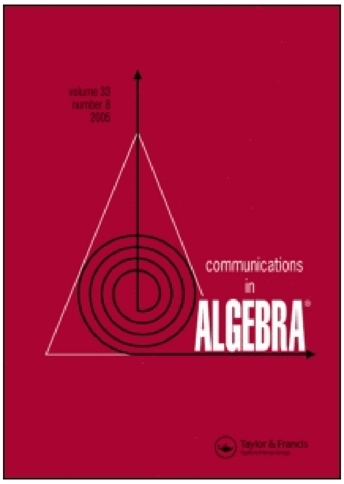

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Maps behaving like exponentials and maximal unipotent subgroups of groups of Lie type

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Abstract

Let U be a maximal unipotent subgroup of a finite classical group in good characteristic. We prove the existence of a bijection between U and the associated Lie algebra preserving centralizers. As a consequence, we obtain information on the sizes of the conjugacy classes of U. Similar results are proved in the exceptional cases.

1 Introduction

Let $G = {}^m G(K)$ be a group of Lie type defined over a finite field K of characteristic p and order q^m. Let U be a Sylow p-subgroup of G. Aim of this paper is to prove that, under some mild restriction on p, the conjugacy classes sizes of U are q-powers. For conciseness we will call such groups **q-power-size** groups. More precisely, we show that if G is classical or of type G_2, and p is a good prime (see [Ca2], pag. 28), then U is a q-power-size group. Our approach is Lie theoretic in flavor since we try to determine maps behaving like exponentials from Lie(U) onto U. Similar investigations were led by Kazhdan, the author, and Taoufik to analyze, among other features, the degrees of the irreducible complex characters of U (see [Ka], [Pr], and [Ta]).

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We establish first that power series induce maps **preserving centralizers**, i.e. $f(C_{\mathfrak{u}}(x)) = C_{f(\mathfrak{u})}(f(x))$, where $\mathfrak{u}$ =Lie(U), $C_{\mathfrak{u}}(x)$, and $C_U(X)$ are the Lie and group centralizers. We subsequently determine sufficient conditions assuring that $f(\text{Lie}(U)) = U$ and, using classical Galois theory, build all applications meeting such conditions. The existence of such maps allows one to translate group theoretic problems into a Lie theoretical language, showing a possible way to tackle questions like the determination of the exact number of irreducible characters of given degree . For the other exceptional groups, we devise an algorithm that builds the matrices associated to the elements of a Chevalley basis via an irreducible representation for the corresponding simple algebra. Working for convenience with representations of minimal degree, we show that the only power series turning Lie(U) into a group is the truncated exponential. It follows that if p exceeds the nilpotence index of the associative hull of a minimal representation for Lie(U), then U is a q-power-size group. Such results rely on extensive calculations with MAGMA, GAP ([S$^+$95]), and Maple.

2 Classical Groups

Given a power series $f(x) = \sum_{i\geq 0} f_i x^i$ defined over a field F and a nilpotent element y of an F-algebra A, the evaluation of f at y is a well defined element of A. Moreover, the maps $\rho_y : a \longrightarrow ay$ and $\lambda_y : a \longrightarrow ya$ are F-linear nilpotent operators on A commuting with each other. For every natural integer i and nilpotent elements y, x in A, let $\omega_i(x,y)$ equal $\sum_{k=0}^{i-1} \lambda_x^k \rho_y^{i-1-k}$, so that $\omega_i(x,y)(\lambda_x - \rho_y) = \lambda_x^i - \rho_y^i$.

We finally recall that if μ, ν are commuting operators, μ invertible, ν nilpotent, then $\mu + \nu$ is invertible.

Lemma 1 *Let f, g be power series defined over the same field F with $f_1 g_1 \neq 0$. Let x, y be nilpotent elements of some F-algebra A. Then x, y commute if and only if $f(x)$ and $g(y)$ do.*

Proof. The sufficiency is obvious. Conversely, set $t = f(x)$ and $d = yt - ty$, then

$$0 = [g(y), t] = \sum g_i(\lambda_y^i - \rho_y^i).t = \sum g_i\omega_i(y,y).d.$$

As sum of commuting nilpotent operators, $\omega_i(y,y)$ is nilpotent, for $i \geq 2$. We have $0 = (g_1\text{id}+\nu).d$, where ν is nilpotent, hence $d = 0$. Analogously, $z = xy - yx$ must vanish . □

This result shows that power series preserve centralizers. In order to proceed we need to describe U in the classical case. As is well known (see [Ca], Ch. I), $G = {}^mG(K)$ can be realized as a subgroup of $\mathrm{GL}_d(K)$ leaving invariant a suitable non-degenerate sesquilinear form, for some integer d. (Orthogonal groups in even characteristic are definitely not q-power-size groups, as we will show.) Let $Frob$ denote the automorphism of K mapping x into x^q and set $\bar{x} = Frob(x)$. In matrix terms $G = \{T \in \mathrm{GL}_d(K) \mid T^\alpha ET = E\}$, where $(a_{ij})^\alpha = (\overline{a_{ji}})$ and E is a matrix of the form $\begin{pmatrix} 0 & \pm I \\ I & 0 \end{pmatrix}$ or $\begin{pmatrix} 0 & 0 & I \\ 0 & C & 0 \\ I & 0 & 0 \end{pmatrix}$, $C = 1$ or $\begin{pmatrix} 1 & 0 \\ 0 & -\nu \end{pmatrix}$, ν a non-square in K (see [Ca], Ch. I). Fortunately enough there is a way to single out a Sylow p-subgroup of G from the set of its unipotent elements. Let $\mathfrak{U}$ be the variety in $(K)_d$, the full matrix algebra, defined as $\left\{\begin{pmatrix} a & 0 \\ b & c \end{pmatrix}\right\}$ or $\left\{\begin{pmatrix} a & 0 & 0 \\ b & I & 0 \\ d & e & c \end{pmatrix}\right\}$ according to the block decomposition of E, where a is a lower unitriangular matrix and c is unipotent.

Lemma 2 *$U = \mathfrak{U} \cap G$ is a Sylow p-subgroup of G.*

Proof. It is enough to prove that U is actually a p-group and compare its order with the p-th part of $|G|$. □

This realization of U is also very useful for the determination of $\mathrm{Lie}(U)$.

Corollary 3 $\mathrm{Lie}(U) = \mathrm{Lie}(\mathfrak{U}) \cap \mathrm{Lie}(G)$.

Applying the definition of tangent space, it follows immediately that $\mathrm{Lie}(\mathfrak{U})$ coincides with $\left\{\begin{pmatrix} a & 0 \\ b & c \end{pmatrix}\right\}$ or $\left\{\begin{pmatrix} a & 0 & 0 \\ b & 0 & 0 \\ d & e & c \end{pmatrix}\right\}$, where a is strictly lower triangular and c is nilpotent. In order to guess which properties assure that $f(\mathrm{Lie}(U)) = U$, f a power series, we consider the exponential map evaluated on a nilpotent algebra A over a field whose characteristic is 0 or exceeds the nilpotence index of A. We remark that exp maps nilpotent elements into unipotent ones, $\exp(-x)\exp(x) = 1$, and finally exp is invertible with respect to composition. Define σ on the Laurent series over K as $(\sum_{i\geq -m} f_i x^i)^\sigma = \sum_{i\geq -m}(-1)^i \overline{f_i} x^i$. The following theorem shows that these are the only ingredients needed.

Theorem 4 *Let $f \in K[[x]]$ be such that $f_0 = 1$, $f_1 \neq 0$, and $f^{1+\sigma} = 1$. Then $f(\mathrm{Lie}(U)) = U$.*

Proof. Since any X in Lie(U) is nilpotent, $f(X)$ is defined and is unipotent. if $-X^{\alpha}E = EX$, then $f(-X^{\alpha})E = Ef(X)$ and $E = f(X)^{\alpha}Ef(X)$. Thus $X \in \mathrm{Lie}(\mathcal{U}) \cap \mathrm{Lie}(G)$ forces $f(X) \in \mathcal{U} \cap G = U$. Finally $f_1 \neq 0$ is equivalent to the existence of a power series g such that $g(f(x)-1) = x$. In particular, f induces a monomorphism and, since $|\mathrm{Lie}(U)| = |U|$, f is also surjective . □

One could wonder whether any such f exists. Using Galois theory we completely characterize such power series. Let E be the field of Laurent series over K. Since σ is obtained composing $Frob$ with subsitution of x with $-x$, then $\sigma \in \mathrm{Aut}(E)$ and $\sigma^2 = 1$. Thus $E_0 = C_E(\sigma)$ is a subfield of E and $|E : E_0| = 2$. In particular, E/E_0 is a cyclic Galois extension. Given $f \in E$, Hilbert's Satz 90 (see [Mo]) shows that $N_{E/E_0}(f) = f^{1+\sigma} = 1$ if and only if $f = g^{1-\sigma}$, for some $g \in E$. Since $x^{2m} \in E_0$, for any m, multiplying g with x^{2m}, for m big enough, assures that a power series f has norm 1 if and only if there exists a power series g such that $f = g^{1-\sigma}$. If $g \in K[[x]]$ and $g_0 = 1$, then $f = g^{1-\sigma} \in K[[x]]$ and $f_0 = 1$. So we have to determine g such that $f_1 \neq 0$ for the corresponding f. Now $g^{1-\sigma} \equiv \frac{1+g_1x}{1-\overline{g_1}x} \equiv 1 + (g_1 + \overline{g_1})x$ (mod $x^2K[[x]]$), hence $f_1 = 0$ for any $g_1 \in K$ only if $\mathrm{char}(K) = 2$ and G is not unitary.

Theorem 5 *Let $G = {}^{m}G(K)$ be a classical group, K a finite field of characteristic p, p a good prime, and $U \in \mathrm{Syl}_p(G)$. There exists a map f from* Lie(U) *onto U preserving centralizers. In particular, U is a q-power-size group, where $|K| = q^m$.*

Proof. If G is of type A, then set $f(x) = 1 + x$, otherwise choose f as in Theorem 4. Let F be $C_K(Frob)$, then $C_{\mathrm{Lie}(U)}(x)$ is an F-algebra for any element x of Lie(U). Hence $|C_U(u)|$ and $|u^U|$ are q-powers, for any $u \in U$. □

3 Exceptional Groups

Let $\mathfrak{g}$ be a complex simple Lie algebra, choose a Cartan subalgebra $\mathfrak{h}$ and denote with Φ, Φ^+, Π the irreducible system of roots, the positive roots and the fundamental roots, respectively. A theorem of Chevalley assures the existence of a basis x_α, y_α, $\alpha \in \Phi^+$, h_π, $\pi \in \Pi$, with integral structure constants (see [Ca], 4.4.2, [Hu], VII). Let $\mathfrak{g}_{\mathbb{Z}}$ be the Lie ring generated by such a basis. Given a complex irreducible $\mathfrak{g}$-module V, a result of Kostant asserts the existence of a $\mathbb{Z}$-lattice M in V left invariant by the action of the Chevalley basis; in other words, we may assume that the elements of such a basis are represented by integral matrices. Thus tensoring with a field K,

$M_K = M \otimes_{\mathbb{Z}} K$ becomes a module for $\mathfrak{g}_K = \mathfrak{g}_{\mathbb{Z}} \otimes_{\mathbb{Z}} K$. Let Ω be the set of fundamental weights of $\mathfrak{g}$ and Ω' the subset of weights occurring in V. Let $\Lambda_{\Omega'}$ be the $\mathbb{Z}$-lattice generated by Ω'. Denote by ϕ the representation afforded by M_K. We then define the Chevalley group of type $\Lambda_{\Omega'}$ as the group generated by $\exp(\phi(tx_\alpha))$, where α varies in Φ^+, and t in K. It is a classical result that these groups are central factors of groups of type Λ_Ω and that the order of the normal central subgroup is given by $|\Lambda_\Omega : \Lambda'_\Omega|$. In the exceptional cases this index is a divisor of 6 (see [Hu], pag. 68), so, if $\mathrm{char}(K) \geq 5$, the maximal unipotent subgroups are isomorphic to the one obtained via the adjoint representation. Let $\mathfrak{u}$ be the Lie algebra generated by $\phi(ty_\alpha)$ and let U be the group generated by $\exp(\phi(ty_\alpha))$, $\alpha \in \Phi^+$, $t \in K$. Since the nilpotence index of the associative hull of $\phi(\mathfrak{u})$ is bounded above by the degree of ϕ and we are trying to define a global map f from $\mathfrak{u}$ onto U through a power series, the probability of success for small characteristics increases if we consider representations of minimal degree. We build explicitly such representations for $\mathfrak{u}$ and show that unless $\mathfrak{u}$ is of type G_2 and $\mathrm{char}(K) = 5$, the only such map is the exponential one.

Theorem 6 *Let ${}^mG(K)$ be a Lie group of exceptional type defined over a finite field K of order q^m. Let U be a maximal unipotent subgroup of G. There exists $c = c(G)$ such that whenever* $\mathrm{char}(K) \geq c$, *then U is a q-power size group.*

Proof. As in the classical case we simply need to establish the existence of a global map from $\mathrm{Lie}(U)$ onto U preserving centralizers. Call α and β the fundamental short and long root for the Lie algebra of type G_2, then the irreducible representation associated to the fundamental weight $2\alpha + \beta$ has dimension 7 and the images of y_γ, $\gamma \in \Phi^+$ are $Y_\alpha = e_{12} + e_{34} + 2e_{45} - e_{67}$, $Y_\beta = -e_{23} + e_{56}$, $Y_{\alpha+\beta} = -e_{13} + e_{24} + 2e_{46} + e_{57}$, $Y_{2\alpha+\beta} = e_{14} - e_{25} + e_{36} + 2e_{47}$, $Y_{3\alpha+\beta} = -e_{15} + e_{37}$, and $Y_{3\alpha+2\beta} = e_{16} - e_{27}$, e_{ij} the elementary matrix (see [FH], 22.3). Given a list $w = (w_1, \ldots, w_n)$ of fundamental roots, the corresponding element $Y_{w_1} \cdots Y_{w_n} \neq 0$ in $(K)_7$ if and only if w is a sublist of $(\alpha, \beta, \alpha, \alpha, \beta, \alpha)$. Thus the Y_γ's generate an associative subalgebra A of the nilpotent algebra N of upper triangular matrices in $(K)_7$ of dimension 14 and nilpotence index 7. Let $f(x)$ be $1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \frac{x^5}{144}$, then f induces a bijection from N onto $I + N$, a maximal unipotent subgroup of $\mathrm{GL}(K, 7)$, whose inverse is given by $g(w - I)$, $g(z) = z - \frac{z^2}{2} + \frac{z^3}{3} - \frac{z^4}{4} + \frac{29z^5}{144} - \frac{49z^6}{288}$. Observe that such maps are well defined if we assume $\mathrm{char}(K) \geq 5$. Set $u = \sum_{\gamma \in \Phi^+} u_\gamma Y_\gamma$ and $v = \sum_{\gamma \in \Phi^+} v_\gamma Y_\gamma$, v_γ, $u_\gamma \in K$. We need to check whether $u * v = g(f(u)f(v) - I)$ is an element of $\mathfrak{u}$ for any choice of $u, v \in \mathfrak{u}$. Since f and g are truncation of the exponential and logarithmic maps to the terms of degree 4, we know that $u * v$ coincides with $H(u, v)$ modulo

A^5, where $H(u,v)$ denotes the Baker-Campbell-Hausdorff series. It turns out that the residual term is a scalar multiple of $Y_{3\alpha+2\beta}$. This proves that $d(\mathrm{G}_2) \leq 5$. As we mentioned in the introduction f was determined by brute force using MAGMA, GAP, and Maple. Similarly we have tested representations ϕ of minimal degree for the exceptional Lie algebras of type F_4, ${}^3\mathrm{D}_4$, ${}^2\mathrm{E}_6$, and E_i, $i = 6, 7, 8$ and it turns out that the only global map turning Lie(U) into a group is the exponential one. Given the maximal weight associated to such representation, we determine the directed graph Γ whose vertices are the weights involved in the representation and these are adjacent if and only if their difference is a fundamental root. It is then obvious that if we call d the diameter of such a graph, and A the associative hull of $\phi(u)$, then $A^d = 0$. Unfortunately d also coincides with the minimum integer such that $a^d = 0$ for any element a of A. Let r be the minimal degree of the representation. We now list the values so obtained

- ${}^3\mathrm{D}_4$: $r = 8$, $d = 7$, hence $c \leq 7$;
- F_4: $r = 26$, $d = 16$, hence $c \leq 17$;
- E_6: $r = 27$, $d = 16$, hence $c \leq 17$;
- ${}^2\mathrm{E}_6$: $r = 27$, $d = 16$, hence $c \leq 17$;
- E_7: $r = 56$, $d = 27$, hence $c \leq 29$;
- E_8: $r = 248$, $d = 58$, hence $c \leq 59$;

and the assertion is proved . □

4 Examples

We conclude by showing that in the classical case our result is best possible, that is, the Sylow 2-subgroup of a symplectic or orthogonal group in even characteristic is not always a q-power-size group. We work out counterexamples of minimal rank but we are confident that the next theorem holds for any rank. Our source of inspiration is [Is].

Theorem 7 *Let G be one among* $\mathrm{Sp}(4,q)$ *or* $\mathrm{SO}_8^{\pm}(q)$, *where q is even and greater than 2. Then a Sylow 2-subgroup U of G is not a q-power-size group.*

Proof. In the symplectic case it is not difficult to prove that U may be realized as $\left\{ \begin{pmatrix} t & 0 \\ at & t^* \end{pmatrix} \right\}$, t lower unitriangular, $t^* = (t')^{-1}$, and a symmetric.

Set $u(t,a)$ for a generic element of this group, then $u(s,b)$ centralizes $u(t,a)$ if and only if s centralizes t and $a+t^*at^{-1}=b+s^*bs^{-1}$. Substituting b with $t'bt$ and s with ts^{-1}, these conditions become $[s,t]=1$ and $s'as+a=t'bt+b$. Now $\psi_t : t \to t'bt+b$ defines a linear map on the space of symmetric matrices. Given $s \in C(t)$, let B_s be the set of symmetric matrices b such that $\psi_t(b)=\psi_s(a)$. Then B_s is either empty or is a coset of $\ker\psi_t$. So $|C(u(t,a))| = n|\ker\psi_t|$, where $n=|\{s\in C(t) : B_s \neq \emptyset\}|$. If G has rank 2, $t=1+\tau e_{21}$, $s=1+\sigma e_{21}$, then $\psi_t=\tau^2 e_{13}+\tau e_{23}$, with respect to the basis $v_1=e_{11}$, $v_2=e_{12}+e_{21}$, $v_3=e_{22}$. Hence if $t\neq 1$ and $a_{22}\neq 0$, then $\psi_s(a)\in \operatorname{Im}\psi_t$ if and only if $\sigma^2\tau=\tau^2\sigma$, that is, if and only if $\sigma=\lambda\tau$, where $\lambda\in\mathbb{F}_2$. Thus $|C(u(t,a))|=2q^2$. By definition $O_{2l}^+(q)$ is the group of transformations preserving the quadratic form $Q(x_1,\dots,x_{2l})=\sum_1^l x_i x_{l+i}$. A routine check shows that $U=\left\{\begin{pmatrix} t & 0\\ at & t^*\end{pmatrix}\right\}$, with t lower unitriangular and a strongly antisymmetric, i.e. $a+a'=0$ and $a_{ii}=0$, is a 2-subgroup of $O_{2l}^+(q)$. Now the elements e_i of the standard basis (conceived as column vectors) are singular and W, the totally singular subspace generated by $e_{l+1},\dots,e_{2l}$, is U-invariant. According to the notation of [Di], pag. 909, $Q(e_i)=\alpha_i=0$, $Q(e_{l+i})=\beta_i=0$, and $c_{ij}=0$, $i,j=1,\dots,l$, for any $u\in U$. It follows that the Dickson discriminant of any element of U is 0. Comparison of orders (see [KL], pag. 19) shows that U is a Sylow 2-subgroup of $SO_{2l}^+(q)$. Assume $l=4$ and consider $t=1+e_{21}+e_{32}+e_{43}=1+n$, then $C(t)=1+un+vn^2+zn^3$, $u,v,z\in K$. Order the pairs (i,j), $i<j$, lexicographically and let $v_1=e_{12}-e_{21},\dots,v_6=e_{34}-e_{43}$ be the corresponding basis for the space of strongly antisymmetric matrices. Consider $a=v_6$, then $\psi_s(a)=(zu+v^2, z+uv, v, u^2+v, u, 0)$ and $\psi_t(b)=(b_2+b_4, b_3+b_4+b_5, b_5, b_5+b_6, b_6, 0)$. Thus $\psi_t(b)=\psi_s(a)$ admits a solution b if and only if $u^2+u=0$ which amounts to u belonging to $\mathbb{F}_2$. Since b_1, b_3, v, z vary arbitrarily, $|C(u(t,a))|=2q^4$. Observe that if $l\leq 3$ well known isomorphisms among classical groups imply that U is a q-power-size group (see [KL], pag. 43). Without quoting this result, elementary manipulations with the ψ-map allow to show that for $l=3$ the conjugacy class vector of U equals $[q,(q-1)(q^2+q+1), q(q^2-1), (q-1)(q^2-1)]$, where the i-th component denotes the number of classes of size q^{i-1}. Instead $O_{2l}^-(q)$ preserves the quadratic form $Q(x_1,\dots,x_l,x_{2l},x_{l+1},\dots,x_{2l-1})=\sum_1^{l-1}x_i x_{l+i}+x_l^2+x_l x_{2l}+\nu x_{2l}^2$, with $x^2+x+\nu$ irreducible over $\mathbb{F}_q$. Let U be the group $\left\{\begin{pmatrix} t & 0 & 0\\ v't & 1 & 0\\ at & w & t^*\end{pmatrix}\right\}$, where t is lower unitriangular of dimension l, $w=vJ$, where $J=\begin{pmatrix}0&1\\1&0\end{pmatrix}$ and v is a $(l-1,2)$-block, $a+a'+vJv'=0$, and

$a_{ii} = (vCv')_{ii}$, where $C = \begin{pmatrix} 1 & 1 \\ 0 & \nu \end{pmatrix}$. Again a direct calculation shows that U is a 2-subgroup of $O^-_{2l}(q)$. As before elements of the shape $u(t,0,a)$ have Dickson discriminant equal to 0. Since $c_{ij} = 0$, $i,j = 1,\dots,l$, and $Q(e_i) = Q(e_{l+i}) = 0$ for $i = 1,\dots,l-1$, we have that $D(u) = \sum_i Q(e_{2l})b_{il}d_{il}$ (compare [Di], pag. 909). For elements of the shape $u(1,v,a)$, $b_{ll} = 0$ and $d_{il} = \delta_{il}$, hence they also have zero Dickson discriminant. Since $u(t,0,a)$ and $u(1,v,a)$ generate U, comparison of orders yields that U is a Sylow 2-subgroup of $SO^-_{2l}(q)$. For $l = 4$, set $t = 1 + t_1e_{21} + t_2e_{32} + t_3e_{31}$. If $t_1t_2 \neq 0$, then a direct calculation shows that $|C(u(t,v,a))|$ is a q-power (actually q^5). Hence consider $t = 1 + e_{21}$, $v' = e_{13} + e_{21}$, and $a = \nu e_{11} + e_{13} + e_{33}$, then $u(s,w,b) \in C(u(t,v,a))$ if and only if $s = 1 + s_1e_{21} + s_3e_{31}$, $s_3 \in \mathbb{F}_2$, $w' = \begin{pmatrix} x_1 & s_3 & x_3 \\ y_1 & 0 & y_3 \end{pmatrix}$, $b_{23} = y_1 + x_3 + s_3$. Since $b_{12}, b_{13}, x_1, x_3, y_1, y_3, s_1$ may be chosen arbitrarily, $|C(u(t,v,a))| = 2q^7$. □

Because of their considerable size, we did not try to build counterexamples for the exceptional groups, but there our best guess is that U is actually a q-power-size group whenever the characteristic is a good prime.

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