \alpha(\quad " \quad)$

Inoltre $\alpha(0) = 0 \quad \alpha(1) = 2 \quad \alpha(2) = 1$

$$\rho : \begin{array}{l} 0 \longmapsto 1 \\ 1 \longmapsto 2 \\ 2 \longmapsto 0 \end{array}$$

se penso ai nomi dei vertici come classi di resto modulo 3 posso descrivere $\rho : i \longmapsto i+1$

$i \bmod 3 \longmapsto (i+1) \bmod 3$

$$\sigma_0 : \begin{array}{l} 0 \longmapsto 0 \equiv -0 \\ 1 \longmapsto 2 \equiv -1 \\ 2 \longmapsto 1 \equiv -2 \end{array} \bmod 3$$

$\sigma : i \longmapsto -i$

$$\rho^2 : \begin{array}{l} 0 \longmapsto 2 \\ 1 \longmapsto 0 \\ 2 \longmapsto 1 \end{array}$$

$\rho\sigma : i \xrightarrow{\sigma} -i \xrightarrow{\rho} -i+1$

è una riflessione infatti non preserva l'orientazione e fissa C centro del triangolo

Basta trovare il vertice fissato $i \equiv_3 -i+1$

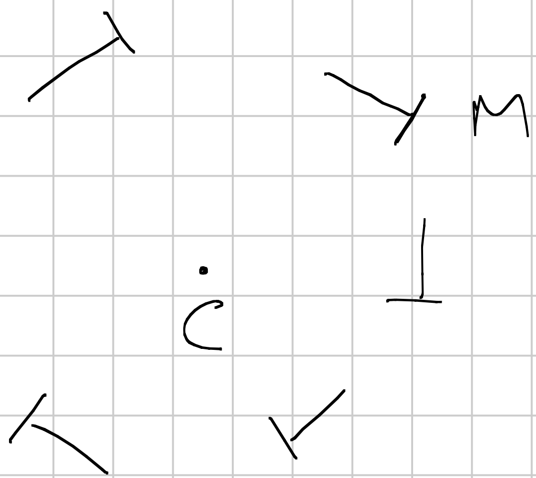
$2i \equiv_3 1 \qquad 2 \cdot 2 \equiv_3 1 \qquad 2 \cdot 2i \equiv_3 2 \qquad i \equiv_3 2$

$2 \equiv_3 -2+1 \qquad \boxed{\rho^2 \sigma = \sigma_1}$

Come faccio a sapere che non ci sono altre isometrie? In quanti modi posso scambiare $\{0,1,2\}$? Quanti sono le funzioni biunivoche da $\{0,1,2\}$ in sé

$3 \cdot 2 \cdot 1 = 6$

$$F = \bigcup_{\rho \in \mathcal{G}} \rho(M)$$



$$\mathcal{G} = \{\rho_{C,k60^\circ} : k \in \mathbb{Z}\}$$

$$\rho'(F') = \rho'\Big(\bigcup_{\mathcal{G}} \rho(M)\Big) = \bigcup \rho'\rho(M) = F'$$

$$\mathcal{G}(F') = \{\underbrace{\rho_1}_{=id}, \dots, \rho_n, \sigma, \sigma\rho_2, \dots, \sigma\rho_n\}$$